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On M-Projective Curvature Tensor of Lorentzian α-Sasakian Manifolds

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Abstract. In this paper, we study the nature of Lorentzian α -Sasakian manifolds admitting M -projective curvature tensor. We show that M -projectively flat and irrotational M -projective curvature tensor of Lorentzian α -Sasakian manifolds are locally isometric to unit sphere $S^n(c)$, where $c=\alpha^2$. Next we study Lorentzian α -Sasakian manifold with conservative M -projective curvature tensor. Finally, we find certain geometrical results if the Lorentzian α -Sasakian manifold satisfying the relation $\mathsf{M}(X,Y)\cdot R=0$.

1. Introduction

If a differentiable manifold has a Lorentzian metric g, i.e., a symmetric non-degenerate (0,2) tensor field of index 1, then it is called a Lorentzian manifold. The notion of Lorentzian manifold was first introduced by Matsumoto [12] in 1989. The same notion was independently studied by Mihai and Rosca [13]. Since then several geometers studied Lorentzian manifold and obtained various important properties. Our present note deals with a special kind of manifold i.e., Lorentzian α -Sasakian manifold. At first we give some introduction about the development of such manifold. An almost contact metric manifold with structure tensors (ϕ,ξ,η,g), where ϕ is a tensor field of type (1, 1), ξ a vector field, η a 1-form and g is a Riemannian metric on M is called trans-Sasakian structure [16] if $(M\times\mathsf{R},J,G)$ belongs to the class W_4 [7] of the Hermitian structure, where J is the almost complex structure on $(M\times\mathsf{R})$ defined by

$$(J,X\frac{d}{dt})=(\phi X-f,\eta(X)\frac{d}{dt}),$$

for all vector fields X on M and smooth functions f on $(M\times\mathsf{R})$, G is the product metric on $M\times\mathsf{R}$. This may be expressed by the condition

$$(\nabla_X\phi)(Y)=\alpha[g(X,Y)\xi+\eta(Y)X]+\beta[g(\phi X,Y)-\eta(Y)\phi X],$$

for smooth functions α and β on M in [2], and we say that the trans-Sasakian structure is of type (α,β). A trans-Sasakian structure of type (α,β) is α -Sasakian, if $\beta=0$ and α a nonzero constant [8], if $\alpha=1$, then α -Sasakian manifold is a Sasakian manifold. Also in 2005, Yildiz and Murathan [23] introduced and studied Lorentzian α -Sasakian manifolds. In [19], Prakasha and his coauthors investigated Weyl-pseudosymmetric and partially Ricci-pseudosymmetric Lorentzian α -Sasakian manifolds. In [24], some classes of Lorentzian α -Sasakian manifolds were studied. Also, three-dimensional Lorentzian α -Sasakian manifolds have been studied in [25]. Further Lorentzian α -Sasakian manifolds were also studied by Prakasha and Yildiz [20], Bhattacharyya and Patra [1]. Recently Dey and Bhattachryya have studied some curvature properties of Lorentzian α -Sasakian manifolds [6] and many others (see, [9, 10]).

The M-projective curvature tensor of Riemannian manifold M^n was defined by Pokhariyal and Mishra [17] is of the following form:

$$\mathrm{M}(X,Y)Z = R(X,Y)Z - \frac{1}{2(n-1)}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY], \tag{1.1}$$

where Q is the Ricci operator defined on $S(X,Y) = g(QX,Y)$. The authors extensively studied the properties of M-projective curvature tensor on the various manifolds (see, [3, 11, 14, 15, 18, 22, 26, 27]). In this paper, we have studied some special properties of Lorentzian α-Sasakian manifold.

The purpose of this paper is to study the properties of M-projective curvature tensor in Lorentzian α-Sasakian manifolds. The paper is organized as follows. Section 2 is concerned with preliminaries of Lorentzian α-Sasakian manifolds. In section 3, we study the M-projectively flat of Lorentzian α-Sasakian manifold. Section 4 deals with the M-projectively flat Lorentzian α-Sasakian manifold satisfies the condition $R(X,Y)\cdot S = 0$. In section 5, we study conservative M-projective curvature tensor of Lorentzian α-Sasakian manifold. In section 6, irrotational M-projective curvature tensor of Lorentzian α-Sasakian manifold are studied. Section 7 is devoted with study of Lorentzian α-Sasakian manifold satisfies the condition $M(X,Y)\cdot R = 0$.

2. Preliminaries

A differential manifold M^n of dimension n is said to be a *Lorentzian α-Sasakian manifold* if it admits a (1, 1)-tensor field ϕ, a vector field ξ, a 1-form η, and Lorentzian metric g which satisfy the conditions

$$\eta(\xi) = -1, \phi^2 = I + \eta \otimes \xi, \phi(\xi) = 0 \; and \; g(X,\xi) = \eta(X), \tag{2.1}$$

$$\eta(\phi X) = 0, g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{2.2}$$

$$(\nabla_X \phi)(Y) = \alpha\{g(X,Y)\xi + \eta(Y)X\}, \tag{2.3}$$

for all X, $Y \in \chi(M^n)$, where $\chi(M^n)$ is the Lie algebra of smooth vector fields on M^n, and ∇ denotes the covariant differentiation operator of Lorentzian metric g. Also, on a Lorentzian α-Sasakian manifold M^n, we have (see [19, 25])

$$\nabla_X \xi = -\alpha\phi X, \; (\nabla_X \eta)Y = -\alpha g(\phi X, Y) = (\nabla_Y \eta)(X). \tag{2.4}$$

Further, on a Lorentzian α - Sasakian manifold M^n the following results holds (see, [23, 19])

$$\eta(R(X,Y)Z) = \alpha^2\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\}, \tag{2.5}$$

$$R(\xi, X)Y = \alpha^2\{g(X,Y)\xi - \eta(Y)X\}, \tag{2.6}$$

$$R(X,Y)\xi = \alpha^2\{\eta(Y)X - \eta(X)Y\}, \tag{2.7}$$

$$S(X,\xi) = (n-1)\alpha^2\eta(X), \tag{2.8}$$

$$S(\phi X, \phi Y) = S(X,Y) + (n-1)\alpha^2\eta(X)\eta(Y). \tag{2.9}$$

Definition 2.1. *A Lorentzian α-Sasakian manifold M^n is said to be η-Einstein if its Ricci tensor S is of the form*

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y), \tag{2.10}$$

where a, b are functions on M^n. If $b = 0$, then η-Einstein manifold becomes Einstein manifold.

In view of (2.1) and (2.10), we have

$$QX = aX + b\eta(X)\xi, \tag{2.11}$$

Let us consider an η-Einstein Lorentzian α-Sasakian manifold. Then putting $X = Y = e_i$ in (2.10), $i = 1,2,\ldots,n$ and taking summation for $1 \le i \le n$, we have

$$r = na - b. \tag{2.12}$$

Now, setting $X = Y = \xi$ in (2.10) and using (2.1) and (2.8), we obtain

$$a - b = (n-1)\alpha^2. \tag{2.13}$$

From the conditions (2.12) and (2.13), gives

$$a = \frac{r}{n-1} - \alpha^2 \quad \text{and} \quad b = \frac{r}{n-1} - n\alpha^2. \tag{2.14}$$

In view of (2.5)-(2.7), it can be easily constructed that in n-dimensional Lorentzian α-Sasakian manifold M^n, the M-projective curvature tensor satisfies the following condition from (1.1):

$$\mathsf{M}(X,Y)\xi = \frac{\alpha^2}{2}\{\eta(Y)X - \eta(X)Y\} - \frac{1}{2(n-1)}\{\eta(Y)QX - \eta(X)QY\}, \tag{2.15}$$

$$\mathsf{M}(\xi,X)Y = \frac{\alpha^2}{2}\{g(X,Y)\xi - \eta(Y)X\} - \frac{1}{2(n-1)}\{S(X,Y)\xi - \eta(Y)QX\}, \tag{2.16}$$

$$\begin{aligned}\eta(\mathsf{M}(X,Y)Z) = {} & \frac{\alpha^2}{2}\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\} \\ & - \frac{1}{2(n-1)}\{S(Y,Z)\eta(X) - S(X,Z)\eta(Y)\}.\end{aligned} \tag{2.17}$$

The above results will be used in the later sections.

3. M-Projectively Flat Lorentzain α-Sasakian Manifold

Definition 3.1. *The Lorentzian α-Sasakian manifold M^n is said to be a M-projectively flat, if we have* $\mathsf{M}(X,Y)Z = 0$.

By taking into account of relation (1.1) and using Definition 3.1, we get

$$R(X,Y)Z = \frac{1}{2(n-1)}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY]. \tag{3.1}$$

Taking $Z = \xi$ in (3.1) and using (2.1), (2.7) and (2.8), we obtain

$$[\eta(Y)X - \eta(X)Y] = \frac{1}{(n-1)\alpha^2}[\eta(Y)QX - \eta(X)QY]. \tag{3.2}$$

Again, putting $Y = \xi$ in (3.2) and using relation (2.1) and (2.8), we obtain

$$QX = (n-1)\alpha^2 X \Leftrightarrow S(X,Y) = (n-1)\alpha^2 g(X,Y). \tag{3.3}$$

Thus, we get the following theorem

Theorem 3.1. *If* an n-dimensional *Lorentzian* α *-Sasakian manifold* M^n *is* M *-Projectively flat, then it is an Einstein manifold and Ricci tensor of* M *has the form* $S(X,Y) = (n-1)\alpha^2 g(X,Y)$.

In this case, by using (3.3) in (3.1), we obtain

$$R(X,Y)Z = \alpha^2\{g(Y,Z)X - g(X,Z)Y\}. \tag{3.4}$$

Theorem 3.2. *If* an n-dimensional *Lorentzian* α *-Sasakian manifold* M^n *is* M *-Projectively flat, then it is an locally isometric to the unit sphere* $S^n(c)$*, where* $c = \alpha^2$.

4. M-Projectively Flat Lorentzian α-Sasakian Manifold Satisfying $R(X,Y)\cdot S = 0$.

In the present section, we consider that M^n is an M-projectively flat Lorentzian α -Sasakian manifold (M^n, g) satisfying the condition $R(X,Y)\cdot S = 0$. Thus we have

$$S(R(X,Y)Z,U) + S(Z,R(X,Y)U) = 0. \tag{4.1}$$

In view of (3.1) in (4.1), we obtain

$$\frac{1}{2(n-1)}[S(QX,U)g(Y,Z) - S(QY,U)g(X,Z)$$
$$+ S(QX,Z)g(Y,U) - S(QY,Z)g(X,U)] = 0. \tag{4.2}$$

Setting $Y = Z = \xi$ in (4.2) and using (2.1) and (2.8), we obtain

$$[S(QX,U) + \eta(X)S(Q\xi,U) - \eta(U)S(QX,\xi) + g(X,U)S(Q\xi,\xi)] = 0. \tag{4.3}$$

Again, by using (2.8) in (4.3), we find

$$-S(QX,U) - (n-1)^2\alpha^4\eta(X)\eta(U) + \eta(U)S(QX,\xi) + (n-1)^2\alpha^4 g(X,U) = 0. \tag{4.4}$$

Let λ be the eigen value of the endomorphism Q corresponding to an eigen vector X. Then putting $QX = \lambda X$ in (4.4) and using relation $g(QX,Y) = S(X,Y)$, we find

$$-\lambda^2 g(X,U) + (n-1)\alpha^2\lambda\eta(X)\eta(U) - (n-1)^2\alpha^4\eta(X)\eta(U)$$
$$+ (n-1)^2\alpha^4 g(X,U) = 0. \tag{4.5}$$

Now, putting $U = \xi$ in (4.5), we obtain

$$[\lambda^2 + (n-1)\alpha^2\lambda - 2(n-1)^2\alpha^4]\eta(X) = 0. \tag{4.6}$$

In this case, since $\eta(X) \neq 0$, the relation (4.6) implies that

$$\lambda^2 + (n-1)\ \alpha^2\lambda - 2(n-1)^2\alpha^4 = 0. \tag{4.7}$$

From (4.7) it follows that the endomorphism Q has two different non-zero eigenvalues, namely, $(n-1)\alpha^2$ and $-2(n-1)\alpha^2$. Thus we can state :

Theorem 4.1. *Let* M^n be an n-dimensional M*-Projectively flat Lorentzian* α *-Sasakian manifold satisfies* $R(X,Y)\cdot S = 0$*, then symmetric endomorphism* Q *of the tangent space corresponding to* S *has two different non-zero eigenvalues.*

5. Conservative M-Projective Curvature Tensor on Lorentzian α-Sasakian Manifold

Definition 5.1. The Lorentzian α-Sasakian manifold (M^n, g) is said to be M-projective conservative if,

$$div\mathsf{M} = 0, \tag{5.1}$$

where div denotes the divergence.

Taking the covariant derivative of (1.1), we get

$$(\nabla_U \mathsf{M})(X,Y)Z = (\nabla_U R)(X,Y)Z - \frac{1}{2(n-1)}[(\nabla_U S)(Y,Z)X - (\nabla_U S)(X,Z)Y$$

$$+ g(Y,Z)(\nabla_U Q)X - g(X,Z)(\nabla_U Q)Y]. \tag{5.2}$$

Contracting with respect to U in (5.2), we obtain

$$(div\mathsf{M})(X,Y)Z = (divR)(X,Y)Z - \frac{1}{2(n-1)}[(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z)$$

$$+ g(Y,Z)divQX - g(X,Z)divQY]. \tag{5.3}$$

We know that

$$divQ(X) = \frac{1}{2}\nabla_X r. \tag{5.4}$$

By virtue of (5.4) in (5.3), we obtain

$$(div\mathsf{M})(X,Y)Z = (divR)(X,Y)Z - \frac{1}{2(n-1)}[(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z)$$

$$+ \frac{1}{2}g(Y,Z)\nabla_X r - \frac{1}{2}g(X,Z)\nabla_Y r]. \tag{5.5}$$

But from [4], we have

$$divR = (\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z). \tag{5.6}$$

Again, by virtue of (5.1) and (5.6) in (5.5), reduces to

$$(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z) = \frac{1}{2(2n-3)}\{g(Y,Z)\nabla_X r - g(X,Z)\nabla_Y r\}. \tag{5.7}$$

Setting $X = \xi$ in (5.7), we obtain

$$(\nabla_\xi S)(Y,Z) - (\nabla_Y S)(\xi,Z) = \frac{1}{2(2n-3)}\{g(Y,Z)\nabla_\xi r - g(\xi,Z)\nabla_Y r\}. \tag{5.8}$$

Further, we know that

$$(\nabla_\xi S)(X,Y) = \xi S(X,Y) - S(\nabla_\xi X,Y) - S(X,\nabla_\xi Y)$$

$$= \xi S(X,Y) - S([\xi,X] + \nabla_X \xi, Y) - S(X,[\xi,Y] + \nabla_Y \xi)$$

$$= \xi S(X,Y) - S([\xi,X],Y) - S(\nabla_X \xi, Y) - S(X,[\xi,Y]) - S(X,\nabla_Y \xi)$$

$$= (L_\xi S)(X,Y) - S(\nabla_X \xi, Y) - S(X,\nabla_Y \xi). \tag{5.9}$$

The Lie derivative of metric g along with vector field X is

$$(L_X g)(Y,Z) = L_X g(Y,Z) - g(L_X Y,Z) - g(Y,L_X Z). \tag{5.10}$$

Setting $X=\xi$ in (5.10) and using (2.4), we obtain

$$(L_\xi g)(Y,Z)=-2\alpha g(\phi Y,Z). \tag{5.11}$$

We now recall that $g(QX,Y)=S(X,Y)$ and using relation (5.11), we get

$$(L_\xi S)(Y,Z)=-2\alpha S(\phi Y,Z). \tag{5.12}$$

Using (2.4) and (5.12) in (5.9), we obtain

$$(\nabla_\xi S)(Y,Z)=0, \tag{5.13}$$

which implies

$$\nabla_\xi r=0. \tag{5.14}$$

By virtue of (5.8) and using (2.2), (2.4),(5.13) and (5.14), we obtain

$$S(\phi Y,Z)=(n-1)\alpha^2 g(\phi Y,Z)+\frac{1}{2\alpha(2n-3)}\eta(Z)dr(Y). \tag{5.15}$$

Replacing $Z=\phi Z$ in (5.15) and using (2.2) and (2.9), we obtain

$$S(Y,Z)=(n-1)\alpha^2 g(Y,Z). \tag{5.16}$$

Contracting the equation (5.16), we obtain

$$r=n(n-1)\alpha^2. \tag{5.17}$$

Thus, we can state the following:

Theorem 5.1. *Let M^n be an n-dimensional M-Projective curvature tensor of Lorentzian α-Sasakian manifold is conservative. Then M^n is an Einstein manifold with a scalar curvature is constant.*

6. Irrotational M-Projective Curvature Tensor on η-Einstein Lorentzian α-Sasakian Manifold

Definition 6.1. *The rotation (curl) of M-projective curvature tensor on a Lorentzian α-Sasakian manifold M^n is defined as*

$$Rot\mathsf{M}=(\nabla_U\mathsf{M})(X,Y)Z+(\nabla_X\mathsf{M})(U,Y)Z$$

$$+(\nabla_Y\mathsf{M})(X,U)Z-(\nabla_Z\mathsf{M})(X,Y)U. \tag{6.1}$$

By taking into account of second Bianchi identity for Riemannian connection ∇, (6.1) becomes

$$Rot\mathsf{M}=-(\nabla_Z\mathsf{M})(X,Y)U. \tag{6.2}$$

If the M-projective curvature tensor is irrotational, then curl $\mathsf{M}=0$ and so by (6.2), we get

$$(\nabla_Z\mathsf{M})(X,Y)U=0,$$

which implies

$$\nabla_Z(\mathsf{M}(X,Y)U)=\mathsf{M}(\nabla_Z X,Y)U+\mathsf{M}(X,\nabla_Z Y)U+\mathsf{M}(X,Y)\nabla_Z U. \tag{6.3}$$

Putting $U=\xi$ in (6.3), we obtain

$$\nabla_Z(\mathsf{M}(X,Y)\xi)=\mathsf{M}(\nabla_Z X,Y)\xi+\mathsf{M}(X,\nabla_Z Y)\xi+\mathsf{M}(X,Y)\nabla_Z\xi. \tag{6.4}$$

Now, substituting $Z=\xi$ in (1.1) and using (2.1), (2.7), (2.8) and (2.11),we obtain

$$\mathsf{M}(X,Y)\xi=\lambda[\eta(X)Y-\eta(Y)X], \tag{6.5}$$

where

$$\lambda=\frac{1}{2(n-1)^2}[n(n-1)\alpha^2-r]. \tag{6.6}$$

By virtue of (6.5) and (2.4) in (6.4), we obtain

$$\mathsf{M}(X,Y)\phi Z=\lambda[g(Y,\phi Z)X-g(X,\phi Z)Y]. \tag{6.7}$$

Replacing Z by ϕZ in (6.7) and simplifying by using (2.1), we get

$$\mathsf{M}(X,Y)Z=\lambda[g(Y,Z)X-g(X,Z)Y]. \tag{6.8}$$

In addition from (1.1) and (6.8), we have

$$\lambda[g(Y,Z)X-g(X,Z)Y]=R(X,Y)Z-\frac{1}{2(n-1)}[S(Y,Z)X-S(X,Z)Y$$

$$+g(Y,Z)QX-g(X,Z)QY]. \tag{6.9}$$

Contracting (6.9) over X and using (6.6), we can find

$$S(Y,Z)=(n-1)\alpha^2 g(Y,Z). \tag{6.10}$$

from (6.10), we obtain

$$r=n(n-1)\alpha^2. \tag{6.11}$$

As a result of (1.1), (6.6), (6.8), (6.10) and (6.11), we obtain

$$R(X,Y)Z=\alpha^2[g(Y,Z)X-g(X,Z)Y]. \tag{6.12}$$

Thus we can state following:

Theorem 6.1. *The* M*-projective curvature tensor in a Lorentzian* α*-Sasakian manifold* M^n *is irrotational, then it is locally isometric to the unit sphere* $S^n(c)$*, where* $c=\alpha^2$.

7. Lorentzain α-Sasakian Manifold Satisfying M·R = 0.

Consider an Lorentzain α - Sasakian manifold satisfying the condition

$$\mathsf{M}(X,Y)\cdot R=0. \tag{7.1}$$

Then from the relation (7.1), it follows that

$$\mathsf{M}(\xi,X)R(Y,Z)U-R(\mathsf{M}(\xi,X)Y,Z)U$$
$$-R(Y,\mathsf{M}(\xi,X)Z)U-R(Y,Z)\mathsf{M}(\xi,X)U=0. \tag{7.2}$$

In view of (2.16), it follows from (7.2) that

$$\frac{\alpha^2}{2}[g(R(Y,Z)U,X)\xi-\eta(R(Y,Z)U)X-g(X,Y)R(\xi,Z)U-\eta(Y)R(X,Z)U$$
$$-g(X,Z)R(Y,\xi)U-\eta(Z)R(Y,X)U-g(X,U)R(Y,Z)\xi-\eta(U)R(Y,Z)X]$$
$$-\frac{1}{2(n-1)}[g(R(Y,Z)U,QX)\xi-\eta(R(Y,Z)U)QX)-S(X,Y)R(\xi,Z)U$$
$$+\eta(Y)R(QX,Z)U-S(X,Z)R(Y,\xi)U+\eta(Z)R(Y,QX)U$$
$$-S(X,U)R(Y,Z)\xi+\eta(Z)R(Y,Z)QX]=0. \tag{7.3}$$

Taking the inner product of the above equation with ξ, we get

$$-\frac{\alpha^2}{2}R'(Y,Z,U,X)-\frac{\alpha^2}{2}[g(X,Y)\eta(R(\xi,Z)U)-\eta(Y)\eta(R(X,Z)U)$$
$$+g(X,Z)\eta(R(Y,\xi)U)-\eta(Z)\eta(R(Y,X)U)+g(X,U)\eta(R(Y,Z)\xi)$$
$$-\eta(U)\eta(R(Y,Z)X)]+\frac{1}{2(n-1)}R'(Y,Z,U,QX)+\frac{1}{2(n-1)}[S(X,Y)\eta(R(\xi,Z)U)$$
$$-\eta(Y)\eta(R(QX,Z)U)+S(X,Z)\eta(R(Y,\xi)U)-\eta(Z)\eta(R(Y,QX)U)$$
$$+S(X,U)\eta(R(Y,Z)\xi)-\eta(U)\eta(R(Y,Z)QX)]=0. \quad (7.4)$$

By virtue of (2.5), (2.6) and (2.7) in (7.4), we obtain

$$-\frac{\alpha^2}{2}R'(Y,Z,U,X)+\frac{1}{2(n-1)}R'(Y,Z,U,QX)$$
$$+\frac{\alpha^4}{2}\{g(X,Y)g(Z,U)-g(X,Z)g(Y,U)\}$$
$$-\frac{\alpha^2}{2(n-1)}\{S(X,Y)g(Z,U)-S(X,Z)g(Y,U)\}=0. \quad (7.5)$$

Setting $Z=U=e_i$ in (7.5) and taking summation over i, $1\le i\le n$, we obtain

$$S^2(X,Y)=(n-1)\alpha^2\{2S(X,Y)-(n-1)\alpha^2 g(X,Y)\}. \quad (7.6)$$

Therefore the S^2 of the Ricci tensor S is the linear combination of the Ricci tensor and the metric tensor g. Here the $(0,2)$-tensor S^2 is defined by $S^2(X,Y)=S(QX,Y)$. Hence we have the following:

Theorem 7.1. *Let M be an n-dimensional Lorentzian α-Sasakian manifold is satisfying the condition $\mathrm{M}\cdot R=0$. Then the S^2 of the Ricci tensor S is the linear combination of the Ricci tensor and the metric tensor g has the form $S^2(X,Y)=(n-1)\alpha^2\{2S(X,Y)-(n-1)\alpha^2 g(X,Y)\}$.*

It is well known that:

Lemma 7.1. *[21] If $\theta=g\wedge A$ be the Kulkarni-Nomizu product of g and A, where g being Riemannian metric and A be a symmetric tensor of type $(0,2)$ at point x of a semi-Reimannian manifold (M^n,g). Then relation $\theta\cdot\theta=\beta Q(g,\theta),\beta\in\mathbb{R}$ is true at x if and only if the condition $A^2=\beta A+\mu g$, $\mu\in\mathbb{R}$ holds at x*

In consequence of Theorem 7.1 and Lemma 7.1, we have the following corollary:

Corollary 7.1. *Let M an n-dimensional Lorentzain α-Sasakian manifold is satisfying the condition $\mathrm{M}(X,Y)\cdot R=0$, then $\theta\cdot\theta=\beta Q(g,\theta)$, where $\theta=g\wedge S$ and $\beta=2(n-1)\alpha^2$.*

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