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On magnetic braking of late-type stars

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Summary. The paper reports on a preliminary study of the variation with angular velocity of the rate of braking of a late-type star. The strength of the dynamo-built field **B** will normally increase with rotation, so that the surface where the wind speed becomes Alfvénic moves further out from the star and the angular momentum transported per unit mass is correspondingly higher. The stronger field tends to trap more gas within a 'dead zone' which does not contribute to the braking, but the extent of the dead zone is limited by the pressure required to balance the higher centrifugal force. These effects are illustrated by a simply parametrized field model. Braking rates are estimated for the two cases with the coronal base density independent of B and proportional to B, respectively.

1 Introduction

The idea of magnetic braking by a stellar wind is now very familiar. In a seminal paper, Schatzman (1962) pointed out that if gas emitted from a star is kept corotating with the star by magnetic torques out to large distances, it will carry off far more angular momentum per unit mass than gas retaining the angular momentum of the stellar surface. The process can be thought of as a competition between the speed at which the outflowing gas tries to set up a state of non-uniform rotation and the speed v_A at which Alfvén waves try to re-establish isorotation along individual field lines (e.g. Cowling 1965). A straightforward application to axisymmetric systems of the hydromagnetic equations [Mestel 1967a, b, 1968 (M68); Weber & Davis 1967 (WD); the Appendix] shows that the angular momentum loss rate is indeed equivalent to that carried by gas kept strictly corotating with the star out to the Alfvénic surface S_A defined by

$$v_p = v_A \equiv B_p/(4\pi\varrho)^{1/2}, \tag{1}$$

where v_p is the wind speed and B_p the meridional (i.e. poloidal) component of the magnetic field.

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In the analysis leading to this basic result, the structure of $\mathbf{B}_p$ is supposed given. Continuity of the flow along $\mathbf{B}_p$ ensures that

$$\tfrac{1}{2}\varrho v_p^2 \lessgtr \frac{B_p^2}{8\pi} \equiv \tfrac{1}{2}\varrho v_A^2$$

well within and well beyond S_A, respectively. At large distances the magnetic torques are weak, so that the gas gains hardly any more angular momentum; and likewise we expect the wind flow to be only weakly affected by $\mathbf{B}_p$, which will rather be strongly distorted so as to follow the streamlines. On the other hand, deep within S_A the same condition $v_p \ll v_A$ which ensures that the gas is kept in near corotation suggests also that the forces – thermal and centrifugal – that drive out the gas against gravity are not strong enough to distort the field, so that $\mathbf{B}_p$ is well described by a nearly curl-free structure, with the flow $\mathbf{v}_p$ channelled parallel to $\mathbf{B}_p$. This leads to the picture (M68) of a multi-component corona. For simplicity, let the flux distribution over the stellar surface be dipolar. Near the star the field is approximately curl-free at all latitudes. Flux-tubes leaving the star near the equator form closed loops, trapping within them hot gas that nevertheless is too weak to overcome the magnetic pressure; they form the 'dead zone'. Along field lines that leave the star sufficiently near to the poles, gas is accelerated outwards, with the field structure changing over well before S_A from the curl-free to the open form; these field lines define the 'wind zone'. As the wind is an energy sink, a latitude-independent heat input at the coronal base should yield a significantly lower temperature for the wind zone than for the dead zone. It can fairly be claimed that the qualitative picture has been vindicated by the X-ray data, with the high-emission regions identified with the dead zones and the 'coronal holes' with the wind zones.

In this paper we are concerned with the dependence of the rate of braking on the rotation of the star. We are interested primarily in late-type stars (including pre-main-sequence stars) with outer convection zones. The magnetic flux emanating from the star is assumed to be maintained by contemporary dynamo action. We parametrize the relation between the pole-strength B_0 of the dipolar component and the stellar rotation rate α by

$$B_0 \propto \alpha^p \tag{2}$$

with p usually taken as unity. With conditions at the coronal base supposed fixed, the stronger the field the further out is the surface S_A, so that the angular momentum loss *per unit mass loss* will increase. However, an increase in B_0 is likely to increase the extent of the dead zone, reducing correspondingly the fraction of the magnetic flux contributing to the braking of the star. In M68 this latter effect was maximized by the adoption of a curl-free structure for $\mathbf{B}_p$ all the way to S_A. It was found that the reduction in the extent of the wind zone more or less compensated for the increased distance of S_A: the braking *efficiency* – measured by the angular momentum transport per unit mass – increased with B_0, but the total angular momentum loss rate tended to saturate.

This model is almost certainly the most conservative; equally the WD radial field model – in which the whole corona contributes – is likely to exaggerate the rate of braking. Okamoto's hybrid field model (1974) illustrates the effect of varying the extent of the dead zone. An accurate estimate requires a mutually self-consistent solution of the equations for both the wind and dead zones. The model of Pneuman & Kopp (1971) does have a two-zone structure but applies only to slow rotators, while Sakurai's work (1985) assumes a WD split monopole structure all the way to the star. Our work is in the spirit of M68: we again adopt a simple field model, with a parameter $\bar{r}$ defining the extent of the dead zone, and use a rough pressure criterion to see how $\bar{r}$ varies with the stellar rotation α. Standard stellar wind theory (summarized in the Appendix) then yields the rate of braking as a function of α.

2 An approximate field model

The essentials of the field structure are as in Fig. 1 (*cf.* M68; Sturrock & Smith 1968; Pneuman & Kopp 1971). The dead zone terminates on the equator at a cusp C, at a distance $\bar{r}$ from the origin. Within $\bar{r}$ the field lines cross the equator normally; beyond $\bar{r}$ the equatorial field is radial, with the jump $B_r \rightarrow -B_r$ maintained by a toroidal component of surface current. Because of the differing dynamical states of gas within the dead (suffix 'd') and wind (suffix 'w') zones,

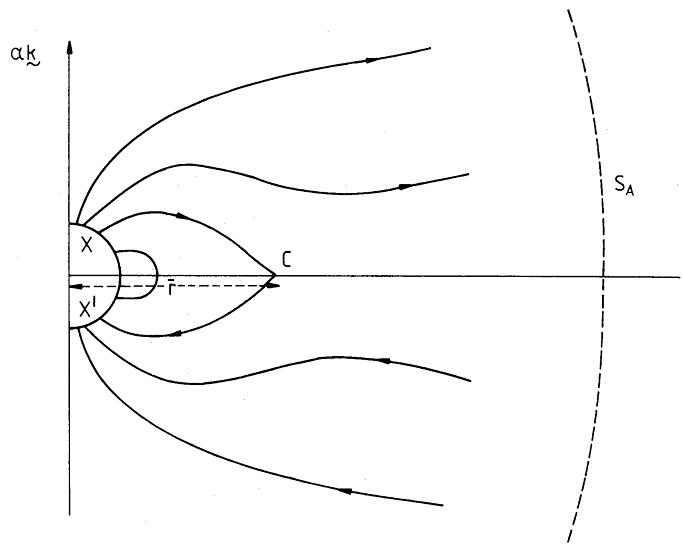

Figure 1. Schematic magnetic field model.

respectively, there is a thermal pressure discontinuity across the separatrix CXX'. In the wind zone the magnetic field has a toroidal component $\mathbf{B}_t$ associated with the non-uniform rotation (*cf.* the Appendix). Dynamical balance requires that on the equator beyond $\bar{r}$ there is a sheet of pinched gas at pressure $p=(B_r^2/8\pi+B_t^2/8\pi)_w$. On CXX' there is a discontinuity in B_p, ensuring that

$$(B_p^2/8\pi+B_t^2/8\pi+p)_w=(B_p^2/8\pi+p)_d. \tag{3}$$

The poloidal currents $\mathbf{j}_p$ maintaining the toroidal field $\mathbf{B}_t$ complete their circuits as a sheet current in the equatorial zone and the interface between the wind and dead zones.

The precise shape of CXX' is not known *a priori*, but will emerge as part of the solution for the field. Even if the toroidal sheet currents on CXX' and the equator are the dominant sources of $\mathbf{B}_p$ outside the star, an accurate computation clearly requires a complicated iterative scheme (*cf.* Pneuman & Kopp 1971). In this paper we are merely concerned to illustrate the effect of a variation in the centrifugal forces on the field structure and so on the rate of magnetic braking. We therefore postpone detailed field construction, adopting instead a simple but reasonably plausible modification of the M68 model. The field is taken as approximately dipolar out to the radius $\bar{r}$ – the radial distance of the cusp – which is now within the point where the Alfvénic surface S_A intersects the equator. Beyond $\bar{r}$ the field is assumed radial. The condition (3) is applied just at C [where $(B_p)_d=0$], and in the approximate form

$$(B_p^2/8\pi)_w=p_d. \tag{4}$$

The neglect of B_t is justified when C is well within S_A. The significant approximation, especially for a rapid rotator, is the neglect of p_w compared with p_d. This is because in the wind zone the centrifugal force helps to accelerate the gas and so reduce the density and pressure, whereas in the dead zone the component of centrifugal force along **B** must be balanced by an outward *increase* in pressure. Thus in the corotating zone, the equation of magnetohydrostatic equilibrium is

$$-\nabla p+\varrho\nabla V+(\nabla\times\mathbf{B})\times\mathbf{B}/4\pi=0, \tag{5}$$

with V the joint gravitational and centrifugal potential

$$V=GM/r+½\alpha^2r^2\sin^2\theta \tag{6}$$

[(r,θ) being spherical polar coordinates]. For simplicity we take the zone to be isothermal with $p=\varrho a_\mathrm{d}^2$, where a_d is the sound speed in the dead zone. The component of (5) along **B** then yields

$$\frac{\varrho}{(\varrho_0)_\mathrm{d}}=\exp\left[-\frac{GM}{Ra_\mathrm{d}^2}\left(1-\frac{R}{r}\right)+\frac{1}{2}\frac{\alpha^2R^2}{a_\mathrm{d}^2}\left(\frac{r^2\sin^2\theta}{R^2}-\sin^2\theta_0\right)\right], \tag{7}$$

where the suffix zero refers to the coronal base $r=R$, and θ_0 is the polar angle of the point where the field line passing through (r,θ) emerges from $r=R$. In a strictly dipolar field, $\sin^2\theta_0=R/\bar{r}$ for the limiting field line CXX'; also $B^2(\bar{r},\pi/2)=B_0^2R^6/4r^6$. Even though it turns out that $(\bar{r}/R)$ given by equation (4) is not very sensitive to factors such as 4, we in fact use the form $B\simeq B_0R^3/r^3$ between R and $\bar{r}$, for it is the contrast between $1/r^3$ and the $1/r^2$ dependence for a radial field that is important rather than numerical factors which are unlikely to persist in an accurate computation of the field. Thus condition (4) takes the form

$$\left(\frac{R}{\bar{r}}\right)^6=\frac{8\pi(\varrho_0)_\mathrm{d}a_\mathrm{d}^2}{B_0^2}\exp\left[-\frac{GM}{Ra_\mathrm{d}^2}\left(1-\frac{R}{\bar{r}}\right)\right]\exp\left[\frac{1}{2}\frac{\alpha^2R^2}{a_\mathrm{d}^2}\left(\frac{\bar{r}^2}{R^2}-\frac{R}{\bar{r}}\right)\right]. \tag{8}$$

As noted, the exponential term involving $\alpha^2R^2/a_\mathrm{d}^2$ will be crucial in restricting the allowed value of $\bar{r}/R$ for a rapid rotator.

It is convenient to introduce parameters similar to those used in M68:

$$l_\mathrm{d}=GM/Ra_\mathrm{d}^2,\qquad \varkappa=\alpha^2R^3/GM,\qquad \varkappa l_\mathrm{d}=\alpha^2R^2/a_\mathrm{d}^2,\qquad \zeta_\mathrm{d}=B_0^2/8\pi(\varrho_0)_\mathrm{d}a_\mathrm{d}^2. \tag{9}$$

For definiteness, we consider stars of solar mass but with rotation rates $\alpha=k\alpha_\odot$, where k varies between unity and 80. Our dynamo parametrization (2) implies $B_0=(B_0)_\odot(\alpha/\alpha_\odot)^p$. The simplest model will have $(\varrho_0)_\mathrm{d}$ and a_d both independent of α, yielding $\zeta_\mathrm{d}\propto\alpha^{2p}$, but this is not obviously consistent with X-ray observations, which show a luminosity $L_x\propto\alpha^2$ (Pallavicini *et al.* 1981; Micela *et al.* 1984). The accuracy with which the exponent in this relation is known is limited by the small number of stars of a given spectral type with known rotation rates. It has been suggested (Mangeney & Praderie 1984; Noyes *et al.* 1984; Simon, Herbig & Boesgaard 1985) that a measure of rotation nearly independent of spectral type could be a Rossby number based on the surface rotation rate and a theoretical estimate of the turnover time of convective cells near the bottom of the convective zone – *cf.* Durney & Latour 1978. However, the value of using such a Rossby number has recently been questioned in two preprints (Rutten & Schrijver 1986; Simon & Fekel 1986). The results obtained for late-type stars seem consistent with the relation $L_x\propto\alpha^2$ found by Pallavicini *et al.*, which we shall therefore provisionally adopt. The variation of coronal emission is determined mostly by the variation in density, with $L_x\propto\varrho_0^2$ (optically thin emission); the variation in temperature has a smaller influence. Thus we must have $\varrho_0\propto\alpha$ approximately. The X-ray luminosity depends on the degree of activity of the star, of which the field strength is a measure. Assuming $L_x\propto B_0^n$, the observational requirement $L_x\propto\alpha^2$ implies that n has to be chosen equal to $2/p$: if the dependence of B_0 on rotation is steep, the relation between coronal emission and field strength must be correspondingly less steep in order to satisfy the observed dependence of L_x on rotation rate. With $p=1$, we would have $L_x\propto B_0^2$, which in fact is the expected dependence of the coronal heating rate on field strength for 'DC heating' models, in which the energy input rate is a certain fraction of the magnetic energy density itself (e.g. Rosner *et al.* 1979; Parker 1983).

With this choice for p,

$$\zeta_d=(\zeta_d)_\odot \frac{[B_0/(B_0)_\odot]^2}{(\varrho_0)_d/(\varrho_0)_{d\odot}}=(\zeta_d)_\odot\left(\frac{\alpha}{\alpha_\odot}\right) \tag{10}$$

instead of $\zeta_d\propto\alpha^2$. Equation (8) for $x\equiv\bar{r}/R$ becomes

$$x^6\exp(l_d/x)\exp[\tfrac{1}{2}\varkappa_\odot l_d(\alpha/\alpha_\odot)^2(x^2-1/x)]=(\zeta_d)_\odot(\alpha/\alpha_\odot)\exp(l_d). \tag{11}$$

We adopt standard solar parameters:

$$T_\odot=2\times10^6,\qquad \mu_\odot=0.6,\qquad \alpha_\odot=2.5\times10^{-6},$$

so that

$$a_d^2=\mathscr{R}T/\mu=2.75\times10^{14},\qquad l_d=GM/Ra_d^2=6.93,\qquad \varkappa l_d=1.45\times10^{-4}(\alpha/\alpha_\odot)^2. \tag{12}$$

In Table 1 we list the solutions of equation (8) as α increases from the solar value ($P\simeq30$ day) to $P\simeq\tfrac{1}{2}$ day for the cases $(\zeta_d)_\odot=4$, 60. In all cases the results show that for slow rotation the centrifugal effect on the pressure in the dead zone is weak, so $\bar{r}/R$ increases with α because the stronger field traps more gas, but after $\alpha/\alpha_\odot\simeq15$ the centrifugal effect takes over and the dead zone begins to shrink.

Table 1.

$\alpha/\alpha_\odot$	$(\zeta_d)_\odot=4$		$(\zeta_d)_\odot=60$	
	$\bar{r}/R$ ($\zeta_d\propto\alpha$)	$\bar{r}/R$ ($\zeta_d\propto\alpha^2$)	$\bar{r}/R$ ($\zeta_d\propto\alpha$)	$\bar{r}/R$ ($\zeta_d\propto\alpha^2$)
1	2.5	2.5	5	5
2	3.1	3.5	5.8	6.6
3	3.4	4.1	6.2	7.75
5	3.9	5.5	6.8	9.2
7	4.14	6.2	7.2	10
10	4.4	6.9	7.4	10.65
20	4.6	7.2	6.9	9.6
30	4.4	6.5	6.1	8.2
60	3.4	4.7	4.3	5.4
80	3.0	3.9	3.6	4.5

3 The Alfvénic surface and the rate of braking

Along the open field lines of the model we apply standard isothermal wind theory (summarized in the Appendix). For a rapid rotator, the wind speed near S_A is well approximated by expression (A15). The wind zone is expected to have a sound speed a_w different from that of the dead zone a_d. The field-streamline $\theta=$constant cuts S_A at the radial distance r_A given by combining (1) with (A15) and the continuity equation (A3):

$$4\pi\eta=\left(\frac{B_p}{v_p}\right)_{r_A}=\frac{B_0(R/\bar{r})^3(\bar{r}/r_A)^2}{a_w[\![\{[v_{th}(r_A/R)]/a_w\}^2+(\alpha^2r_A^2/3a_w^2)\sin^2\theta]\!]^{1/2}}, \tag{13}$$

or

$$\left(\frac{r_A}{R}\right)^2\left[\left(\frac{v_{th}(r_A/R)}{a_w}\right)^2+\frac{\alpha^2r_A^2}{3a_w^2}\sin^2\theta\right]^{1/2}=\left(\frac{R}{\bar{r}}\right)\frac{2\zeta_w}{(v_0/a_w)}, \tag{14}$$

where v_0 is the wind speed at the coronal base. With standard solar parameters, the gas reaches the sound speed a_w above the coronal base, and v_0 is found to be $\simeq 0.15 a_w$ [*cf.* the discussion following equation (A9)]. The parameter $\zeta_w \equiv B_0^2/[8\pi(\varrho_0)_w a_w^2]$ will in general differ from ζ_d, partly through the lower temperature but more through the lower density in the wind zone. A complete theory of the chromosphere and the corona would predict the temperature and density at the base of the corona for given B_0 and so also the value of ζ_w. In the absence of such a theory we choose $\zeta_w \simeq 120$ for the Sun, since (14) then yields $r_A/R \simeq 12$ in rough agreement with observation (Pizzo *et al.* 1983). We again adopt the dynamo relation (2) with $p=1$, and as with ζ_d, we consider the two cases $\zeta_w \propto \alpha$ and $\zeta_w \propto \alpha^2$, corresponding to $(\varrho_0)_w \propto B_0$ and $(\varrho_0)_w$ independent of B_0, respectively. As an example, we take the wind zone temperature to be one-half that in the dead zone, so that (14) becomes

$$\left(\frac{r_A}{R}\right)^2 \left[\left(\frac{v_{th}}{a_w}\right)^2_{r_A} + 7.3\times 10^{-5}\left(\frac{\alpha}{\alpha_\odot}\right)^2 \left(\frac{r_A}{R}\right)^2 \sin^2\theta\right]^{1/2} = \frac{1600}{(\bar{r}/R)}\left(\frac{\alpha}{\alpha_\odot}\right)^n \tag{15}$$

with $n=1$ or 2. Equatorial values of r_A/R are listed in Table 2, use being made of the respective values of $\bar{r}/R$ in Table 1 and of v_{th}/a_w in Table A1.

Table 2.

$\alpha/\alpha_\odot$	$(r_A/R)_{\theta=\pi/2}$ $(\zeta_w \propto \alpha)$	$(r_A/R)_{\theta=\pi/2}$ $(\zeta_w \propto \alpha^2)$	
1	12.3	12.3	
2	15	19	$(\zeta_w)_\odot = 120$
5	20	36	
10	25	52	$(\zeta_d)_\odot = 60$
20	29	72	
30	31	88	
60	35	127	
80	37	149	

From equation (A5), the transport of angular momentum by the flow and by the magnetic stresses jointly is equivalent to that carried by gas kept corotating with the star out to S_A:

$$-\dot{J} = 2(2\pi)\alpha \int_0^{\pi/2} (\varrho_A v_A r_A^2) r_A^2 \sin^2\theta \sin\theta \, d\theta$$

$$= 4\pi\alpha_\odot R^4 \left(\frac{\alpha}{\alpha_\odot}\right)(\varrho_0)_w \left(\frac{v_0}{a_w}\right) a_w \int_0^1 \frac{\varrho(\bar{r})}{(\varrho_0)_w}\frac{v(\bar{r})}{v_0}\left(\frac{\bar{r}}{R}\right)^2 \left(\frac{r_A}{R}\right)^2 (1-\mu^2)\, d\mu. \tag{16}$$

By continuity, $\varrho(\bar{r})v(\bar{r})/(\varrho_0)_w v_0 = \bar{B}/B_0 \simeq (R/\bar{r})^3$. A lower limit for $-\dot{J}$ is given by using the equatorial value for r_A/R for all values of θ. This should in fact be a tolerable approximation even for rapid rotators, as (r_A/R) given by (15) increases markedly over the equatorial values in Table 2 only on field-streamlines on which θ is small and so which make only a modest contribution to the total angular momentum transport. Expression (16) then yields

$$-\dot{J} = \left[\frac{8\pi}{3}\alpha_\odot(\varrho_0)_\odot R^4 a_w\left(\frac{v_0}{a_w}\right)\right] K\left(\frac{\alpha}{\alpha_\odot}\right) \tag{17}$$

where

$$K(\alpha/\alpha_\odot) = (R/\bar{r})(r_A/R)^2(\alpha/\alpha_\odot)^q, \tag{18}$$

with $q=1$ corresponding to $n=2$ in (15) (ϱ_{0w} independent of B_0) and $q=2$ to $n=1$ in (15) ($\varrho_{0w}\propto B_0$). Thus the α-dependence of $-\dot{J}$ is contained in the function K which is listed in Table 3. We note that braking is significantly more efficient when $\varrho_{0w}\propto B_0$. This is due primarily to the increase in the mass loss, which more than compensates for the reduction in the Alfvénic radius shown in Table 2. The smaller extent of the dead zone for the $\varrho_{0w}\propto B_0$ case (*cf.* Table 1) also contributes significantly.

Table 3.

$\alpha/\alpha_\odot$	$K\equiv(R/\bar{r})(r_A/R)^2(\alpha/\alpha_\odot)^q$ ($q=2$, $\varrho_{0w}\propto B_0$)	($q=1$, ϱ_{0w} independent of B_0)
1	30.5	30.5
2	155	110
5	1.5×10^3	7.0×10^2
10	8.4×10^3	2.5×10^3
20	4.8×10^4	1.1×10^4
30	1.4×10^5	2.8×10^4
60	10^6	1.8×10^5
80	2.4×10^6	4.0×10^5

4 Discussion

A braking law $\dot{\alpha}\propto-\alpha^p$ yields the asymptotic time-dependence $\alpha\propto t^{-1/(p-1)}$. Inspection of Table 3 shows that the case $\varrho_{0w}\propto B_0$ yields $p\simeq2.6$ over the whole range $1\leqslant\alpha/\alpha_\odot\leqslant80$, corresponding to $\alpha\propto t^{-0.625}$. The case with ϱ_{0w} independent of B_0 has $p\simeq1.91$ when $1\leqslant\alpha/\alpha_\odot\leqslant10$, yielding $\alpha\propto t^{-1.1}$; when $10\leqslant\alpha/\alpha_\odot\leqslant80$, $p\simeq-2.5$, yielding $\alpha\propto t^{-0.69}$. For comparison, we note that a purely thermally driven wind with no dead zone and with the $B_0\propto\alpha$ dynamo law yields the well-known law $\dot{\alpha}\propto-\alpha^3$, $\alpha\propto t^{-0.5}$ (Skumanich 1972) whatever the dependence of ϱ_{0w} on B_0.

It is no surprise that different cases yield different power-law approximations (*cf.* Mestel 1984). Our very preliminary study serves to illustrate the sensitivity of the braking law to poorly understood features of coronal phenomena, such as the mass-loss rate and the associated extent of the dead zone. A dependence on α of temperature in both wind and dead zones will also modify the law (Vilhu & Moss 1986). We hazard the guess that our estimates for the instantaneous rate of braking will turn out correct as to order of magnitude. More precise numbers must await refinement of our rather crude magnetic field model. Well within S_A the current sheets on the equator and on the boundary between dead and wind zones may indeed be the dominant sources of B_p, and techniques used in the star formation context (Mestel & Ray 1985) when suitably generalized may be applicable (Fitzpatrick & Mestel, in preparation). Further out, volume currents probably cannot be ignored. The quasi-radial field beyond $\bar{r}$ is likely to remain a fair approximation out to S_A, but ultimately the pinching toroidal component forces the poloidal field into a cylindrical structure (Sakurai 1985).

Clearly, much more work needs to be done, both for main-sequence and pre-main-sequence stars. A complete wind theory may require inclusion of Alfvén wave driving to supplement the thermal pressure and centrifugal force (*cf.* Lago 1979; De Campli 1981; Hartmann, Edwards & Avrett 1982). The complexity of both the physical basis and of the necessary mathematical analysis suggests that most insight is likely to come from theoretical studies carried out in close contact with observational work.

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Appendix: Résumé of magnetic stellar wind theory

The perfect conductivity approximation $\nabla\times(\mathbf{v}\times\mathbf{B})=0$ yields the integrals

$$\mathbf{v}_\mathrm{p}=\varkappa\mathbf{B}_\mathrm{p}, \qquad \Omega-\varkappa B_\phi/\varpi=\alpha, \tag{A1}$$

or

$$\mathbf{v}=\varkappa\mathbf{B}+\varpi\alpha\mathbf{t}, \tag{A2}$$

where (ϖ, ϕ, z) are cylindrical polar coordinates, $\mathbf{t}$ the unit toroidal vector; α is a constant on each field-streamline, to be identified with the assumed uniform stellar angular velocity. The continuity equation relates the density ϱ to the scalar $\varkappa$:

$$\frac{\varrho v_\mathrm{p}}{B_\mathrm{p}}=\varrho\varkappa=\eta=\text{constant on each field line.} \tag{A3}$$

The toroidal component of the equation of motion yields the torque integral

$$-\frac{\varpi B_\phi}{4\pi}+\varrho\varkappa\Omega\varpi^2=-\frac{\beta}{4\pi} \tag{A4}$$

where $-\beta/4\pi$ is the constant rate of flow of angular momentum along a unit flux tube, carried jointly by the streaming gas and the moment of the Maxwell stresses. Equations (A1, A3, A4) would yield singularities in Ω and B_ϕ at the Alfvénic point $P_{\rm A}$ where $v_{\rm p}=v_{\rm A}=B_{\rm p}/(4\pi\varrho)^{1/2}$ unless

$$-\beta/4\pi=\eta\alpha\varpi_{\rm A}^2=(\varrho v_{\rm p}/B_{\rm p})\alpha\varpi_{\rm A}^2, \tag{A5}$$

showing that steady angular momentum transport is equivalent to effective corotation out to $S_{\rm A}$. At any point the actual values of Ω and B_ϕ are given by (Mestel 1967a, b, 1968)

$$\Omega/\alpha=(1-\varrho_{\rm A}\varpi_{\rm A}^2/\varrho\varpi^2)/(1-\varrho_{\rm A}/\varrho), \tag{A6}$$

$$\varpi B_\phi=-4\pi\eta\alpha\varpi_{\rm A}^2(1-\varpi^2/\varpi_{\rm A}^2)/(1-\varrho_{\rm A}/\varrho). \tag{A7}$$

Of the two other components of the equation of motion, one balances the poloidal components of the magnetic force against gravitation, pressure and inertia. In a complete theory (e.g. Sakurai 1985) this component is used to fix the detailed structure of $\mathbf{B}_{\rm p}$, a problem bypassed by the adoption of the approximate field model of this paper. The last component is conveniently taken as the generalized Bernoulli equation for a rotating isothermal system of sound speed $a_{\rm w}$:

$$H\equiv\tfrac{1}{2}v_{\rm p}^2+\tfrac{1}{2}\Omega^2\varpi^2+a_{\rm w}^2\log\varrho-GM/r-\alpha\Omega\varpi^2$$

$$=E=\text{constant on streamline}. \tag{A8}$$

The term $-\alpha\Omega\varpi^2$ comes from the rate of working $(\mathbf{j}\times\mathbf{B}/c)\cdot\mathbf{v}=\varpi(\mathbf{j}\times\mathbf{B}/c)\cdot\alpha\mathbf{t}$ of the magnetic torque density that gives the outstreaming gas its angular momentum. Once the condition that Ω and B_ϕ be finite at $P_{\rm A}$ has led to the forms (A6, A7) then all non-singular solutions pass automatically through $P_{\rm A}$. Substitution from (A3) and (A6) – with the magnetic field structure assumed known – converts (A8) into a relation $H(\varrho,r)=E$ between ϱ and the convenient monotonic coordinate r. The critical points of this curve are at the intersections $(r_{\rm s},\varrho_{\rm s})$, $(r_{\rm f},\varrho_{\rm f})$ of $\partial H/\partial r=0$ and $\partial H/\partial\varrho=0$, corresponding respectively to the slow and fast magnetosonic wave speeds $v_{\rm s}$ and $v_{\rm f}$. Smoothness of the flow through these points yields the two conditions

$$H(r_{\rm s},\varrho_{\rm s})=E, \qquad H(r_{\rm f},\varrho_{\rm f})=E \tag{A9}$$

which suffice to fix the solution along each field-streamline in terms of the non-dimensionalized coronal temperature, stellar rotation and magnetic flux (Weber & Davis 1967; Goldreich & Julian 1970; Sakurai 1985).

In most cases of interest the slow magnetosonic speed reduces effectively to the sound speed. Unless the rotation is very rapid, this speed is reached at the point $r_{\rm s}\simeq GM/3a^2$ (within the dipolar field region). The value E of H is then fixed from (A9) to be $O(a_{\rm w}^2)$; with standard solar values inserted the velocity v_0 at the coronal base [to be used in (14)] is $\simeq 0.15a_{\rm w}$. When $\alpha/\alpha_\odot$ approaches 40, the terms in Ω in (A8) begin to be significant for streamlines not close to the axis, forcing the sonic point in closer to the star, and increasing v_0 somewhat; however, as $r_{\rm A}$ from (14) is then $\propto v_0^{-1/3}$, the consequent error in the estimate (17) for the angular momentum loss will not be serious.

We are interested especially in rapidly rotating models for which both the Alfvénic point and *a fortiori* the fast magnetosonic point are in the radial field domain, and $\alpha^2r_{\rm f}^2>\alpha^2r_{\rm A}^2\gg(GM/r,\,a_{\rm w}^2)$. In the limit $a_{\rm w}^2/\alpha^2r_{\rm A}^2\sin^2\theta\to 0$ it can be shown that $r_{\rm f}\to\infty$, $\varrho\to 0$, with

$$v_{\rm f}^3\simeq v_{\rm A}\alpha^2r_{\rm A}^2\sin^2\theta, \tag{A10}$$

$$\Omega_f \varpi_f^2 \simeq \alpha \varpi_A^2 (1 - v_A/v_f), \tag{A11}$$

$$\varpi B_\phi \simeq -4\pi\eta\alpha\varpi_A^2 (v_A/v_f), \tag{A12}$$

$$E \simeq \tfrac{1}{2} v_f^2 - (\alpha\varpi_A)^2 (1 - v_A/v_f). \tag{A13}$$

Since $E = O(a_w^2) \to 0$ in this limit, (A13) together with (A10) yields

$$v_f \simeq 3v_A/2, \qquad v_A \simeq (2\sqrt{6}/9)\alpha\varpi_A \simeq 0.54\alpha\varpi_A. \tag{A14}$$

At r_A, equation (A8) then yields $\Omega/\alpha = (1 - \sqrt{19/27}) \simeq 0.16$. The corrections to these values for a_w finite are quite modest when $a_w^2/\alpha^2 r_A^2 \ll 1$.

However, large the ratio $\alpha^2 r^2/a_w^2$, there is always a cone about the axis within which $\alpha^2 r_A^2 \sin^2\theta/a_w^2 \ll 1$, so that the Ω-terms in (A8) are small, and the flow then reduces to a simple stellar wind (Parker 1963). Once gas is well past the sonic point, the subsequent acceleration by thermal pressure steadily declines, in contrast to the centrifugal driving term which increases out to the Alfvénic surface. These purely thermal wind speeds v_{th} depend slightly on the value of $\bar{r}$ where the field changes from dipolar to radial. Typical estimates are given in Table A1.

Table A1.

r/R	v_{th}/a_w
12	2.1
13	2.3
15	2.4
20	2.8
25	2.85
30	2.9
50	3.2
90	3.6

In order to determine the position of the Alfvénic surface S_A and so to estimate the rate of braking, we need the value of the wind speed v_A on S_A to use in equation (14). We adopt the approximate form

$$\left(\frac{v_A}{a_w}\right) = \left[\left(\frac{v_{th}(r_A/R)}{a_w}\right)^2 + \frac{\alpha^2 r_A^2}{3a_w^2}\sin^2\theta\right]^{1/2} \tag{A15}$$

with v_{th} given by Table A1, and the purely centrifugal part from the approximation (A14). Once a more accurate field has been constructed, there will be little difficulty in replacing (A15) by a more accurate expression; for the present (A15) spans the two limits of purely thermal and purely centrifugal driving.