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# On Magnetorheologic Elastomers for Vibration Isolation, Damping, and Stress Reduction in Mass-varying Structures

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**ABSTRACT:** This article considers two devices based on a magnetorheological elastomer (MRE): an MRE isolator under a frequency-varying harmonic excitation and a MRE Dynamic Vibration Absorber (DVA) mounted on a frequency-varying structure under a random excitation. In the first case, it is shown that the commandability of the elastomer improves the reduction of the RMS value of the body displacement by 10%. In the second case, it is shown on a simple example that a MRE DVA, while not optimal, can reduce the stress in the structure about 50% better than a classical DVA when the mass of the structure changes 35%. This makes them suitable to avoid high stress in mass-varying structures, and delay some damage mechanisms like the emergence of cracks and fatigue.

*Key Words:* magnetorheologic elastomer, vibration isolation, vibration damping, stress reduction, mass-varying structure.

## INTRODUCTION

CONSIDER a structure fixed on a support through a set of mounts. Often, the mechanical properties of a structure are evolving in time (e.g., changes in mass, temperature, load cycle). If the stiffness and the damping properties of the mounts remain constant, their isolation performances are not always optimal. A technique whose spring constant and damping factor can be changed represents a considerable improvement of the vibration isolation potential. Along with piezoelectric ceramics and shape memory alloys, the magnetorheological (MR) materials have those capabilities and are more adapted to extreme conditions (Carlson and Jolly, 2000; Bellan and Bossis, 2002).

This study focuses on MR elastomers. Measured characteristics from published studies (Kallio, 2005) are used to evaluate their isolation and damping performances in two cases. The first is a tunable isolator supporting a structure and subjected to a narrow band excitation  $w$  (Figure 1(a)). The problem is addressed in the 'MRE Isolator' section, with a single degree of freedom (d.o.f.) MRE isolator. The second is a tunable dynamic vibration absorber (DVA) mounted on a

structure whose mass is varying, and that is subjected to a broad band random excitation  $w$  (Figure 1(b)). The problem is addressed in 'MRE Dynamic Vibration Absorber' section, with an MRE DVA appended on a flexible structure. In the numerical example, a  $n$ -story like structure has been considered for its simplicity. However, the procedure is applicable to any type of flexible structure. The effect of the MRE DVA on the stress reduction in the structure is discussed.

The next section presents the theoretical model of a magnetorheological elastomer (MRE), along with its potential applications.

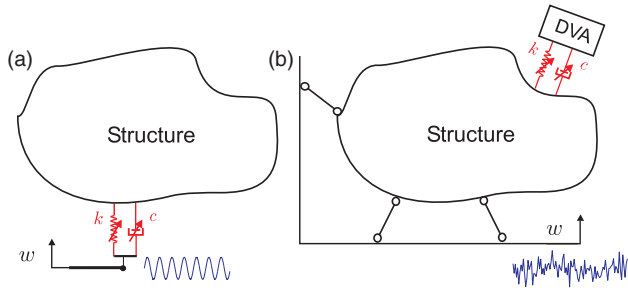
## MODEL OF MRE

### Structure of MRE

An MRE is a composite material, whose mechanical properties are modified when it is facing an external magnetic field. It is made of two main components: an elastomer matrix and dispersed particles. The elastomer matrix is magnetically non-conductive and has the role of hosting the particles. They are magnetically conductive and both their presence and their movement inside the elastomer matrix explains the MR effect.

The steps for producing a MRE are (Coquelle, 2004): (i) preparation of the elastomer matrix, (ii) dispersion of

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Figures 1 and 3–8 appear in color online: <http://jim.sagepub.com>



**Figure 1.** (a) Tunable isolator mounted on a structure and subjected to a narrow band excitation  $w$ ; (b) tunable DVA mounted on a structure that is subjected to a broad band random excitation  $w$ .

the filler particle inside the matrix, (iii) vacuum preparation–degassing, (iv) cross-linking and curing. All the parameters of the process (curing temperature and duration, kind of agitation, nature of the mixing, and/or curing aid) depend on the base material and on the intended use (Kallio, 2005).

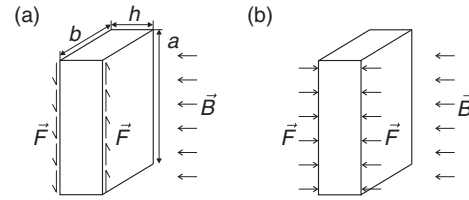
Depending on the curing conditions, two different kinds of MRE can be produced: aligned and isotropic. If a magnetic field is applied to the compound during curing and cross-linking, the particles move inside the matrix and align themselves with the magnetic lines. When the composite material is totally cured, the particle chains are trapped inside the elastomer matrix. Such *aligned* MRE have anisotropic mechanical properties, depending on the orientation of the force toward the particle chains. If no field is applied during curing, the *isotropic* MRE has the same mechanical properties in every direction.

In Lokander and Stenberg, (2002), it is shown that the nature of the matrix has no influence on the apparition of the MR effect. However, higher the stiffness of the base matrix, lower the relative increase in stiffness due to the introduction of magnetically conductive particles. The choice of the nature of the elastomer matrix also depends on the kind of environment the isolator will face. Actually, the influencing parameters on the stiffness variations of an MRE are the filler volume fraction and the average size of the filler particles (Davis, 1998).

### Applications of MRE

A comprehensive study of the MR effect and its potential applications is given by Carlson and Jolly (2000). The MR materials have been widely studied in academic and industrial institutions. All presented applications of an MRE aim at reducing vibrations, thanks to its varying properties.

MREs have potential applications in automotive domain, where it can play a part in the improvement of suspension performances (Watson, 1997; Stewart et al., 1998; Takeshita and Wakita, 2005). Different kinds of adaptive tuned vibration absorber (ATVA) have also been developed (Ginder et al., 2001; Lerner, 2005; Lerner and



**Figure 2.** Two modes of use of a MRE: (a) shear mode; (b) traction-compression mode.

Cunefare, 2005; Deng et al., 2007; Deng and Gong, 2008; Lerner and Cunefare, 2008). The MR effect is the key to the tunability of such devices.

### Loading Cases

The MRE can be used in shear mode or intraction–compression mode (Carlson and Jolly, 2000; Farshad and Benine, 2003). In the former case, the magnetic field applied to the elastomer is perpendicular to the applied force (Figure 2(a)); in the latter case, they are parallel (Figure 2(b)). Only the shear mode is considered in this article, as experimental studies have shown that it offers the highest interval of stiffness variation (Kallio, 2005).

### Model Description

The behavior of the MREs has been investigated for nearly 20 years and some studies introduced a model of that behavior (Walsh and Lamancusa, 1992; Davis, 1998; Coquelle, 2004; Shen et al., 2004; Gong et al., 2005; Kallio, 2005; Lerner, 2005; Guan et al., 2006; Stepanov et al., 2006). In the following part, the expressions that are significant for designing an isolator made of MRE are shown and explained.

Viscoelastic materials are characterized by a complex shear modulus:

$$G^* = G' + iG'', \quad (1)$$

where  $G'$  is the *storage modulus* and  $G''$  is the *loss modulus*, representative of the capacity of the material to either store and give back or dissipate the energy.

Using a simple Kelvin–Voigt model (spring and damper in parallel), the equivalent stiffness and damping properties of the material can be expressed as:

$$k = \frac{G'A}{h}; \quad \xi = \frac{G''}{2G'}, \quad (2)$$

where  $k$  and  $\xi$  are the stiffness and the damping ratio of the elastomer,  $A = ab$  and  $h$  are as depicted in Figure 2.

Dispersing some particles inside the host matrix engenders a composite material, whose modulus is greater than the stiffness of the host alone. Moreover, the modulus

further increases when the composite material is facing an external magnetic field.

In the absence of a magnetic field, the shear modulus of the MRE composite is given by (Guth, 1945):

$$G_0 = G_{\text{unfilled}}(1 + 2.5\phi + 14.1\phi^2), \quad (3)$$

where  $\phi$  is the volume fraction of the particles.

As the magnetic field increases, more and more filler particles try to align themselves with the magnetic field lines. These microscopic movements modify the intrinsic structure of the matrix and engender an increasing stiffening in the direction of the magnetic lines. The shear modulus of the MRE material is assumed to increase linearly from the zero-field state to the saturation state (Davis, 1998). Its expression is given by:

$$G = G_0 + \Delta G, \quad (4)$$

where  $\Delta G$  is the linear increase of the shear modulus due to the increase of the magnetic field. The detailed expression of  $\Delta G$  is given in (Davis, 1998; Kallio, 2005).

Experiments (Kallio, 2005) have shown that a similar behavior for the damping coefficient and the equivalent parameters of the elastomer can be rewritten as:

$$\begin{cases} k(B) = a_k + b_k B \\ \xi(B) = a_\xi + b_\xi B, \end{cases} \quad (5)$$

where  $B$  is the magnetic field and  $a_k$ ,  $b_k$ ,  $a_\xi$ ,  $b_\xi$  are constant quantities (Figure 3). The following parameters have been used in this study:  $a_k = 2000$  N/m;  $b_k = 4.55$  kN/T/m;  $a_\xi = 0.15$ ;  $b_\xi = 0.6364$   $T^{-1}$ ;  $B_s = 0.11$  T.

The magnetic saturation appears when nearly all the filler particles are magnetized. At saturation, the MR effect is at its maximum and the values of the equivalent stiffness and damping coefficients remain constant if the magnetic field increases further (Figure 3).

Producing the desired magnetic field can be difficult or impossible, due to installation contingencies (e.g., available space, allowed current). Moreover, one can make the composite material both stiffer and softer during its use. Introducing a permanent magnet inside the magnetic circuit offers a bias that could help reach the desired field values (Figure 3).

The model described above is based on assumptions made for simplification. This level of modeling is sufficient to explain the design strategies expressed in the following two sections.

## MRE ISOLATOR

The model used in this section is a single d.o.f isolator (Figure 4) subjected to narrow band harmonic excitation. It consists of a spring with a MRE at one of its end, like in some advanced car suspension systems

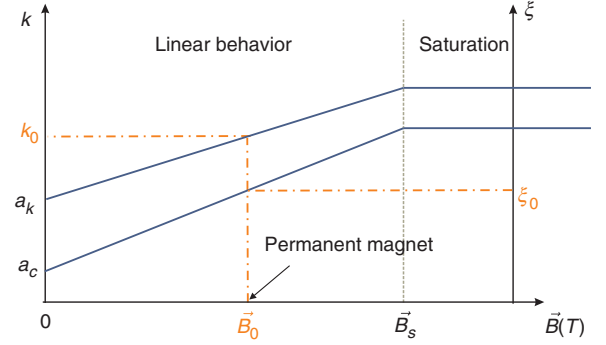


Figure 3. Equivalent stiffness and damping coefficients of a MRE as a function of the applied magnetic field.

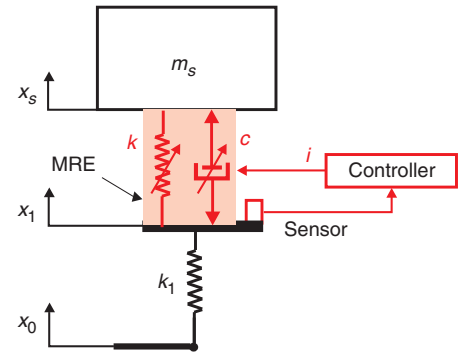


Figure 4. Model of MRE isolator.

(Watson, 1997; Stewart et al., 1998). The numerical values used in this study are  $m_s = 100$  kg and  $k_1 = 100$  kN/m. The dynamics of the system read:

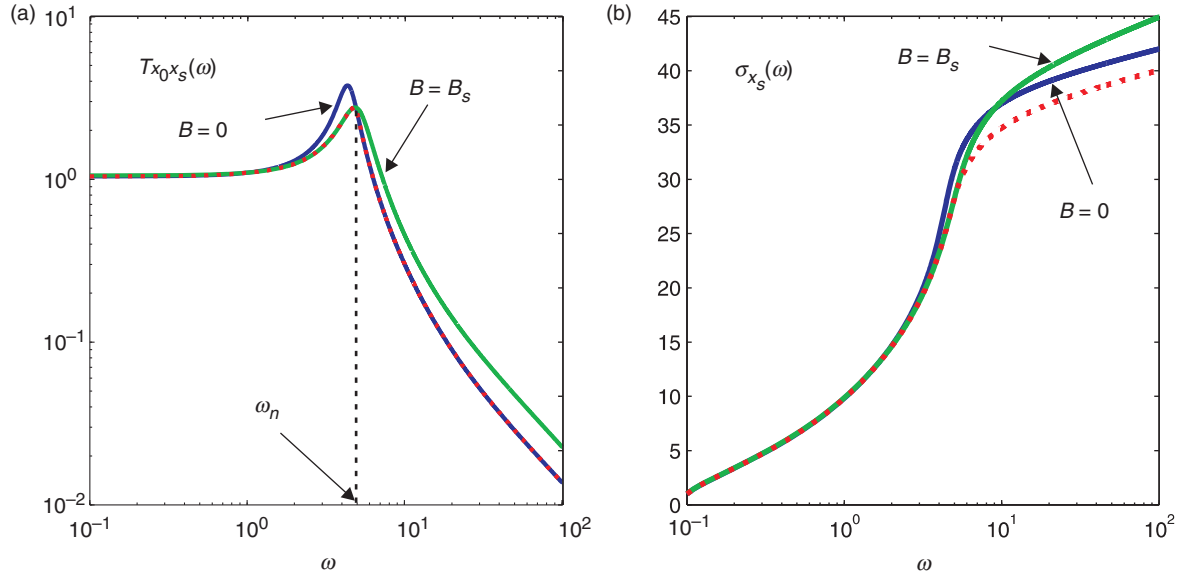
$$\begin{cases} m_s \ddot{x}_s + c(\dot{x}_s - \dot{x}_1) + k(x_s - x_1) = 0 \\ c(\dot{x}_s - \dot{x}_1) + k(x_s - x_1) = k_1(x_1 - x_0), \end{cases} \quad (6)$$

where  $m_s$  is the mass of the oscillator,  $k_1$  is the stiffness of the isolator spring,  $k$  and  $c = 2\xi\sqrt{km_s}$  the equivalent stiffness and damping coefficients of the elastomer given by Equation (5).

The isolator should be designed to keep the motion of the mass,  $m_s$ , as small as possible, whatever the excitation. This is very difficult when the frequency of the excitation is close to the natural frequency of the isolator. In the case of commendable elastomer, the stiffness of the isolator can be modified to maintain the resonance frequency of the isolator as far as possible from the frequency of the excitation.

Consider a periodic chirp excitation with a unit amplitude applied in  $x_0$ . Figure 5(a) shows the absolute value of the transmissibility  $T_{x_0 x_s}(\omega)$  of the isolator for two extreme values of the magnetic field :  $B = 0$  (blue curve) and  $B = B_s$  (green curve). Figure 5(b) shows the cumulative RMS integral of the transmissibilities defined as:

$$\sigma_{x_s}(\omega) = \left[ \int_0^\omega |T_{x_0 x_s}(v)|^2 dv \right]^{1/2}, \quad (7)$$



**Figure 5.** (a) Transmissibility  $T_{x_0 x_s}(\omega)$  and (b) RMS value  $\sigma_{x_s}(\omega)$  of a MRE isolator for two extremal values of the magnetic field:  $B = 0$  (blue curve) and  $B = B_s$  (green curve).

which describes how the various frequencies contribute to the RMS of the mass displacement.

When the frequency of the excitation is lower than  $\omega_n$ , the transmissibility obtained for  $B = B_s$  is the lowest; above this value,  $B = 0$  gives the lowest transmissibility. This suggests the following control strategy:

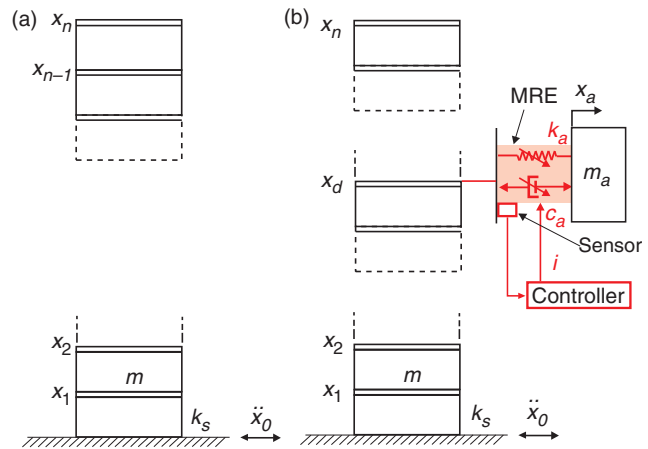
$$\begin{aligned} \text{If } \omega \leq \omega_n \text{ then } B &= B_s, \\ \text{If } \omega > \omega_n \text{ then } B &= 0. \end{aligned}$$

The control strategy is implemented and shown as a dotted red line in Figure 5. Figure 5(b) shows that the frequency shift can reduce the RMS value of the mass displacement by 10%.

The principle of controlling the behavior of the isolator by changing the magnetic field it is facing, only works for narrow-band excitation. It is well adapted for periodic chirp excitation or if the structure has two significantly different ranges of function (e.g., landing/take-off and cruise for an airplane). For random excitation, as the frequency content is spread over a wide frequency range, there is no benefit (and potentially a worsening effect) to use a commandable MRE isolator.

### MRE DYNAMIC VIBRATION ABSORBER

The principle of the DVA has been introduced in 1909 (Frahm, 1909a, b) and the first theoretical background has been developed in Den Hartog and Ormondroyd (1928). Since that time, DVAs have shown their efficiency to decrease the vibrations of various types of structures. However, as they are tuned to oscillate at a specified frequency, they become less efficient when the



**Figure 6.** (a)  $n$ -story flexible structure; (b)  $n$ -story flexible structure on which is appended a MRE DVA on story  $d$ .

frequency of the structure varies in time. The aim of this section is to investigate the ability of an MRE DVA to follow the frequency variations of the resonance of a structure.

Consider an  $n$ -story frame (Figure 6) subjected to a horizontal excitation of acceleration  $\ddot{x}_0$ . The variables of the system are the lateral displacements of each stage  $x = (x_1, \dots, x_n)^T$ . The mass and stiffness matrices are, respectively,  $M = mI_n$  and

$$K = k_S \begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & & & \\ & & \ddots & & \\ & & & 2 & -1 \\ & & & -1 & 1 \end{pmatrix}. \quad (8)$$

In order to distinguish between rigid body and flexible motion of the structure, the absolute coordinates are decomposed into:

$$x = \mathbf{1}x_0 + y_i, \quad (9)$$

where  $x_0$  is the support motion and  $\mathbf{1}$  is the unit column vector. Using these notations, the equation of motion can be expressed in relative coordinates as:

$$M\ddot{y} + C\dot{y} + Ky = -M\mathbf{1}\ddot{x}_0, \quad (10)$$

where  $C$  is the damping matrix.

Consider a DVA attached on floor  $d$  and tuned to mode  $k$  according to the equal peak design procedure described hereafter.

When the DVA is mounted on the structure, Equation (10) becomes:

$$M\ddot{y} + C\dot{y} + Ky = -M\mathbf{1}\ddot{x}_0 - D, \quad (11)$$

where  $D$  is the damper force,  $D(i) = \delta_{id}F_a$  and

$$F_a = m_a\ddot{x}_a = c_a(\dot{x}_d - \dot{x}_a) + k_a(x_d - x_a), \quad (12)$$

where  $m_a$ ,  $k_a$ , and  $c_a = 2\xi_a\sqrt{k_a m_a}$  are, respectively, the mass, stiffness, and damping coefficient of the DVA. In modal coordinates, the vector of relative coordinates is:

$$y = \Phi z, \quad (13)$$

where  $\Phi = (\phi_1, \phi_2, \dots, \phi_n)$  is the matrix of mode shapes of the structure. The modes satisfy the orthogonality conditions:

$$\Phi^T M \Phi = \text{diag}(\mu_i); \quad \Phi^T K \Phi = \text{diag}(\mu_i \omega_i^2), \quad (14)$$

where  $\mu_i$  and  $\omega_i$  are, respectively, the modal mass and the natural frequency of mode  $i$ . Assuming a modal damping ratio  $\xi_i$  defined by:

$$\Phi^T C \Phi = \text{diag}(2\xi_i \mu_i \omega_i). \quad (15)$$

The equation of motion is transformed into a set of decoupled equations:

$$\mu_i \ddot{z}_i + 2\xi_i \mu_i \omega_i \dot{z}_i + \mu_i \omega_i^2 z_i = -\Gamma_i \ddot{x}_0 - \Delta_i \quad (16)$$

where  $\Gamma_i = \phi_i^T M \mathbf{1}$  is the modal participation factor of mode  $i$  and  $\Delta_i = \phi_i^T D$ . Assuming that, in the vicinity of  $\omega_k$ , the response of the structure at the location of the DVA is dominated by mode  $k$ , the displacement can be approximated as:

$$y_d \simeq \phi_k(d) z_k, \quad (17)$$

reducing the system (16) at a single equation. Then, optimum values of the stiffness and damping coefficient of the DVA are found by reducing the two peaks appearing on the transmissibility  $T_{fdyd}$  between a force applied on d.o.f.  $d$  and the displacement of the structure at this point as much as possible (Den Hartog and Ormondroyd, 1928). It gives:

$$k_a = m_a \left( \frac{\omega_k}{1 + \lambda} \right)^2, \quad (18)$$

and:

$$\xi_a = \sqrt{\frac{3\lambda}{8(1 + \lambda)}}, \quad (19)$$

where:

$$\lambda = \frac{m_a}{\mu_k} \phi_k^2(d). \quad (20)$$

In such a structure subjected to a seismic excitation, a relevant quantity is the shear force at the base, given by:

$$F_0 = k_s(x_1 - x_0). \quad (21)$$

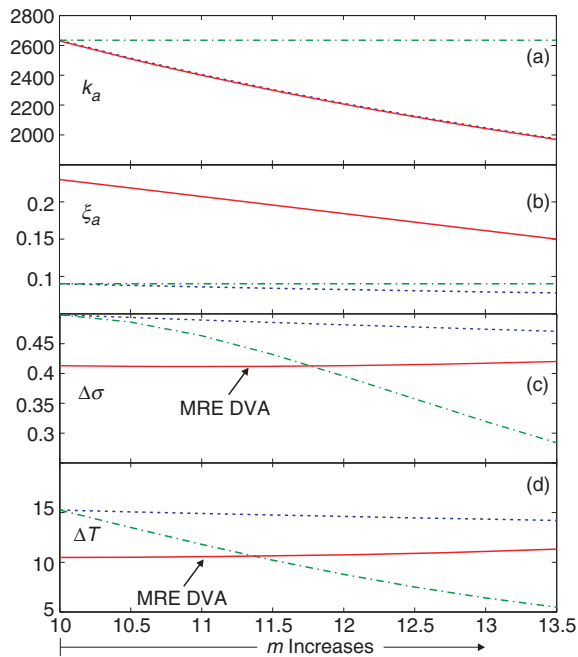
In the numerical example,  $m = 10$  kg,  $k_s = 750$  kN/m,  $d = 7$ ,  $m_a = 1\%$  of the total mass ( $m_T = 7m$  in this case) and  $k = 1$ .

Using Equations (18) and (19), the optimal values of the stiffness and damping coefficient of the DVA are  $k_a = 2620$  N/m and  $\xi_a = 0.09$ . Unfortunately, for an actual material, it is not possible to choose independently the stiffness and damping coefficient. Choosing a stiffness of  $k_a = 2620$  N/m gives automatically a damping of  $\xi_a = 0.23$  (Figure 3, data from (Kallio, 2005)). As this value is greater than the optimal value, the DVA is over-damped, providing a lower efficiency than the optimal one.

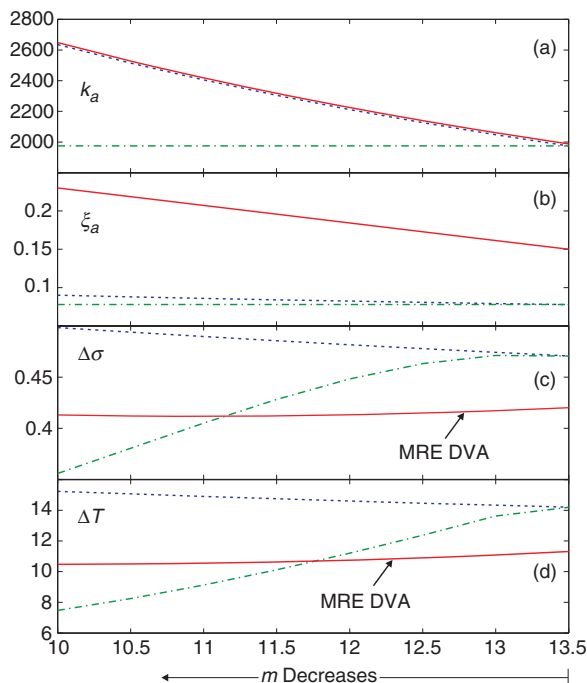
Now consider that the mass of each story of the structure increases from  $m = 10$  kg to  $m = 13.5$  kg. Three cases are compared: (i) a DVA with constant properties, optimal for  $m = 10$  kg; (ii) a fictive DVA that would have optimal values for the stiffness and damping coefficient; (iii) an MRE DVA. Figure 7(a)–(d) compare, respectively, the stiffness and damping of the DVA, the reduction ( $\Delta\sigma$ ) of the RMS value of the shear force at the base of the frame:

$$\sigma_{F_0} = \left[ \int_0^\infty |T_{x_0 F_0}(\omega)|^2 d\omega \right]^{1/2}, \quad (22)$$

and the reduction ( $\Delta T$ ) of the maximum value of the transmissibility  $|T_{x_0 F_0}|$  calculated in dB.



**Figure 7.** Comparison between three DVAs appended on an increasing mass structure: DVA with constant properties (dashed dotted line), DVA with optimally tunable properties (dotted line), and MRE DVA (solid line). (a) Stiffness of the DVA; (b) damping coefficient of the DVA; (c) reduction of the RMS value of the shear force at the base of the structure; (d) reduction of the maximum amplitude of the shear force to excitation transmissibility  $|T_{x,F_0}|$ .



**Figure 8.** Comparison between three DVAs appended on a decreasing mass structure: DVA with constant properties (dashed dotted line), DVA with optimally tunable properties (dotted line), and MRE DVA (solid line). (a) Stiffness of the DVA; (b) damping coefficient of the DVA; (c) reduction of the RMS value of the shear force at the base of the structure; (d) reduction of the maximum amplitude of the shear force to excitation transmissibility  $|T_{x,F_0}|$ .

When the mass of each story is 13.5 kg, the performances of the three DVAs are as follows. The fictive DVA maintains  $\Delta\sigma$  above 45% and  $\Delta T=15$  dB. The DVA with constant properties is not tuned to the first resonance anymore, and  $\Delta T$  and  $\Delta\sigma$  decrease rapidly to 5 dB and 28% respectively. The MRE DVA, while not optimal for a structure with constant properties, maintains  $\Delta\sigma$  at 41% and  $\Delta T=11$  dB, which is about 50% better than the DVA with constant properties. In other words, using an MRE DVA instead of a classical DVA for multi-supported mass-varying structures can reduce the stress close to the supporting points, and accordingly delay some damage mechanisms like the emergence of cracks and fatigue. Similar results are presented in Figure 8 when the mass of each storey decreases from  $m=13.5$  kg to  $m=10$  kg.

## CONCLUSIONS

In this article, the isolation and damping properties of the MRE have been investigated separately, using measured mechanical properties. Firstly, a single isolator, made out of MRE, was subjected to a narrow band excitation. It has been shown that the MRE isolator can decrease the RMS value of the body displacement by 10%. Secondly, an n-story mass-varying structure has been considered, on which is appended an MRE DVA. It has been shown that a proper application of the magnetic field to the MRE can maintain a good efficiency compared to a DVA with constant properties. In the example considered, an MRE DVA, while not optimal, can reduce the stress in the structure about 50% better than a classical DVA when the mass of the structure changes 35%. Then the adaptability of the MRE DVA is found to be particularly suited for multi-supported structures whose mass and/or stiffness is varying in a fixed range of values in order to avoid high stress close to the supporting points, and delay some damage mechanisms like the emergence of cracks and fatigue.

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