both sides of (4.1) to produce

$$\left[\begin{array}{c} \dot{x}^1 - f^1(x^1, a^1, r^1_{\text{nom}}) \\ b^1 - g^1(x^1, a^1, r^1_{\text{nom}}) \\ \hline \begin{bmatrix} -I & L^{11}_{11} & 0 & L^{12}_{11} \\ 0 & L^{21}_{11} & -I & L^{22}_{12} \\ 0 & (L^2_{21})^{-L} L^1_{21} & 0 & I \end{bmatrix} \begin{bmatrix} a^1 \\ b^1 \\ a^2 \\ b^2 \end{bmatrix} \end{array}\right] = \left[\begin{array}{c} \theta \\ \theta \\ \hline -L^1_{12}u \\ -L^2_{12}u \\ (L^2_{21})^{-L}[y^m - L_{22}u] \end{array}\right]. \quad (4.2)$$

Clearly there is an explicit expression for b^2 in terms of a^1 and b^1 which coincides with the equations of the pseudo circuit approach. Further reductions produce a similar expression for a^2 which produces a set of equations identical to those of the pseudo circuit approach.

A final point addresses the ability to solve the fault diagnosis equations at each iteration of the procedure. In the case of the pseudo circuit approach this depends on the left invertibility of L^2_{21}—i.e., the relative dimensions and maximal column rank of L^2_{21}. In the case of the Pseudo Nominal Tableau equations, computability of (a^2, b^2) depends on the relative dimensions of L^2_{21} and the full column rank of the Jacobian of (4.1)—i.e., the left invertibility of the Jacobian:

$$\begin{bmatrix} d_0 I - \dfrac{\partial f^1}{\partial x^1} & -\dfrac{\partial f^1}{\partial a^1} & 0 & 0 & 0 \\ -\dfrac{\partial g^1}{\partial x^1} & -\dfrac{\partial g^1}{\partial a^1} & I & 0 & 0 \\ 0 & -I & L^{11}_{11} & 0 & L^{12}_{11} \\ 0 & 0 & L^{21}_{11} & -I & L^{22}_{11} \\ 0 & 0 & L^1_{21} & 0 & L^2_{21} \end{bmatrix}. \quad (4.3)$$

A sufficient but not necessary condition for the maximal rank of this matrix is the left invertibility of L^2_{21}. Hence flexibility in choosing Group one and Group two partitions is greater in the tableau formulation. To see this consider the circuit of Fig. 1 whose CCM can be written as

Component Equations

$$\dot{x}_1 = \frac{1}{C_3} a_3 \qquad b_1 = \frac{1}{R_1} a_1 \qquad b_2 = R_2 a_2$$

$$b_3 = x_1 \qquad b_4 = a_4 \left(a_4 - \frac{1}{R_4}\right)^2.$$

Connection Equations

$$\begin{bmatrix} a_1 \\ a_4 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_4 \\ b_2 \\ b_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_1$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_4 \\ b_2 \\ b_3 \end{bmatrix}.$$

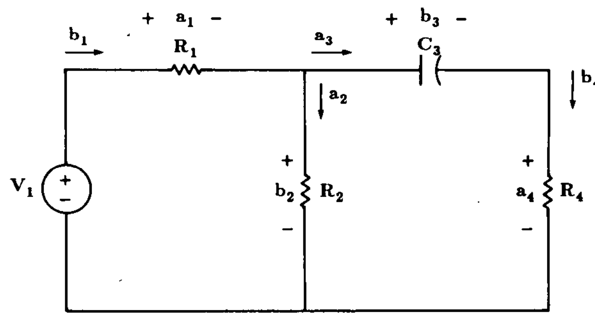

Fig. 1.

Observe that if components 2 and 3 are in Group 2, then

$$L^2_{21} = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}$$

and $[L^2_{21}]^{-L}$ do not exist. However, the associated Jacobian (equation (4.3)) has full rank where the Jacobian takes the from

$$\begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & q(a_4) & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

for appropriate $q(\cdot)$ where all parameter values are given a nominal value of unity.

As a final point, we note that (4.2) indicates that the component equations are rearranged to conform to the Group 1/Group 2 partitioning. This particular form arises for expository reasons and is clearly unnecessary. It is simply necessary to delete the Group 2 components from the usual tableau (lexiographically ordered) and solve for the x^i's, b^i's, and a^i's. Hence the part of the Jacobian dependent as the connection information remains invariant for the various partitions into Group 1 and Group 2 components.

Acknowledgment

The authors would like to acknowledge Professor R. Saeks of Arizona State University for his helpful and valuable inputs during the course of this research.

References

[1] C. C. Wu, K. Nakajima, C. L. Wey, and R. Saeks, "Analog fault diagnosis with failure bounds," *IEEE Trans. Circuit Syst.*, vol. CAS-29, pp. 277–284, May 1982.

[2] R. DeCarlo and R. Saeks, *Interconnected Dynamical Systems*. New York: Marcel Dekker, 1981.

[3] L. Rapisarda and R. DeCarlo, "Fault diagnosis of nonlinear analog circuits," vol. II, Tech. Rep. TR-EE 82-22, Purdue University, W. Lafayette, IN, July 1982.

On Maximally-Flat Linear-Phase FIR Filters

P. P. VAIDYANATHAN

Abstract—An implementation for maximally-flat FIR filters is proposed that requires a much smaller number of multiplications than a direct form structure. The values of the multiplier coefficients in the implementation are conveniently small, and do not span a huge dynamic range, unlike in a direct form implementation.

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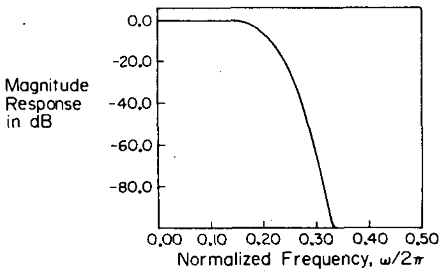

Fig. 1. FIR maximally-flat filter with $K=17$, $L=9$.

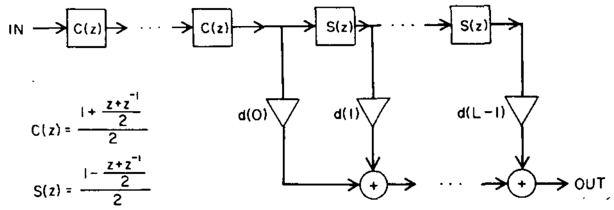

Fig. 2. (L-1)-multiplier implementation.

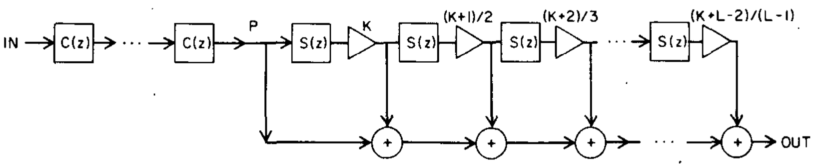

Fig. 3. An improved (L-1)-multiplier implementation.

Digital FIR filters with a maximally-flat frequency response at frequencies $\omega=0$ and π have been studied by Herrmann [1] and Kaiser [2]. As pointed out by Kaiser [2], the order of these filters varies approximately as the square of the reciprocal of the transition bandwidth, and consequently, the number of digital multipliers required for a direct form implementation is much larger than for a standard equiripple design. In addition, the values of the impulse response coefficients span a huge dynamic range. However, unlike equiripple designs, maximally-flat FIR filters have the advantage of having a monotone frequency response, which is desirable in certain applications. (For example, an attenuation exceeding 100 dB almost everywhere in the stopband can be easily achieved.) In addition, the design of maximally-flat transfer functions is very simple, essentially based on closed form formulas, and elegant programs are available for this [2].

In view of these advantages of maximally-flat FIR filters, it is, therefore, of interest to develop efficient implementations for these filters, i.e., requiring the smallest number of multipliers. In this correspondence we indicate a method for achieving this.

A low-pass maximally-flat FIR zero-phase filter has a frequency response of the form:

$$H(e^{j\omega})=\left(\cos\frac{\omega}{2}\right)^{2K}\sum_{n=0}^{L-1}d(n)\left(\sin\frac{\omega}{2}\right)^{2n} \tag{1}$$

where the coefficients $d(n)$ are given by

$$d(n)=\frac{(K-1+n)!}{(K-1)!n!}. \tag{2}$$

The filter order is $N=2(K+L-1)$, where K and L denote the degrees of tangency at $\omega=\pi$ and $\omega=0$, respectively. (The first $2K-1$ derivatives of $H(e^{j\omega})$ are zero at $\omega=\pi$, and the first $2L-1$ derivatives are zero at $\omega=0$). As an example, the frequency response of a maximally-flat low-pass FIR filter with $K=17$ and $L=9$ is shown in Fig. 1. The transition bandwidth (95 to 5 percent width) is 0.2π, whereas the center of transition band is at 0.4π.

In order to obtain a direct form implementation, one computes the impulse response, $h(n)$, by doing a $(N+1)$ point inverse DFT on $H(e^{j\omega})$ where $\omega=2\pi n/(N+1)$. In view of the symmetry of the impulse response, the direct form requires $K+L$ multipliers. Instead, we can try to implement the filter in the following form:

$$H(z)=\left(\frac{1+\frac{z+z^{-1}}{2}}{2}\right)^K\sum_{n=0}^{L-1}d(n)\left(\frac{1-\frac{z+z^{-1}}{2}}{2}\right)^n \tag{3}$$

which is shown in Fig. 2, requiring only $L-1$ multipliers ($d(0)$ is always unity). The building blocks $C(z)$ and $S(z)$ in the figure are multiplierless. The disadvantage of the circuit in Fig. 2 is that the multipliers $d(n)$ tend to be very large, because of the form in (2). In fact, $d(n)$ grows very fast as n increases. For example, with $K=17$, we have

$d(0)=1,\quad d(1)=17,\quad d(2)=153,\quad d(3)=969,\quad d(4)=4845,$
$d(5)=20{,}349,\quad d(6)=74{,}613,\quad d(7)=24{,}5157,$ and so on.

A different implementation can be obtained by observing that $d(n)$ can be written as

$$d(n)=K\cdot\frac{K+1}{2}\cdot\frac{K+2}{3}\cdot\dots\cdot\frac{K+n-1}{n}. \tag{4}$$

This leads to the implementation in Fig. 3, requiring multipliers whose values are conveniently small. Thus for $K=17$, the multiplier values are

$$17,9,6.33,5,4.2,3.66,\cdots$$

and so on, and there are precisely $L-1$ multipliers in the resulting structure. Thus, the frequency response of Fig. 1 can be achieved with only 8 multipliers, as against 26 multipliers required by the direct form.

If $H(z)$ is such that K is less than L, then one can first design a maximally-flat low-pass filter $G(z)$ such that

$$G(e^{j\omega})=\left(\cos\frac{\omega}{2}\right)^{2L}\sum_{n=0}^{K-1}d(n)\left(\sin\frac{\omega}{2}\right)^{2n} \tag{5}$$

and then obtain $H(z)$ as

$$H(z)=1-G(-z) \tag{6}$$

(assuming zero phase), requiring only $K-1$ (rather than $L-1$) multipliers.

The building blocks $C(z)$ tend to have very small gain in the filter stopband, whereas the building blocks $S(z)$ have a small gain in the filter passband, and gain approaching unity in the stopband. Thus stopband signals which are at a "noise level" at the point P in Fig. 3, get amplified because of the multiplier-cascade following point P. This seems to be a disadvantage of the circuit, and we feel that this can be overcome by using sufficient precision for internal signals in the cascade of $C(z)$. However, a careful analysis of noise and dynamic range interactions is necessary in order to understand the circuit further, and this is currently under study.

ACKNOWLEDGMENT

The author wishes to thank Dr. J. F. Kaiser of the Bell Laboratories for useful discussions.

REFERENCES

[1] O. Herrmann, "On the approximation problem in nonrecursive digital Filter design," *IEEE Trans. Circuit Theory*, vol. CT-18, pp. 411–413, May 1971.

[2] J. F. Kaiser, "Design subroutine (MXFLAT) for symmetric FIR lowpass digital filters, with maximally-flat pass and stopbands," in *Programs for Digital Signal Processing*, IEEE Press, 1979.

Corrections to "Composite Amplifier Structures for Use in Active *RC* Biquads"

F. W. STEPHENSON

In the above paper,[1] the following correction should be made: Equation (1b) should read

$$A(s) = \frac{\omega_t^2}{s^2(1+\alpha)}. \tag{1b}$$

Manuscript received June 2, 1984.

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[1] *IEEE Trans. Circuits Syst.*, vol. CAS-31, pp. 420–423, Apr. 1984.