ORIGINAL ARTICLE

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On mechanical behavior of traditional timber shear wall in Taiwan II: simplified calculation and experimental verification

Abstract In the previous report of this ongoing study, results of an extensive field survey were collated and a theoretical model was proposed to predict the mechanical behavior of timber shear walls of traditional design in Taiwan. The initial objective of the present report was to propose a simplified calculation method for estimating the initial stiffness and yield strength of traditional timber shear walls. Based on the results of the field survey, a total of 15 full-scale specimens were tested to verify the theoretical model and simplified calculation proposed previously. Good agreement was found from comparison of analytical and experimental results. The results of this study show that the friction behavior between board units and beams plays the major role in resisting the lateral force applied on the timber shear wall, followed by the resistance supplied by embedment. The resistance provided by bamboo nails is minor due to the small section. Another trend found was that for set dimensions of a timber shear wall, the board width can be increased to obtain higher stiffness and strength of the shear wall.

Key words Timber shear wall · Initial stiffness · Experiments

Introduction

For the purpose of structural evaluation of historic timber structures, it is important to understand the behavior of structural elements subjected to external force. Because shear walls usually play an important role in withstanding the lateral force in the structure, it is important to improve

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knowledge of the mechanical performance of these shear walls. In our previous report,[1] we developed a theoretical model to predict the load–displacement relation of traditional timber shear walls in Taiwan, based upon the extensive field investigation. The objectives of this study were not only to simplify the calculation method for estimation of initial stiffness and yield strength, but also to examine the validity of the theoretical model by experiments. The factors considered in this study include the material properties of top and bottom beams, and dimensions of both timber shear walls and exterior frames.

Simplified calculation

The initial stiffness

The theoretical model proposed in our previous report[1] can help to predict the load–displacement curve of traditional timber shear walls in Taiwan. However, in many cases, the stiffness and yield strength of shear walls are of interest. As described in the previous report,[1] the moment resistance of an entire timber shear wall is made up of contributions from embedment, friction, and dowel action of bamboo nails. In the other words, the total resistance at the elastic stage should be

$$M(\theta) = n_{\mathrm{u}} \cdot \left(M_{\mathrm{E}} + M_{\mathrm{F}} + M_{\mathrm{B}} \right) \tag{1}$$

$$M(\theta) = n_{\mathrm{u}} \cdot \left(F_{\mathrm{E,t}} \cdot L_{\mathrm{E}} + \mu \cdot F_{\mathrm{E,t}} \cdot H_{\mathrm{b}} + n_{b} \cdot W_{\mathrm{b}}^{2} \cdot k_{\mathrm{b}} \cdot \tan\theta \right) \tag{2}$$

where n_{u} is the number of board units in the timber shear wall, M_{E}, M_{F}, and M_{B} are moment resistance induced by embedment, friction, and bamboo nails, respectively, $F_{\mathrm{E,t}}$ is the resultant force due to embedment at top beam at elastic stage, L_{E} is the lever arm for embedment, μ is the friction coefficient, H_{b} is the height of the board unit, n_{b} is the number of bamboo nails in the board unit, W_{b} is the width of the board unit, and k_{b} is the dowel stiffness of bamboo nails.

At the elastic stage, the resultant force due to embedment at the top beam can be expressed as

$$F_{E,t} = \frac{lc_t^2 \cdot T_b \cdot E_{\perp,t}}{2 \cdot Bd_t} \cdot \alpha(\theta) \cdot \sin\theta \cdot \cos^2\theta \tag{3}$$

where lc_t is the compression length at the top beam (m), T_b is the thickness of the board unit (m), $E_{\perp,t}$ is the modulus of elasticity (MOE) perpendicular to grain (N/m²), and Bd_t is the depth of the top beam (m). $\alpha(\omega)$ is the compensating factor for calculating the MOE with inclined grain, derived from Hankinson's formula under the assumption that the MOE parallel to grain is 20 times that perpendicular to grain. Setting the constant in Hankinson's formula to $n = 3.1$ for Chinese cedar,[2] the relation becomes

$$\alpha(\theta) = \frac{20}{20 \cdot \cos^{3.1}\theta + \sin^{3.1}\theta} \tag{4}$$

When the timber shear wall is subjected to lateral force, both the exterior frame and the board units will rotate. Embedment occurs at both the top and bottom beams. The total embedment, Δ, should be equal to the difference between the vertical components of the side column and the board unit. We can obtain Δ by

$$\Delta = \sqrt{H_b^2 + W_b^2} \cdot \sin(\phi + \theta) - H_b \cdot \cos\theta \tag{5}$$

where $\phi = \tan^{-1}\dfrac{H_b}{W_b}$.

The total embedment can be further calculated as

$$\begin{aligned}\Delta &= \sqrt{H_b^2 + W_b^2} \cdot (\sin\phi \cdot \cos\theta + \cos\phi \cdot \sin\theta) - H_b \cdot \cos\theta \\ &= W_b \cdot \sin\theta\end{aligned} \tag{6}$$

By substituting the total embedment into compression length, the compression length at the top beam for a certain rotation can be expressed as

$$\begin{aligned}lc_t &= \frac{\sqrt{\dfrac{Bd_t \cdot E_{\perp,b}}{Bd_b \cdot E_{\perp,t}}} \cdot \Delta}{\left(1 + \sqrt{\dfrac{Bd_t \cdot E_{\perp,b}}{Bd_b \cdot E_{\perp,t}}}\right) \cdot \sin\theta \cdot \cos\theta} \\ &= \frac{\xi \cdot \Delta}{(1 + \xi) \cdot \sin\theta \cdot \cos\theta} \\ &= \frac{\xi \cdot W_b}{(1 + \xi) \cdot \cos\theta}\end{aligned} \tag{7}$$

in which ξ is the adjustment coefficient for different beam depths and MOE of the top and bottom beams.

Substituting compression length, Eq. 7, into Eq. 3, the resultant force at the top beam becomes

$$\begin{aligned}F_{E,t} &= \frac{T_b \cdot E_{\perp,t}}{2 \cdot Bd_t} \cdot \left(\frac{\xi \cdot W_b}{(1 + \xi) \cdot \cos\theta}\right) \cdot \alpha(\theta) \cdot \sin\theta \cdot \cos^2\theta \\ &= \frac{T_b \cdot E_{\perp,t} \cdot \xi^2 \cdot W_b^2}{2 \cdot Bd_t \cdot (1 + \xi)^2} \cdot \alpha(\theta) \cdot \sin\theta\end{aligned} \tag{8}$$
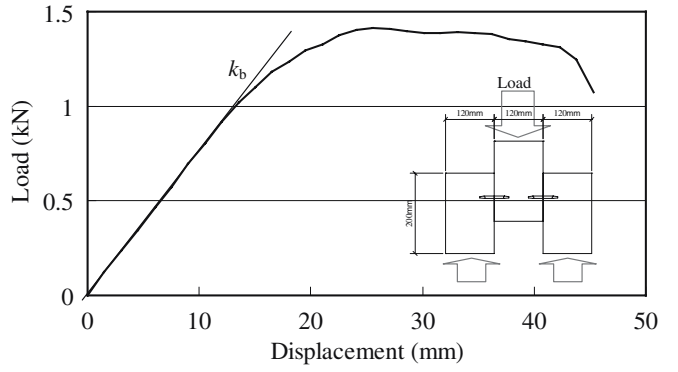

Fig. 1. Load–displacement relation of bamboo nail

The dowel strength data for bamboo nails obtained from the double shear test are illustrated in Fig. 1. From the experiments, the dowel stiffness of bamboo nail is approximately 3.78×10^4 N/m. As described in our previous report,[1] only few literature studies[3–5] have focused on the sliding friction of wood–wood surfaces; the value of 0.5 is used as the friction coefficient in this study. By considering the resultant force in the elastic stage, stiffness of bamboo nails, and friction coefficient, Eq. 2 becomes

$$\begin{aligned}M(\theta) &= \frac{n_u \cdot T_b \cdot E_{\perp,t} \cdot \xi^2 \cdot W_b^2}{2 \cdot Bd_t \cdot (1 + \xi)^2} \cdot \left(\frac{2}{3} \cdot W_b + \frac{1}{2} \cdot H_b\right)\alpha(\theta) \cdot \sin(\theta) \\ &\quad + 3.78 \times 10^4 \cdot n_u \cdot n_b \cdot W_b^2 \cdot \tan\theta\end{aligned} \tag{9}$$

At the early stage, i.e., with very small rotation, the parameter $\alpha(\omega) \doteqdot 1$, we can simplify Eq. 9 as

$$\begin{aligned}M(\theta) \cong M'(\theta) &= \left(\frac{2}{3} \cdot W_b + \frac{1}{2} \cdot H_b\right) \cdot \frac{n_u \cdot T_b \cdot E_{\perp,t} \cdot \xi^2 \cdot W_b^2}{2 \cdot Bd_t \cdot (1 + \xi)^2} \cdot \sin\theta \\ &\quad + 3.78 \times 10^4 \cdot n_u \cdot n_b \cdot W_b^2 \cdot \tan\theta\end{aligned}$$
$$\tag{10}$$

Differentiation can be used to obtain the tangential stiffness at an early stage, and the stiffness becomes

$$\begin{aligned}K_i = \frac{d}{d\theta}M'(\theta) &= \left(\frac{1}{3} \cdot W_b + \frac{1}{4} \cdot H_b\right) \cdot \frac{n_u \cdot T_b \cdot E_{\perp,t} \cdot \xi^2 \cdot W_b^2}{Bd_t \cdot (1 + \xi)^2} \cdot \cos\theta \\ &\quad + 3.78 \times 10^4 \cdot n_u \cdot n_b \cdot W_b^2 \cdot \sec^2\theta\end{aligned}$$
$$\tag{11}$$

Note that Eq. 11 is valid only for the elastic stage.

Yield strength

Figure 2 illustrates the embedded distribution at the beam element. At the yield strength, we can get yield strain by

$$\begin{aligned}\varepsilon_y &= \frac{lc_t \cdot \sin\theta_y \cdot \cos\theta_y}{Bd_t} \\ &= \frac{\xi \cdot W_b \cdot \sin\theta_y}{(1 + \xi) \cdot Bd_t}\end{aligned} \tag{12}$$

where θ_y is the rotation when yield occurs at the top beam.

$$\therefore \theta_y = \sin^{-1}\left(\frac{(1+\xi)\cdot Bd_t \cdot \varepsilon_y}{\xi \cdot W_b}\right) \qquad (13)$$

The yield strength can be gained by substituting the rotation at the yield point, θ_y, into Eq. 9:

$$M_{yield} = M(\theta)\big|_{\theta=\theta_y} \qquad (14)$$

Experimental

A total of fifteen specimens were tested in this study to verify the theoretical models proposed in previous work and this study. The variables discussed include height, width, and number of board units in a timber shear wall, and beam depth and moduli of elasticity of top and bottom beams. The details of the specimens are given in Table 1.
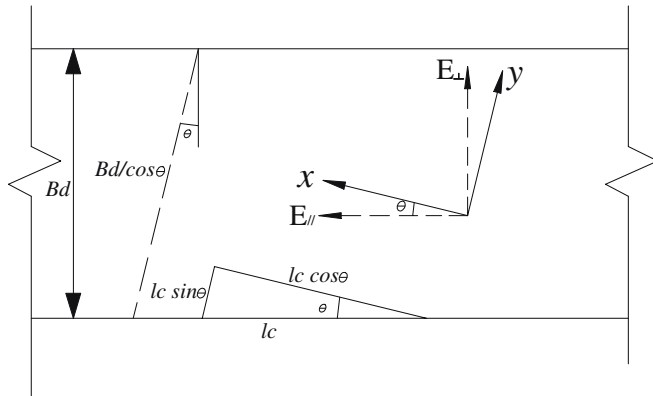
The experiment consisted of five series according to geometries of the timber shear wall and the board unit; however, the material properties and geometry conditions of the top and bottom beams were varied within the series. The top and bottom beams of first specimens in each series had similar moduli of elasticity perpendicular to grain $(E_\perp)$ and similar depth, whereas these two terms were allowed to change in the following two specimens.

The board units in the specimens were connected by bamboo (*Bambusa stenostachya.*) nails that were 4×4 mm in cross section and 75 mm long. For specimens with height over 800 mm, three bamboo nails were used; otherwise, only two bamboo nails were used to connect the board units. Board units and exterior frames were made of Chinese cedar (*Cunninghamia lanceolata*). The thickness of the board unit was 20 mm and the width of the beams was 60 mm for all the specimens, which takes the results of field survey into consideration. All specimens were fabricated by carpenters according to traditional means on a construction site during renovation of an historic timber structure in Taiwan and then moved to the laboratory for testing. The specimens were stored in the laboratory with good natural ventilation for about 1 month before testing, and moisture content was controlled in the range of 18%–19% at testing.

To omit the effect of joint stiffness of the exterior frame, pin connections were used to connect the beams and columns. The detail of the specimen is shown in Fig. 3, and the experimental setup is illustrated in Fig. 4. As illustrated in Fig. 3, the groove depths are 10 mm in both beams and columns. The gap between columns and the end board unit is about 2 mm. Monotonic load was applied by displacement control with the rotational speed of 5×10^{-3} rad/min. The experiments were terminated when the maximum stroke of the hydraulic jack was reached. The moment and story drift of the wall are defined as:

Fig. 2. Strain distribution due to embedment

Table 1. Details of the specimens

Experiment	Top beam			Bottom beam			Board height (mm)	Board width (mm)	No. of boards
	Beam depth (mm)	$E_{//}$[a] (Gpa)	$E_\perp$[b] (Gpa)	Beam depth (mm)	$E_{//}$ (Gpa)	$E_\perp$ (Gpa)			
A-1	150	6.59	0.27	150	6.25	0.28	900	200	3
A-2	150	6.33	0.23	150	5.98	0.19	900	200	3
A-3	120	5.13	0.21	150	6.91	0.18	900	200	3
B-1	150	7.69	0.20	150	5.54	0.21	700	100	6
B-2	150	9.04	0.33	150	8.58	0.43	700	100	6
B-3	150	4.67	0.21	180	6.04	0.29	700	100	6
C-1	180	6.65	0.22	180	6.79	0.23	700	200	2
C-2	150	4.88	0.18	180	9.48	0.23	700	200	2
C-3	150	10.16	0.44	180	7.79	0.34	700	200	2
D-1	180	6.19	0.22	180	6.36	0.22	900	100	4
D-2	150	7.89	0.33	150	7.03	0.29	900	100	4
D-3	150	6.25	0.22	150	7.83	0.19	900	100	4
E-1	150	5.49	0.27	150	6.28	0.27	900	150	4
E-2	180	5.98	0.19	180	7.24	0.29	900	150	4
E-3	150	4.69	0.22	180	5.07	0.25	900	150	4

$E_{//}$, Modulus of elasticity parallel to the grain; $E_\perp$, modulus of elasticity perpendicular to the grain
[a] Obtained from four-point bending tests
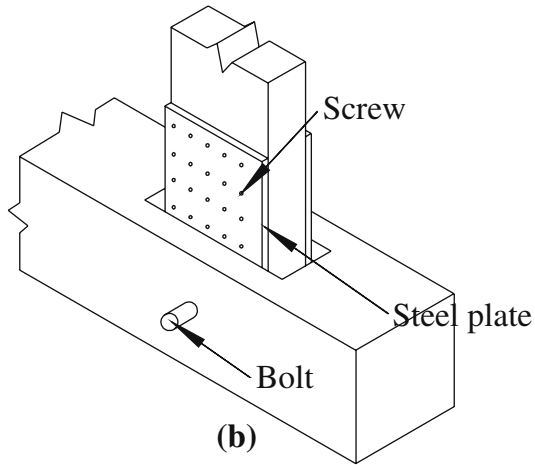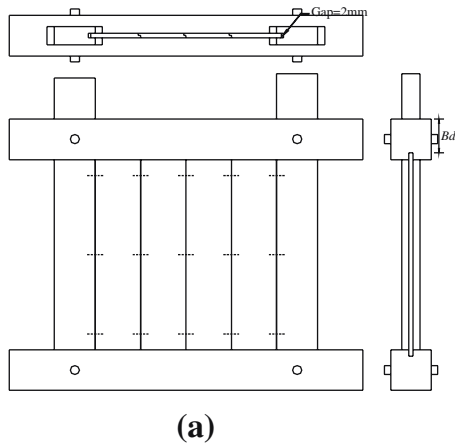[b] Obtained from compressive test

(a)

(b)

Fig. 3. Detail of **a** the specimen tested and **b** the pin connection and its reinforcement
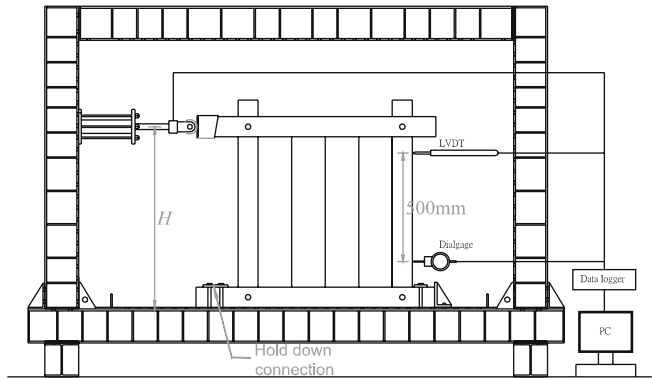

Fig. 4. Experimental apparatus

$$M = P \times H \tag{15}$$

$$\theta = \frac{\delta_1 - \delta_2}{500} \tag{16}$$

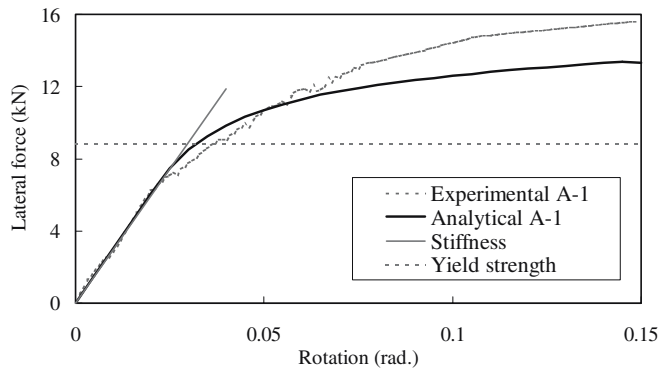where P is the applied load (N), and δ_1 and δ_2 are relative displacement (mm).

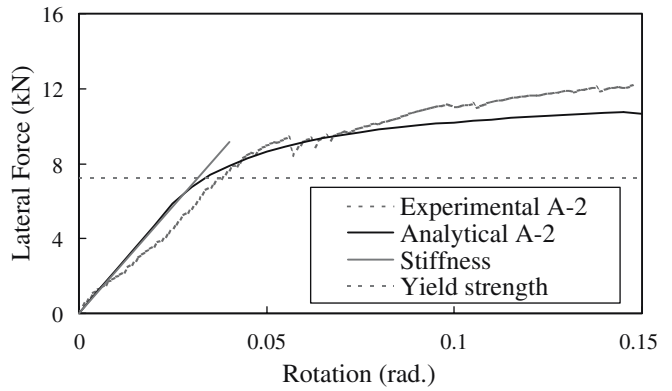**Fig. 5.** Comparison of results for specimen A-1
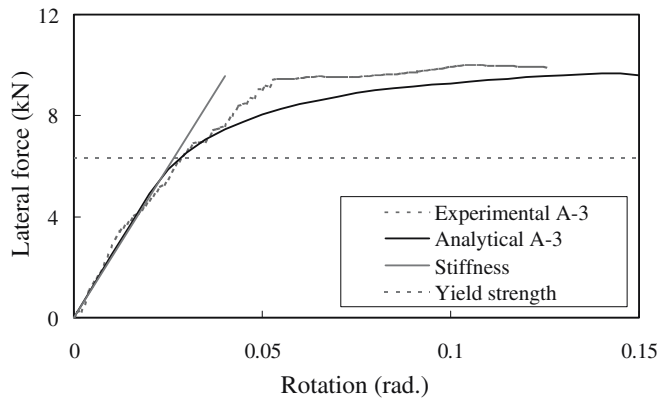

Fig. 6. Comparison of results for specimen A-2

Fig. 7. Comparison of results for specimen A-3

Results and discussion

Verification

The proposed models considered the moment resistance of a timber shear wall contributed from the embedment, friction, and bamboo nail. Comparisons between experimental results, analytical results proposed in a previous report, the initial stiffness at a rotation of 1/150 rad, and yield strength proposed in this study, are shown in Figs. 5–19. Figures 5–19 demonstrate that the proposed model in the previous report
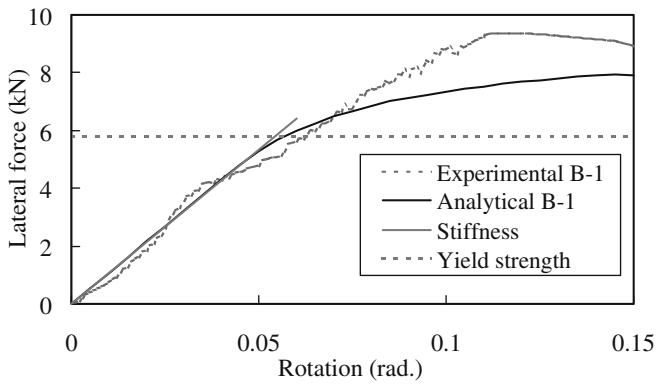
Fig. 8. Comparison of results for specimen B-1
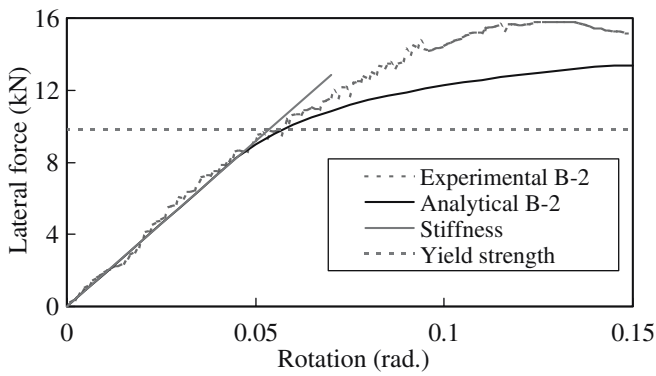

Fig. 9. Comparison of results for specimen B-2

Fig. 10. Comparison of results for specimen B-3

Fig. 11. Comparison of results for specimen C-1
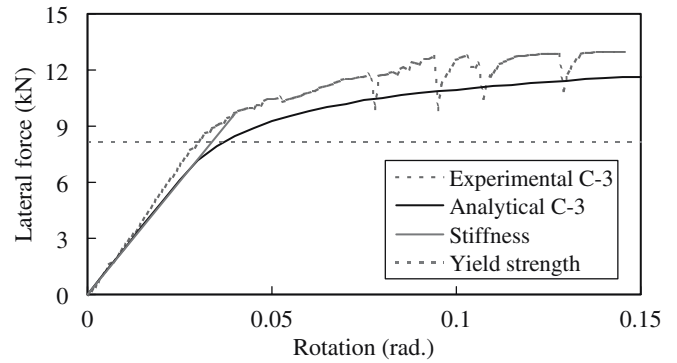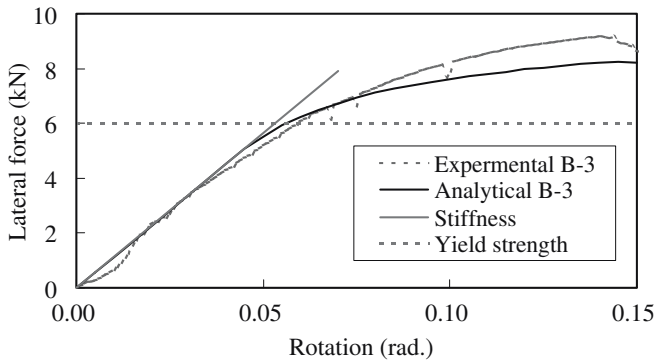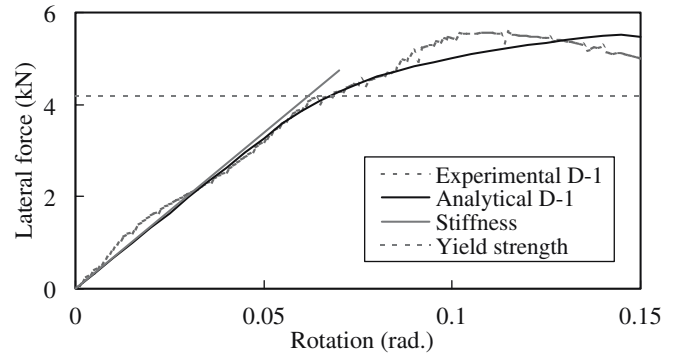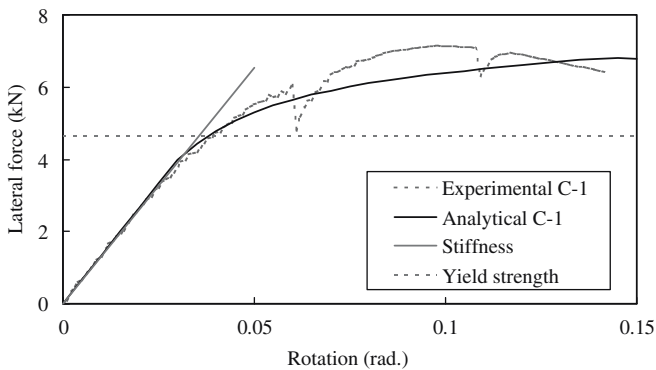

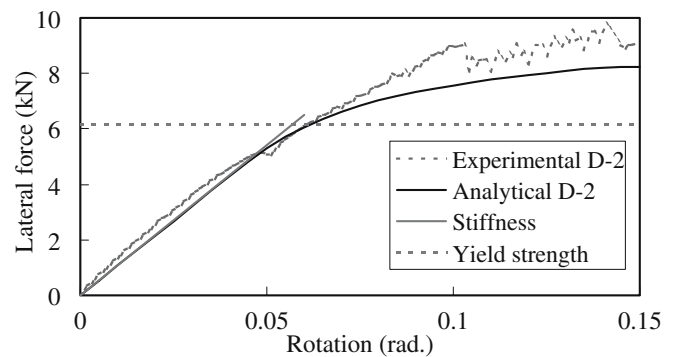**Fig. 12.** Comparison of results for specimen C-2

Fig. 13. Comparison of results for specimen C-3

Fig. 14. Comparison of results for specimen D-1

Fig. 15. Comparison of results for specimen D-2
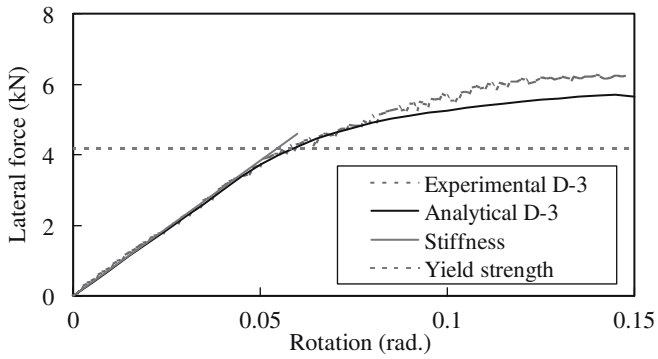
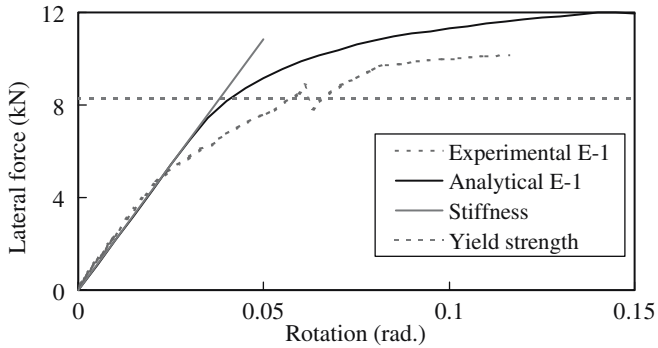**Fig. 16.** Comparison of results for specimen D-3

Fig. 17. Comparison of results for specimen E-1
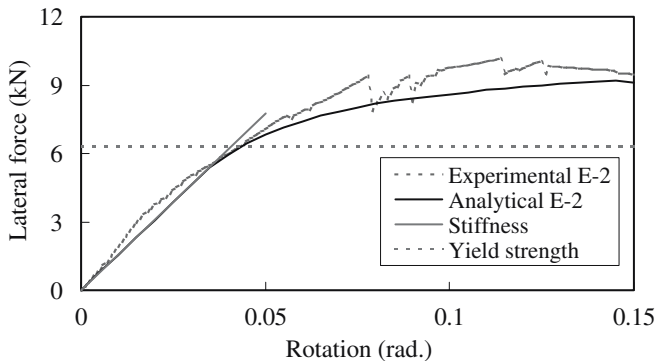

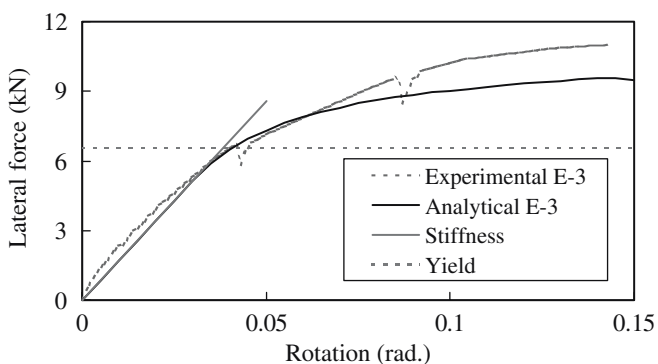**Fig. 18.** Comparison of results for specimen E-2

Fig. 19. Comparison of results for specimen E-3

and this study correspond well with the experimental results, thus demonstrating the validity of the proposed model. It is noteworthy that the proposed theoretical models are valid in the dimension range of this study, as the flexural behavior of top and bottom beams are neglected. For wider specimens, the flexural deformation of the beam might become significant and further modification might be needed.

Comparison of effects of embedment, friction, and bamboo nail

The three components of embedment, friction, and bamboo nails contribute to the lateral force resistance of the timber shear walls. Figure 20 compares the lateral force resistance contribution made by various components of specimen D-3. It is found that the friction effect plays an important role in resisting the lateral force, followed by the embedment effect and bamboo dowel. In the case of specimen D-3, the friction action contributes 81.8% to the initial stiffness; embedment and bamboo nails contribute 12.1% and 6.1%, respectively.

The contributions to initial stiffness made by each component of the tested specimens calculated from the theoretical model are given in Table 2. It is found that the effect of friction behavior plays a significant role in every specimen, and its contribution ranges from 70% to 84%. By contrast, the bamboo nails make a minor contribution to lateral force resistance, ranging from 3% to 7%, in the shear wall. This is because the lever arms for friction action are usually much larger than those for embedment and dowel action of bamboo nails. Furthermore, it is observed that the effect of bamboo nails decreases with the increase of MOE perpendicular to grain of the top and bottom beams of the shear wall. This is because the contributions made by friction and embedment are directly related to the MOE of the beams. Compared with the effect induced by friction, the effect of embedment will increase in the case that board units have smaller aspect ratio. From Eq. 11, it is obvious that the ratio of lateral force resistances supplied by friction and embedment depends on the aspect ratio, H_b/W_b, of the board unit.
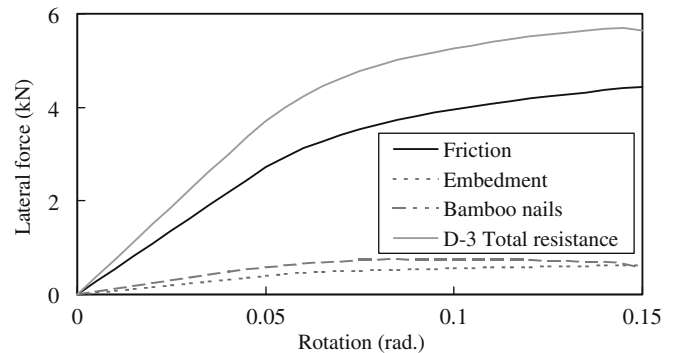

Fig. 20. Lateral resistance induced by friction, embedment, and bamboo nails for specimen D-3

Table 2. Contribution to stiffness made by three various components

Specimen no.	Contribution to initial stiffness (%)			Aspect ratio of board unit
	Friction	Embedment	Bamboo	
A-1	74.0	21.9	4.1	4.5
A-2	73.1	21.6	5.3	4.5
A-3	73.2	21.7	5.1	4.5
B-1	79.8	15.2	5.0	7.0
B-2	81.6	15.6	2.8	7.0
B-3	80.1	15.3	4.6	7.0
C-1	69.0	26.2	4.8	3.5
C-2	68.9	26.3	4.8	3.5
C-3	70.5	26.9	2.6	3.5
D-1	81.3	12.0	6.7	9.0
D-2	83.5	12.4	4.1	9.0
D-3	81.8	12.1	6.1	9.0
E-1	78.2	17.1	4.4	6.0
E-2	76.9	17.1	6.0	6.0
E-3	77.4	17.2	5.4	6.0
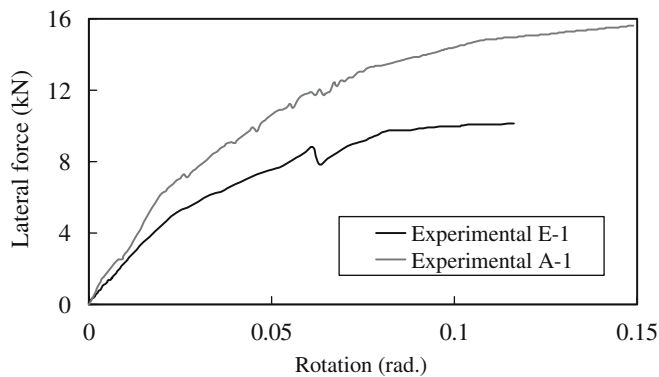
Fig. 21. Comparison of lateral resistance of timber shear wall with similar material properties and dimensions but with different board widths

Effect of board width

The width of board units used in a timber shear wall depends on the dimensions of the logs that the carpenters can obtain and the method used to saw the logs. In many cases, carpenters can only obtain small board units, and they need more board units to make the timber shear wall. Figure 21 was plotted to explore the effect of board width on the mechanical performance of timber shear walls. The specimens A-1 and E-1 have similar material properties and geometrical conditions, but have different board widths. Figure 21 indicates that when the timber shear walls have the same dimensions, the initial stiffness and yield strength will increase if wider board units are used. A similar trend is obtained by using the theoretical model proposed in this and previous studies. In other words, higher strength and stiffness can be obtained in timber shear walls by using wider board units.

Conclusions

Simplified calculation methods for initial stiffness and yield strength of traditional timber shear walls were proposed in this study. A total of 15 specimens were fabricated and tested to verify the theoretical model proposed in our previous report and this study. Experiments showed that the theoretical models can correspond well with experimental results, which implies the validity of the theoretical models established within the dimensions studied. The results of experiments and the theoretical model show that friction behavior plays an important role in resisting the lateral force, followed by the effect of embedment. The bamboo nails can only take about 5% of the lateral force in many cases. Furthermore, the width of the board unit is another important factor that affects the mechanical performance of traditional timber shear walls in Taiwan. Stiffness and strength increase when wide board units are used in fabrication of a timber shear wall.

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