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**TITLE** ON MONITORING NUCLEAR POWER PLANT EMERGENCY DIESEL GENERATOR RELIABILITY

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# On Monitoring Nuclear Power Plant Emergency Diesel Generator Reliability

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# 1. INTRODUCTION

If offsite power is interrupted, the availability of onsite alternating current power supplies is a major factor in assuring acceptable safety at commercial light-water-cooled nuclear power plants. To control the risk of severe core damage during station blackout accidents at a given plant, the reliability of the emergency diesel generators (EDGs) to start and load-run upon demand must be maintained at a sufficiently high level. The minimum EDG reliability, which we denote by RT, is targeted at either 0.95 or 0.975 per nuclear unit consistent with the reliability level that the plant operator assumed in the coping analysis for station blackout.

In 1992 the US Nuclear Regulatory Commission (NRC) considered an amendment that would require licensees to test and monitor EDG reliability against performance-based criteria that indicate possible degradation from the EDG target reliability levels. They originally proposed the following set of fixed sample-size triggers for use in monitoring EDG reliability:

> **EARLY WARNING:** If there are 3 failures in the last 20 demands for either an individual EDG or for all EDGs assigned to a nuclear unit, this is an early indicator of deterioration of EDG reliability.
>
> **PROBLEM DIESEL:** If there are 4 failures in the last 25 demands of an EDG, this is further indication of EDG reliability deterioration. Following corrective action, this EDG is to be subjected to accelerated testing to demonstrate effectiveness of corrective actions (i.e., 7 consecutive failure-free tests).
>
> **DOUBLE TRIGGER:** If there are 5 failures within the last 50 demands and 8 failures within the last 100 demands (RT = 0.95), or 4 failures within the last 50 demands and 5 failures within the last 100 demands (RT = 0.975), then this is reasonable evidence that the EDG reliability level has degraded below the selected target.

For convenience, the early warning criterion will be denoted simply as 3/20, which we read as "3 failures in the last 20 demands." Similarly, the problem diesel criterion will be indicated as 4/25, while the double trigger criteria will be denoted as 5/50 and 8/100 or 4/50 and 5/100, respectively. In the remainder of this paper, we will refer to these criteria collectively as the **proposed** triggers or procedure.

We note here that these proposed triggers are so-called **fixed sample-size** triggers in that the number of prior demands is precisely identified as an integral part of the stated trigger procedures (e.g., 20, 25, 50, or 100 demands). Such triggers contrast with so-called **variable sample-size** triggers in which the number of demands for use in determining whether or not a trigger condition exists varies from month-to-month. The alternative triggers described in Section 3 are of the variable sample-size type.

The overall NRC goal is to develop a method that maximizes the probability of detecting a significant decrease in EDG reliability while minimizing the probability of indicating a decrease when none has actually occurred (a false alarm). It is recognized that these are competing requirements.

The purpose of this report is to compare the performance of the proposed triggers with corresponding alternative sequential variable sample-size triggers which potentially permit earlier detection of EDG reliability degradation without significantly increasing the false alarm rate. The comparison is to be done in a simulated use environment by means of Monte Carlo simulation. We are also interested in the inverse conditional probabilities of reliability degradation given that a trigger has occurred.

Section 2 describes the Monte Carlo simulation used to assess the performance of both procedures. The alternative trigger procedure is described in Section 3, and performance comparisons of both procedures are discussed in

Section 4. The inverse probabilities of degradation given a trigger are considered in Section 5, while Section 6 presents our conclusions.

## 2. MONTE CARLO SIMULATION

We developed a Monte Carlo simulation for use in examining the performance of the proposed and alternative trigger procedures in a simulated use environment. The computer program was written in standard FORTRAN 77 and executed on both a Sun workstation using a SunOS operating system under a Sun FORTRAN compiler and a Macintosh IIfx using System 7.0 under an Absoft MacFortran/020 compiler.

For this initial simulation, we assumed 2 EDGs per nuclear unit, in which each EDG is routinely tested each month and alternately from month-to-month. We also assumed that the monthly tests were conditionally independent of each other given the underlying reliability of each diesel.

The early warning trigger is applied in three separate ways: to the test data for each individual diesel and the combined test data from both diesels. The problem diesel trigger is applied separately to the test data for each individual diesel, while the double trigger is applied only to the combined test data from both diesels.

Thus, the proposed trigger values were applied as follows:

| Trigger | Applies To |
|---|---|
| Early Warning (3/20) | Individual and all EDGs |
| Problem Diesel (4/25) | Individual EDGs |
| Double (5/50 and 8/100) or (4/50 and 5/100) | All EDGs |

Following a problem diesel trigger, the corresponding EDG is subjected to confirmatory accelerated testing until there are 7 consecutive failure-free tests prior to the next routine test on the diesel. All test results, including the confirmatory test results following a problem diesel trigger, are counted for the triggers.

Because we are interested in the length of time to first detect an EDG degraded reliability condition [the number of months that elapse before the degraded condition is first detected by the appropriate trigger(s)], we did not model the improvement in EDG reliability that the confirmatory testing is designed to produce. In other words, the confirmatory testing was conducted at the same degraded EDG reliability level that triggered the problem diesel condition in the first place. This was done in order to examine the performance of the double trigger in detecting EDG degradation which has not been corrected.

Each simulation replication consists of a maximum of 500 routine monthly tests on each diesel. Generally, we found that 500 tests are quite sufficient to first detect the levels of degradation we are interested in, including spurious false alarm detections in which no degradation has occurred.

Each replication continues until the desired triggers of interest have each occurred for the first time after which the current replication is terminated and a new replication is begun. For example, for a given EDG reliability level, suppose that all three triggers are of interest. Further, suppose that the first early warning condition occurs on the fifth month following the degradation, the first problem diesel condition on the eleventh month, and the first double trigger on the seventeenth month. These values would be recorded for the corresponding triggers for this replication, after which a new replication begins.

For purposes of initializing the proposed triggers so that they can be used at month1, each diesel is tested for 100 months prior to month 1 at some specified initial reliability level, which we denote by RI, which is taken to be the same for each diesel. Although 100 months is sufficient, this value may be changed as an input parameter for a given run of the computer program. We typically considered values of RI of 0.95, 0.975, and 0.99. The initial values of 0.95 and 0.975 correspond to the target reliability levels, while 0.99 roughly corresponds to the industry average EDG reliability to start and accept load upon demand.

The EDG degradation (the "shift" in reliability) for both diesels is programmed to occur at a specified month MS. This value is likewise an input parameter for a given computer run. The program is written such that zero, one, or two EDGs experience a degradation in reliability at month MS. A **two-diesel shift** occurs when the reliability of both diesels has shifted from their initial value, while a **single-diesel shift** occurs when the reliability of only one diesel has shifted. The reliability of both EDGs from month 1 through month MS-1 is assumed to be equal to the specified initial reliability RI.

The simulation was conducted using 10,000 replications. The corresponding sampling error was observed to be less than or equal to 0.01 for all probability calculations. For example, if the probability of detecting a shift in EDG reliability of a given magnitude by a certain number of months was calculated to be, say, 0.78, changing only the random number seed could change this computed output value by no more than 0.01 (between 0.77 and 0.79). This error is acceptably small; thus, 10,000 replications are sufficient.

As mentioned above, for each replication we are interested in observing the first month after the EDG reliability degradation occurs in which the desired trigger condition(s) exist. We calculate and record the corresponding number of months after the shift occurs.

For example, suppose that MS = 20. For a given trigger of interest, suppose that we have the following situation:

Rep 1 : Observe degradation at month 25 -> Save 25 - 20 + 1 = 6 months
Rep 2 : Observe degradation at month 23 -> Save 23 - 20 + 1 = 4 months
•
•
•
Rep 10,000: Observe degradation at month 27 -> Save 27 - 20 + 1 = 8 months

Because we are interested in rapidly detecting EDG reliability degradation, the appropriate random variable (rv) of interest is the number of months to first detect the degradation. On a given run of the computer program, we have 10,000 empirical observations of this rv from which we compute moments and quantiles of the distribution of this rv. Specifically, we compute the sample mean, standard deviation, skewness, and kurtosis of this rv. Also, we compute selected quantiles using standard nonparametric techniques [Conover (1971)].

In this study, we are interested in the uncertainty inherent in detecting a given EDG reliability degradation, **not** simply the average (or mean) performance, such as the average number of months to detect a shift of a specified magnitude. Thus, we compute the cumulative probabilities of detecting the degradation for a specified vector of months (as input on a given computer run). Because we are interested in studying rapid detection of degradation, we commonly specify 25 different months; namely, months 1 through 15, 20, 25, 30, 35, 40, 45, 50, 60, 75, and 100. We then calculate the cumulative probability of detection for each of these specified months by counting the fraction of the 10,000 replications in which the first detection occurred no later than the specified month (i.e., by the desired month or earlier).

We then plot these cumulative probabilities for each of the desired months to form a cumulative probability distribution associated with the rv **months to first detection after the shift occurs**, or, more simply, **months after shift occurs**. Section 4 contains several plots of such distributions. It is these cumulative probability of detection distributions which we will consider when we compare the performance of the alternative trigger procedure in Section 3 to the proposed trigger.

## 3. SEQUENTIAL TRIGGER PROCEDURE

The performance of the proposed triggers is quite sensitive to the initial reliability RI prior to degradation. This is illustrated in Fig. 1 in which we have plotted the cumulative probability of detecting a two-diesel shift from reliability RI to reliability R = 0.90, when using the proposed 5/50 and 8/100 double trigger, for three different values of RI. A target reliability RT = 0.95 is also assumed. We observe that these probabilities are quite sensitive to the value of RI. The detection probabilities are inversely proportional to RI. Small values of RI yield the largest detection probabilities because, in this case, there are more failures in the initial test data prior to the degradation thus allowing the double trigger condition to be more rapidly satisfied once the degradation occurs. A similar situation exists for the other proposed triggers as well.

It would be more desirable if the detection probabilities were less dependent on RI. In this case the performance would be more uniform in industry-wide implementation of the triggers. This shortcoming could be avoided by using a trigger procedure in which the previous data collected before the shift doesn't have as much influence. The question then becomes: Is there a trigger procedure which periodically recycles (i.e., resets, restarts, or reinitializes) in the sense that , once recycling occurs, **all** the past performance data are ignored and the trigger statistics begin anew? The answer is affirmative.

In addition, these alternative triggers should have higher probabilities of more rapidly detecting degradation in EDG reliability without increasing the false alarm rates because less weight is given to the data prior to the degradation.

Wald (1947) developed the notion and use of item-by-item sequential sampling based on the sequential probability ratio test (SPRT). It is well known that the use of these variable sample-size plans usually require less sampling for the same detection probabilities than corresponding fixed sample-size plans. In the EDG context considered here, this statement equates to more rapid anticipated detection of EDG reliability degradation than that of the proposed triggers.

Vesely et al (1982) also developed an SPRT approach for monitoring component failure rates in nuclear power plants. Based on Monte Carlo simulation, they concluded that SPRT-based procedures can be quite effective in detecting unacceptably high component failure rates or unacceptable increases (shifts) in the failure rate. While their procedure is similar to the approach we consider here, it differs in two important aspects: (1) they use a more complicated set of criteria for establishing their control limits; and (2) they graphically implement their procedure in the form of a control chart, while we choose a simple tabular format.

The SPRT procedure was initially developed for lot-by-lot acceptance sampling, in which lots of some product are submitted to item-by-item sequential sampling. If the accumulated number of defective items in a sequential sample of size n exceeds a stated upper rejection limit, then the entire lot is rejected as having a defect (or failure) rate that is unacceptably large. On the other hand, if the accumulated number of defective items falls below a stated acceptance limit, then the entire lot is accepted as having a defect rate that is acceptably small. Wald also shows that, with probability 1, the procedure eventually converges to one of these two states. The acceptance and rejection limits are calculated by specifying four parameters which together determine a pair of desired risk criteria -- the so-called consumer and producer risks. The performance-based statistic required for using these SPRT triggers is the cumulative (or total) number of EDG failures in n tests (or demands) as n increases month-to-month.

In the case of EDG testing, we have a continuous process as the data become continually available; thus, the notion and use of "lots", as required by Wald, is not present. In this case, the SPRT procedure can be modified in a straightforward way to incorporate **recycling** (restarting, re initialization, or resetting) as discussed by Lorden and Eisenberger (1968). The SPRT, when used with recycling, is similar to CUSUM testing, although the resulting control chart can be much different [Van Dobben de Bruyn (1968) and Lucas (1976)].

Recycling is a simple notion. Suppose that a lower acceptance limit has been established. If the cumulative number of EDG failures falls on or below this limit at some month n, say, then at month n + 1 the entire procedure is recycled. By recycling we mean that the SPRT procedure starts anew at month n + 1 as though month n + 1 is now month "1". Correspondingly, the cumulative number of failures is also reset to zero after month n. Because of our particular notion and use of this lower limit as a recycling trigger, as opposed to an acceptable lot quality limit, we refer to the lower SPRT acceptance limit as the **recycling limit**. Apart from its use in permitting SPRT process control, the advantage in recycling is that it makes the SPRT procedure less dependent on past data, hence more sensitive to reliability degradation.

On the other hand, if the cumulative number of EDG failures falls on or above the upper rejection limit, then the trigger condition is said to exist, thereby indicating a degradation in EDG reliability. For example, as for the proposed procedure, the SPRT early warning trigger procedure is used in conjunction with the test and operational data for each diesel separately as well as for the combined data. If the cumulative number of failures falls on or above the upper rejection limit for one or more of these early warning triggers, this is taken as an early warning indication that EDG degradation has occurred. In our particular EDG application of the SPRT procedure, we refer to the upper rejection limit as the **detection limit**, as it is this limit which indicates that a degradation in EDG reliability has in fact occurred.

The SPRT limits are determined based on four specified parameters -- $\alpha$, $\beta$, $p_0$, and $p_1$. In the original Wald development, the $\alpha$ and $\beta$ parameters represent Type I and Type II statistical errors, respectively, while $p_0$ and $p_1$ represent the quality levels (in terms of lot fraction defective) at which the Type I and Type II errors occur. Thus, $p_0$ is an acceptable quality level for which lots are to be accepted, while $p_1$ is an unacceptable quality level at which lots are to be rejected. It is thus required that $p_1$ must be larger than $p_0$. The pair $(\alpha, p_0)$ defines the so-called producer's risk point and the pair $(\beta, p_1)$ defines the so-called consumer's risk point on the operating characteristic (OC) curve. However, because of the use of combination plans along with recycling, these designations no longer hold, and the four parameters no longer have this simple interpretation. Thus, we treat the four parameters as simply that -- four parameters that must be specified in order to define the SPRT procedure **without** any particular interpretation being attached to these parameters.

For the four specified parameters $(\alpha, \beta, p_0, p_1)$, the SPRT detection and recycling limits are given by

$$\begin{array}{ll} \text{DETECTION LIMIT:} & D = A + Bn \\ \text{RECYCLING LIMIT:} & R = C + Bn \end{array} \qquad (1)$$

where

$$
\begin{aligned}
U &= \ln[p_1(1-p_0)] \\
V &= \ln[p_0(1-p_1)] \\
G &= U - V \\
A &= \ln[(1-\beta)/\alpha]/G \\
B &= \ln[(1-p_0)/(1-p_1)]/G \\
C &= \ln[\beta/(1-\alpha)]/G
\end{aligned}
\qquad (2)
$$

and where n denotes the number of EDG tests.

To illustrate this procedure, consider the SPRT problem diesel trigger. By employing the philosophy discussed below, the four specified parameters are found to be $\alpha = 0.05$, $\beta = 0.38$, $p_0 = 0.05$, and $p_1 = 0.20$. From (1) and (2) , the corresponding detection and recycling limits are given by $D = 1.6158 + 0.1103n$ and $R = -0.588 + 0.1103n$, respectively. These limits are plotted in Fig. 2.

By using these limit equations, it is much simpler to implement the SPRT procedure by constructing a table of the detection and recycling values as a function of n over an appropriately large range of n. We have done this in Table 1 which is used as follows: If the cumulative number of failures in n tests is equal to or greater than the corresponding value in the column labeled **D** (for Detection), then a problem diesel condition is declared for the EDG for which the data apply and the SPRT procedure would be recycled at the next scheduled monthly test. If the cumulative number of failures in n tests equals or is less than or equal to the corresponding value in the column labeled **R** (for Recycling), then the procedure would simply be recycled at the next scheduled monthly test with no associated EDG declaration being made. If the cumulative number of failures in n tests falls within the **D** and **R** values, the SPRT procedure would likewise make no declaration (insufficient evidence for either a problem diesel condition or for recycling) and the procedure would simply continue by further accumulating next month's test results and comparing the new accumulated failure total to the tabled values at n+1.

We observe in Table 1 that detection of a problem diesel condition requires at least 2 EDG tests on a given diesel in which both tests are failures. There are, of course, many other pathways in which detection can occur. We also observe that recycling requires at least 6 EDG tests on a given diesel in which there are no failures. Although they do not affect the implementation of the SPRT trigger procedure, many of the values reported in Table 1 are superfluous. For example, it is not possible to recycle the procedure at $n = 7$ with 0 failures because the procedure will already have recycled at $n = 6$.

The confirmatory tests are not considered in the proposed SPRT procedures as 7 consecutive successful EDG tests will often lead to recycling anyway. Thus, following a problem diesel condition, the accelerated test results are not considered in any of the proposed SPRT procedures and, if such testing is to remain a part of the proposed rule, the confirmatory test results are only exogenously used to ensure that a degraded EDG condition has been corrected.

The philosophy used to determine the SPRT triggers is as follows. Recall that the detection probabilities associated with the proposed triggers vary according to RI. We choose to determine SPRT triggers (using Monte Carlo simulation as the appropriate tool) that closely match the two-diesel degradation false alarm probability distribution associated with the corresponding worse-case proposed trigger procedure. Recall from Fig. 1 that the highest false alarm probabilities occur for the smallest feasible value of RI, i.e., when RI is equal to the target reliability. Thus, we choose to match the false alarm probability distribution for two-diesel degradation when RI is equal to the target reliability. This method assumes that the highest false alarm detection probabilities associated with the proposed triggers are acceptably small and ensures that the

SPRT triggers will not significantly exceed these false alarm probabilities. The required four parameters are found by a direct search method of observing the output cumulative detection probability distributions from the Monte Carlo program. It is hoped that the probability of rapidly detecting actual reliability degradation using the SPRT triggers will then exceed that of the proposed triggers. That this is indeed the case will now be illustrated.

Figure 3 illustrates the comparative performance of both problem diesel trigger procedures for the case when R = 95% (where detection indicates a false alarm) and R = 80% (significant degradation) when both diesels degrade to these levels perhaps due to some common cause. Figure 3 thus illustrates the match used to determine the SPRT problem diesel parameters, thus identifying the SPRT procedure. When RI is 0.95, we observe the close match in the false alarm distributions as desired. Although only 35 months of data are displayed in Fig. 3, the false alarm distributions continue to match through 100 months after the shift occurs. However, in this case, note that the SPRT procedure has higher probabilities of more rapidly detecting the shift in reliability to R = 80% than the proposed trigger. When RI is 0.99 (closer to the industry average), the proposed trigger has significantly smaller false alarm probabilities and significantly smaller probabilities of rapidly detecting the shift to R = 80% than the corresponding SPRT procedure. As claimed earlier, the performance of the SPRT trigger is less sensitive to the initial reliability.

Applying this same philosophy in conjunction with (1) and (2) yields the following SPRT trigger parameters:

| Trigger | $\alpha$ | $\beta$ | $p_0$ | $p_1$ | A | B | C |
|---|---|---|---|---|---|---|---|
| Early Warning | 0.15 | 0.28 | 0.05 | 0.20 | 1.0067 | 0.1103 | -0.713 |
| Problem Diesel | 0.05 | 0.38 | 0.05 | 0.20 | 1.6158 | 0.1103 | -0.588 |
| Double (RT = 0.95) | 0.07 | 0.20 | 0.05 | 0.20 | 1.5635 | 0.1103 | -0.986 |
| Double (RT = 0.975) | 0.025 | 0.28 | 0.025 | 0.20 | 1.4756 | 0.0869 | -0.548 |

The corresponding tabular format (analogous to Table 1) for easy use in implementing these SPRT triggers is given in Tables 2 - 4 for the remaining triggers. However, because the double triggers are only used in conjunction with the combined EDG test data, only the even values of n are required in Tables 3 and 4.

Although the tables are quite similar, a close examination reveals differences which significantly alter their performance. The performance of these SPRT triggers relative to the corresponding proposed triggers will now be illustrated.

## 4. PERFORMANCE COMPARISONS

In this section we compare the performance of the proposed and SPRT problem diesel triggers under various simulated use conditions. The performance results for the early warning and double triggers are comparable. In particular, we consider RI values of 0.95, 0.975, and 0.99; two-diesel and single diesel degradation; and step and ramp (gradual) degradation profiles. For convenience, we have included the important parameters in either the figure caption or on the figures themselves in all of the illustrations referenced in this section.

In order to compare the performance of both methods, we must choose a month MS in which the reliability degradation occurs. The performance of neither trigger procedure significantly depends on the particular month in which the degradation (or shift) occurs, thus, in our simulation study we generally chose to introduce the EDG degradation arbitrarily at month 20.

Figure 4 illustrates the comparative results for detecting a two-diesel degradation to reliability R when RI is 0.99. The SPRT procedure clearly outperforms the proposed procedure in early detection of the degradation.

Similarly, Fig. 5 considers a single-diesel degradation to reliability R, and, as in the case of two-diesel degradation, the SPRT outperforms the proposed procedure in early detection of the degradation.

The only type of EDG degradation considered thus far has been of the **step** variety. That is, the degradation to level R immediately occurs at some month in the form of a step function. We also consider another pattern of degradation, which we denote as **ramp** degradation. By ramp degradation we mean that the degradation begins some month at the RI level and linearly degrades to level R by some specified period of months later (we consider periods of 3, 6, 12, and 18 months). Thus, ramp degradation models the situation where the degradation in EDG reliability is gradual and constant from month-to-month, due to some persistent cause.

Figure 6 illustrates the comparative results obtained for detecting two-diesel degradation to R = 0.80 for both step and ramp patterns of degradation for RI = 0.99. The SPRT performance is excellent relative to the proposed trigger.

## 5. INVERSE PROBABILITIES OF RELIABILITY DEGRADATION

We calculate the desired inverse probability of EDG reliability degradation given a trigger using Bayes' theorem. Let $T_k$ denote the event that a trigger condition (of the desired type under consideration) occurs at month k. Let $D_n$ denote the event that the underlying reliability R of a single diesel degrades to a specified value below the chosen target reliability RT for the first time at month $n \leq k$. Further, let $D^k$ denote the event that single-diesel degradation occurs on or before month k, and let $\bar{D}^k$ denote the complement of event $D^k$ (that is, degradation does not occur on or before month k).

Of particular interest here is the realistic case in which (1) only one diesel degrades in reliability, and (2) the initial reliability RI of both diesels prior to degradation is equal to the industry average of approximately 0.99.

In earlier sections we have estimated the conditional probabilities of triggering given degradation $P(T_k \mid D_n)$ and the false alarm probabilities $P(T_k \mid D^k)$ under various conditions, including those of interest here. The desired inverse probability of interest is $P(D^k \mid T_k)$ as k ranges between 1 and 100.

For simplicity, we consider only the case in which the underlying diesel reliability R has two possible values; either $R = RI \geq RT$ or $R = RD < RT$. Here RD denotes a specified degraded reliability level, and we consider two cases; namely, $RD = 0.80$ and $RD = 0.90$. Also, recall that we are only interested in the case in which $RI = 0.99$. If no degradation has occurred on each operational or test demand, each diesel has either a satisfactory reliability of 0.99 with probability $1 - r$ or an unsatisfactory (degraded) reliability of RD with probability $r$. Once the diesel reliability degrades to level RD, it remains at this degraded level until it is either "caught" by a corresponding trigger or the simulation ceases.

Thus, we consider the two-point unconditional discrete probability distribution for R (a two-point prior reliability distribution) given by

$$P(R) = \begin{cases} \varepsilon, & R = RD \\ 1 - \varepsilon, & R = RI \end{cases}. \tag{3}$$

It follows from the geometric distribution that $P(D_n) = \varepsilon(1 - \varepsilon)^{n-1}$ and $P(\bar{D}^k) = (1 - \varepsilon)^k$.

Using Bayes' theorem, we have

$$P(D^k \mid T_k) = \frac{\sum_{n=1}^{k} \varepsilon(1 - \varepsilon)^{n-1} P(T_k \mid D_n)}{\sum_{n=1}^{k} \varepsilon(1 - \varepsilon)^{n-1} P(T_k \mid D_n) + (1 - \varepsilon)^k P(T_k \mid \bar{D}^k)}. \tag{4}$$

For example, consider the case of single-diesel degradation when using the sequential problem diesel trigger procedure, $RD = 0.80$, $RI = 0.99$, $\varepsilon = 0.01$, and $k = 2$ months. From earlier work we know that $P(T_2 \mid D_1) = 0.0374$, $P(T_2 \mid D_2) = 0.0018$, and $P(T_2 \mid \bar{D}^2) = 0.0002$. From (4), we calculate

$$P(D^2 \mid T_2) = \frac{0.01(0.0374) + 0.01(1 - 0.01)(0.0018)}{0.01(0.0374) + 0.01(1 - 0.01)(0.0018) + (1 - 0.01)^2(0.0002)}$$

$$= 0.67.$$

Thus, given a problem diesel sequential trigger condition at month 2, the probability is 0.67 that the corresponding EDG reliability has degraded to 0.80 on or before month 2.

We present the comparative results for the proposed fixed sample-size and sequential trigger procedures. Because we do not know the precise value of $\varepsilon$, we consider three values for $\varepsilon$; namely, 0.01, 0.001, and 0.0001.

Figure 7 gives the conditional probabilities $P(D^k \mid T_k)$ of single-diesel degradation to reliability RD = 0.8 and 0.9 as a function of the month k at which the trigger occurs (for k between 1 and 30) for the problem diesel trigger procedures and for $\varepsilon = 0.01$, where BC denotes "base-case". Note that, in the figure caption, the term "prior to month k" means "on or before month k". The sequential trigger procedure yields the highest conditional probabilities in the early months, while the proposed fixed sample-size procedure produces higher probabilities in the later months. Similarly, Figs. 8 and 9 consider $\varepsilon = 0.001$ and 0.0001, respectively.

In most cases, the steepest portion of the curves in Figs. 7 - 9 occurs prior to $k = 30$ months; the curves are relatively flat beyond 30 months. Thus, the results are given here only for $k \leq 30$. However, we also calculated results for k as large as 100 months.

## 6. CONCLUSIONS

We have presented a performance-based alternative trigger procedure based on the Wald sequential SPRT for use in detecting EDG reliability degradation. These SPRT triggers are just as easy to use as the proposed triggers. From our simulation results, we conclude that the variable sample size SPRT triggers: (1) are generally more powerful for rapid detection of EDG reliability degradation than the proposed fixed sample-size triggers without significantly increasing the false alarm detection probabilities; and (2) have probabilistic performance characteristics which are less dependent on the initial

EDG reliability value(s) (prior to degradation) than the proposed triggers. Also, unlike the proposed triggers, the SPRT triggers require no past data for their initial implementation. They can be implemented beginning at month 1 using only the EDG test results for that month.

For the conditions we investigated here, the conditional probabilities of diesel reliability degradation on or before month k given a trigger condition at month k are largest for the sequential trigger procedures in the early months (that is, when k is small). The sequential triggers produce especially advantageous results in the early months, while the proposed trigger procedures are generally superior for large values of k.

Both the proposed and SPRT methods described here may be used for monitoring process quality in other cases as well. In general, these methods can be used to monitor potential changes (shifts) in a binomial parameter p over time for the case in which only a single item is periodically available at any one time for Bernoulli testing. Such a situation miight occur if items are either unavailable or expensive to test, such as in the case of destructive testing of expensive items.

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## TABLE 1

## SEQUENTIAL PROBLEM DIESEL TRIGGER

| n | D | R | n | D | R | n | D | R |
|---|---|---|---|---|---|---|---|---|
| 1 | † | * | 34 | 6 | 3 | 67 | 10 | 6 |
| 2 | 2 | * | 35 | 6 | 3 | 68 | 10 | 6 |
| 3 | 2 | * | 36 | 6 | 3 | 69 | 10 | 7 |
| 4 | 3 | * | 37 | 6 | 3 | 70 | 10 | 7 |
| 5 | 3 | * | 38 | 6 | 3 | 71 | 10 | 7 |
| 6 | 3 | 0 | 39 | 6 | 3 | 72 | 10 | 7 |
| 7 | 3 | 0 | 40 | 7 | 3 | 73 | 10 | 7 |
| 8 | 3 | 0 | 41 | 7 | 3 | 74 | 10 | 7 |
| 9 | 3 | 0 | 42 | 7 | 4 | 75 | 10 | 7 |
| 10 | 3 | 0 | 43 | 7 | 4 | 76 | 10 | 7 |
| 11 | 3 | 0 | 44 | 7 | 4 | 77 | 11 | 7 |
| 12 | 3 | 0 | 45 | 7 | 4 | 78 | 11 | 8 |
| 13 | 4 | 0 | 46 | 7 | 4 | 79 | 11 | 8 |
| 14 | 4 | 0 | 47 | 7 | 4 | 80 | 11 | 8 |
| 15 | 4 | 1 | 48 | 7 | 4 | 81 | 11 | 8 |
| 16 | 4 | 1 | 49 | 8 | 4 | 82 | 11 | 8 |
| 17 | 4 | 1 | 50 | 8 | 4 | 83 | 11 | 8 |
| 18 | 4 | 1 | 51 | 8 | 5 | 84 | 11 | 8 |
| 19 | 4 | 1 | 52 | 8 | 5 | 85 | 11 | 8 |
| 20 | 4 | 1 | 53 | 8 | 5 | 86 | 12 | 8 |
| 21 | 4 | 1 | 54 | 8 | 5 | 87 | 12 | 9 |
| 22 | 5 | 1 | 55 | 8 | 5 | 88 | 12 | 9 |
| 23 | 5 | 1 | 56 | 8 | 5 | 89 | 12 | 9 |
| 24 | 5 | 2 | 57 | 8 | 5 | 90 | 12 | 9 |
| 25 | 5 | 2 | 58 | 9 | 5 | 91 | 12 | 9 |
| 26 | 5 | 2 | 59 | 9 | 5 | 92 | 12 | 9 |
| 27 | 5 | 2 | 60 | 9 | 6 | 93 | 12 | 9 |
| 28 | 5 | 2 | 61 | 9 | 6 | 94 | 12 | 9 |
| 29 | 5 | 2 | 62 | 9 | 6 | 95 | 13 | 9 |
| 30 | 5 | 2 | 63 | 9 | 6 | 96 | 13 | 9 |
| 31 | 6 | 2 | 64 | 9 | 6 | 97 | 13 | 10 |
| 32 | 6 | 2 | 65 | 9 | 6 | 98 | 13 | 10 |
| 33 | 6 | 3 | 66 | 9 | 6 | 99 | 13 | 10 |
|  |  |  |  |  |  | 100 | 13 | 10 |

† Detection requires at least 2 diesel tests in which both tests are failures

* Recycling requires at least 6 diesel tests in which there are no failures

## TABLE 2

## SEQUENTIAL EARLY WARNING TRIGGER

| n | D | R | n | D | R | n | D | R |
|---|---|---|---|---|---|---|---|---|
| 1 | † | * | 34 | 5 | 3 | 67 | 9 | 6 |
| 2 | 2 | * | 35 | 5 | 3 | 68 | 9 | 6 |
| 3 | 2 | * | 36 | 5 | 3 | 69 | 9 | 6 |
| 4 | 2 | * | 37 | 6 | 3 | 70 | 9 | 7 |
| 5 | 2 | * | 38 | 6 | 3 | 71 | 9 | 7 |
| 6 | 2 | * | 39 | 6 | 3 | 72 | 9 | 7 |
| 7 | 2 | 0 | 40 | 6 | 3 | 73 | 10 | 7 |
| 8 | 2 | 0 | 41 | 6 | 3 | 74 | 10 | 7 |
| 9 | 2 | 0 | 42 | 6 | 3 | 75 | 10 | 7 |
| 10 | 3 | 0 | 43 | 6 | 4 | 76 | 10 | 7 |
| 11 | 3 | 0 | 44 | 6 | 4 | 77 | 10 | 7 |
| 12 | 3 | 0 | 45 | 6 | 4 | 78 | 10 | 7 |
| 13 | 3 | 0 | 46 | 7 | 4 | 79 | 10 | 8 |
| 14 | 3 | 0 | 47 | 7 | 4 | 80 | 10 | 8 |
| 15 | 3 | 0 | 48 | 7 | 4 | 81 | 10 | 8 |
| 16 | 3 | 1 | 49 | 7 | 4 | 82 | 11 | 8 |
| 17 | 3 | 1 | 50 | 7 | 4 | 83 | 11 | 8 |
| 18 | 3 | 1 | 51 | 7 | 4 | 84 | 11 | 8 |
| 19 | 4 | 1 | 52 | 7 | 5 | 85 | 11 | 8 |
| 20 | 4 | 1 | 53 | 7 | 5 | 86 | 11 | 8 |
| 21 | 4 | 1 | 54 | 7 | 5 | 87 | 11 | 8 |
| 22 | 4 | 1 | 55 | 8 | 5 | 88 | 11 | 8 |
| 23 | 4 | 1 | 56 | 8 | 5 | 89 | 11 | 9 |
| 24 | 4 | 1 | 57 | 8 | 5 | 90 | 11 | 9 |
| 25 | 4 | 2 | 58 | 8 | 5 | 91 | 12 | 9 |
| 26 | 4 | 2 | 59 | 8 | 5 | 92 | 12 | 9 |
| 27 | 4 | 2 | 60 | 8 | 5 | 93 | 12 | 9 |
| 28 | 5 | 2 | 61 | 8 | 6 | 94 | 12 | 9 |
| 29 | 5 | 2 | 62 | 8 | 6 | 95 | 12 | 9 |
| 30 | 5 | 2 | 63 | 8 | 6 | 96 | 12 | 9 |
| 31 | 5 | 2 | 64 | 9 | 6 | 97 | 12 | 9 |
| 32 | 5 | 2 | 65 | 9 | 6 | 98 | 12 | 10 |
| 33 | 5 | 2 | 66 | 9 | 6 | 99 | 12 | 10 |
|  |  |  |  |  |  | 100 | 13 | 10 |

† Detection requires at least 2 diesel tests in which both tests are failures

* Recycling requires at least 7 diesel tests in which there are no failures

## TABLE 3

## SEQUENTIAL DOUBLE TRIGGER (RT = 95%)

| n | D | R | n | D | R | n | D | R |
|---|---|---|---|---|---|---|---|---|
| 1 | † | * | 34 | 6 | 2 | 67 | 9 | 6 |
| 2 | 2 | * | 35 | 6 | 2 | 68 | 10 | 6 |
| 3 | 2 | * | 36 | 6 | 2 | 69 | 10 | 6 |
| 4 | 3 | * | 37 | 6 | 3 | 70 | 10 | 6 |
| 5 | 3 | * | 38 | 6 | 3 | 71 | 10 | 6 |
| 6 | 3 | * | 39 | 6 | 3 | 72 | 10 | 6 |
| 7 | 3 | * | 40 | 6 | 3 | 73 | 10 | 7 |
| 8 | 3 | * | 41 | 7 | 3 | 74 | 10 | 7 |
| 9 | 3 | 0 | 42 | 7 | 3 | 75 | 10 | 7 |
| 10 | 3 | 0 | 43 | 7 | 3 | 76 | 10 | 7 |
| 11 | 3 | 0 | 44 | 7 | 3 | 77 | 11 | 7 |
| 12 | 3 | 0 | 45 | 7 | 3 | 78 | 11 | 7 |
| 13 | 3 | 0 | 46 | 7 | 4 | 79 | 11 | 7 |
| 14 | 4 | 0 | 47 | 7 | 4 | 80 | 11 | 7 |
| 15 | 4 | 0 | 48 | 7 | 4 | 81 | 11 | 7 |
| 16 | 4 | 0 | 49 | 7 | 4 | 82 | 11 | 8 |
| 17 | 4 | 0 | 50 | 8 | 4 | 83 | 11 | 8 |
| 18 | 4 | 0 | 51 | 8 | 4 | 84 | 11 | 8 |
| 19 | 4 | 1 | 52 | 8 | 4 | 85 | 11 | 8 |
| 20 | 4 | 1 | 53 | 8 | 4 | 86 | 12 | 8 |
| 21 | 4 | 1 | 54 | 8 | 4 | 87 | 12 | 8 |
| 22 | 4 | 1 | 55 | 8 | 5 | 88 | 12 | 8 |
| 23 | 5 | 1 | 56 | 8 | 5 | 89 | 12 | 8 |
| 24 | 5 | 1 | 57 | 8 | 5 | 90 | 12 | 8 |
| 25 | 5 | 1 | 58 | 8 | 5 | 91 | 12 | 9 |
| 26 | 5 | 1 | 59 | 9 | 5 | 92 | 12 | 9 |
| 27 | 5 | 1 | 60 | 9 | 5 | 93 | 12 | 9 |
| 28 | 5 | 2 | 61 | 9 | 5 | 94 | 12 | 9 |
| 29 | 5 | 2 | 62 | 9 | 5 | 95 | 13 | 9 |
| 30 | 5 | 2 | 63 | 9 | 5 | 96 | 13 | 9 |
| 31 | 5 | 2 | 64 | 9 | 6 | 97 | 13 | 9 |
| 32 | 6 | 2 | 65 | 9 | 6 | 98 | 13 | 9 |
| 33 | 6 | 2 | 66 | 9 | 6 | 99 | 13 | 9 |
|  |  |  |  |  |  | 100 | 13 | 10 |

† Detection requires at least 2 diesel tests in which both tests are failures

* Recycling requires at least 9 diesel tests in which there are no failures

# TABLE 4

## SEQUENTIAL DOUBLE TRIGGER (RT = 97.5%)

| n | D | R | n | D | R | n | D | R |
|---|---|---|---|---|---|---|---|---|
| 1 | † | * | 34 | 5 | 2 | 67 | 8 | 5 |
| 2 | 2 | * | 35 | 5 | 2 | 68 | 8 | 5 |
| 3 | 2 | * | 36 | 5 | 2 | 69 | 8 | 5 |
| 4 | 2 | * | 37 | 5 | 2 | 70 | 8 | 5 |
| 5 | 2 | * | 38 | 5 | 2 | 71 | 8 | 5 |
| 6 | 2 | * | 39 | 5 | 2 | 72 | 8 | 5 |
| 7 | 3 | 0 | 40 | 5 | 2 | 73 | 8 | 5 |
| 8 | 3 | 0 | 41 | 6 | 3 | 74 | 8 | 5 |
| 9 | 3 | 0 | 42 | 6 | 3 | 75 | 8 | 5 |
| 10 | 3 | 0 | 43 | 6 | 3 | 76 | 9 | 6 |
| 11 | 3 | 0 | 44 | 6 | 3 | 77 | 9 | 6 |
| 12 | 3 | 0 | 45 | 6 | 3 | 78 | 9 | 6 |
| 13 | 3 | 0 | 46 | 6 | 3 | 79 | 9 | 6 |
| 14 | 3 | 0 | 47 | 6 | 3 | 80 | 9 | 6 |
| 15 | 3 | 0 | 48 | 6 | 3 | 81 | 9 | 6 |
| 16 | 3 | 0 | 49 | 6 | 3 | 82 | 9 | 6 |
| 17 | 3 | 0 | 50 | 6 | 3 | 83 | 9 | 6 |
| 18 | 4 | 1 | 51 | 6 | 3 | 84 | 9 | 6 |
| 19 | 4 | 1 | 52 | 6 | 3 | 85 | 9 | 6 |
| 20 | 4 | 1 | 53 | 7 | 4 | 86 | 9 | 6 |
| 21 | 4 | 1 | 54 | 7 | 4 | 87 | 10 | 7 |
| 22 | 4 | 1 | 55 | 7 | 4 | 88 | 10 | 7 |
| 23 | 4 | 1 | 56 | 7 | 4 | 89 | 10 | 7 |
| 24 | 4 | 1 | 57 | 7 | 4 | 90 | 10 | 7 |
| 25 | 4 | 1 | 58 | 7 | 4 | 91 | 10 | 7 |
| 26 | 4 | 1 | 59 | 7 | 4 | 92 | 10 | 7 |
| 27 | 4 | 1 | 60 | 7 | 4 | 93 | 10 | 7 |
| 28 | 4 | 1 | 61 | 7 | 4 | 94 | 10 | 7 |
| 29 | 4 | 1 | 62 | 7 | 4 | 95 | 10 | 7 |
| 30 | 5 | 2 | 63 | 7 | 4 | 96 | 10 | 7 |
| 31 | 5 | 2 | 64 | 8 | 5 | 97 | 10 | 7 |
| 32 | 5 | 2 | 65 | 8 | 5 | 98 | 10 | 7 |
| 33 | 5 | 2 | 66 | 8 | 5 | 99 | 11 | 8 |
|  |  |  |  |  |  | 100 | 11 | 8 |

† Detection requires at least 2 diesel tests in which both tests are failures

* Recycling requires at least 7 diesel tests in which there are no failures

Figure 1

### The Effect Of Initial Reliability RI On The Performance Of The Proposed Double (5/50 and 8/100) Trigger For Detecting A Two-Diesel Degradation To Reliability R = 90%

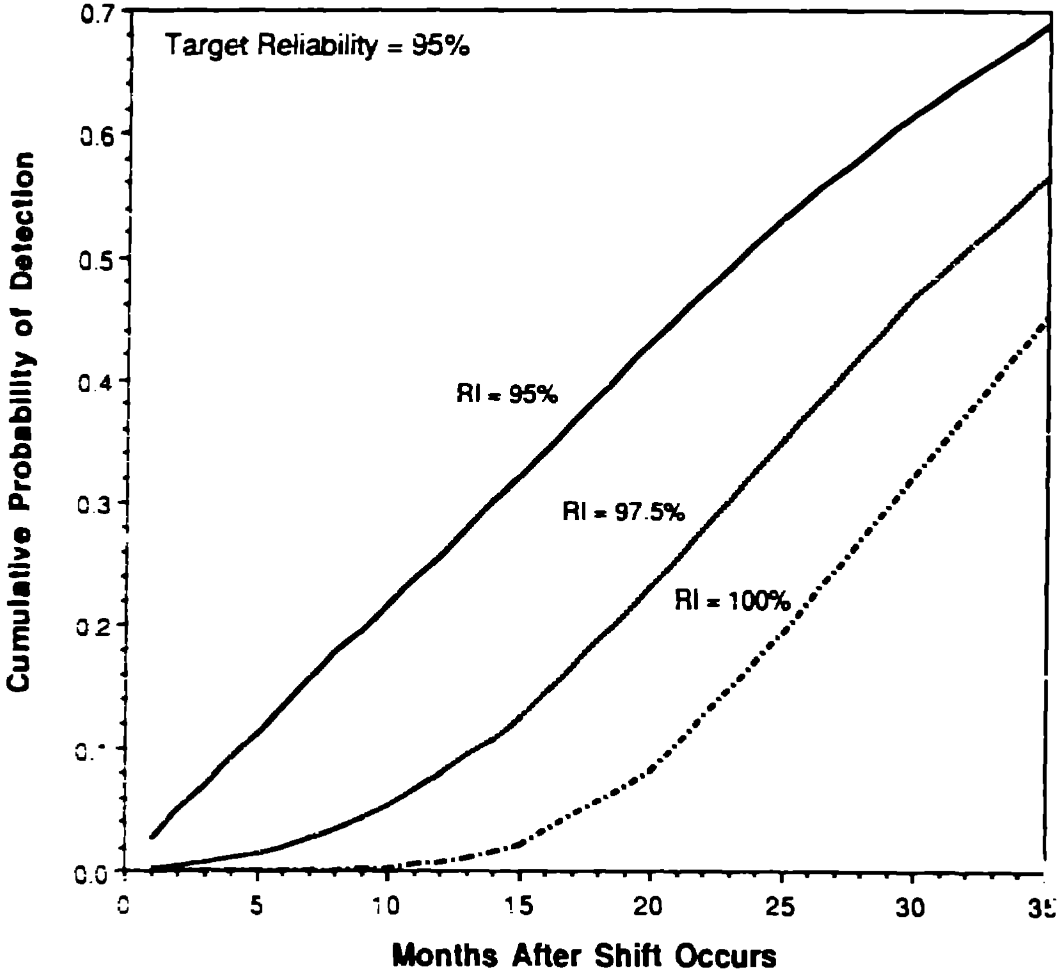
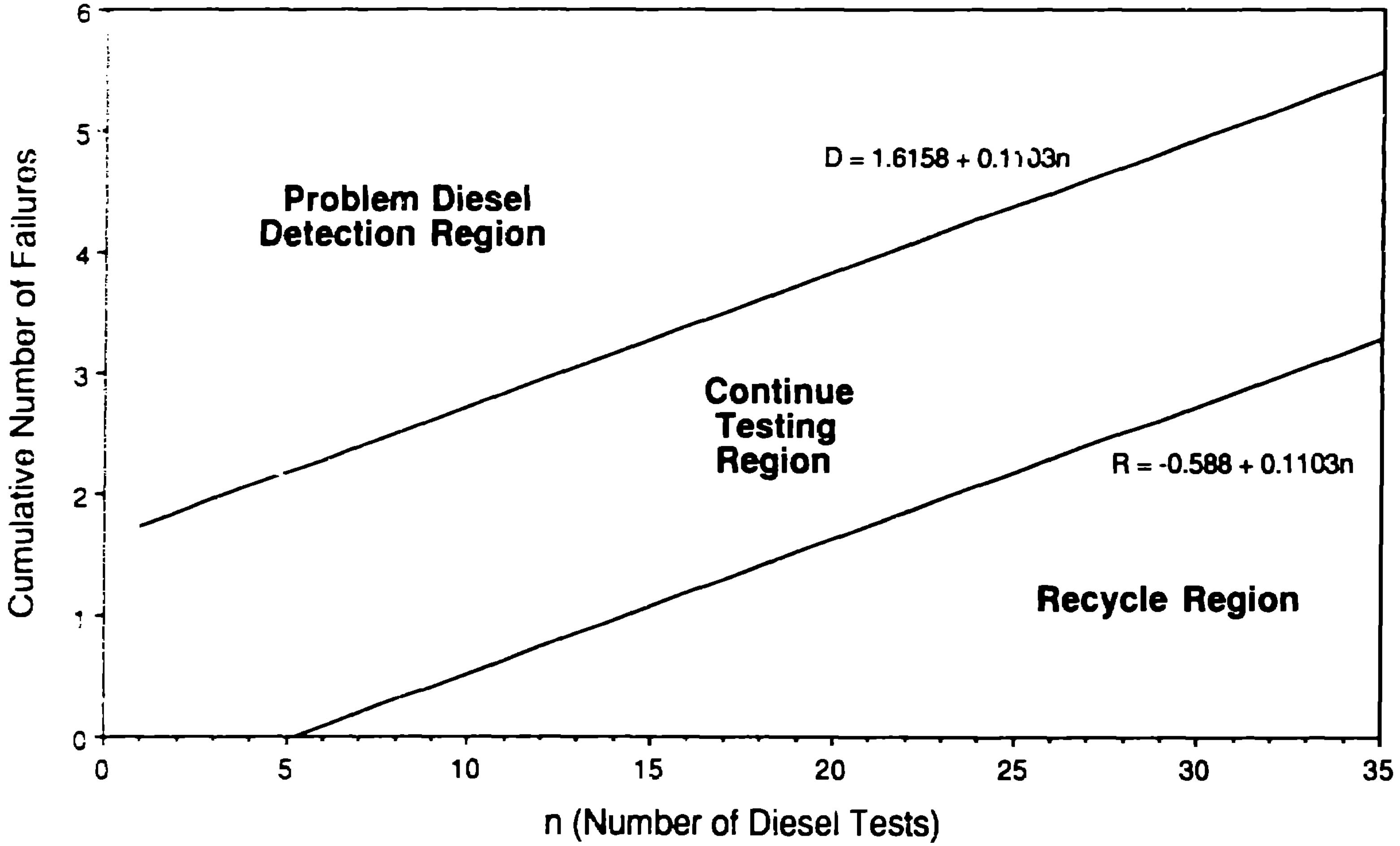


Figure 2

**Sequential Problem Diesel Trigger**



## Figure 3

### The Effect Of Initial Reliability On The Performance Of The Proposed Problem Diesel (4/25) and Wald Sequential Triggers For Detecting A Two-Diesel Shift To Reliability R At Month 20

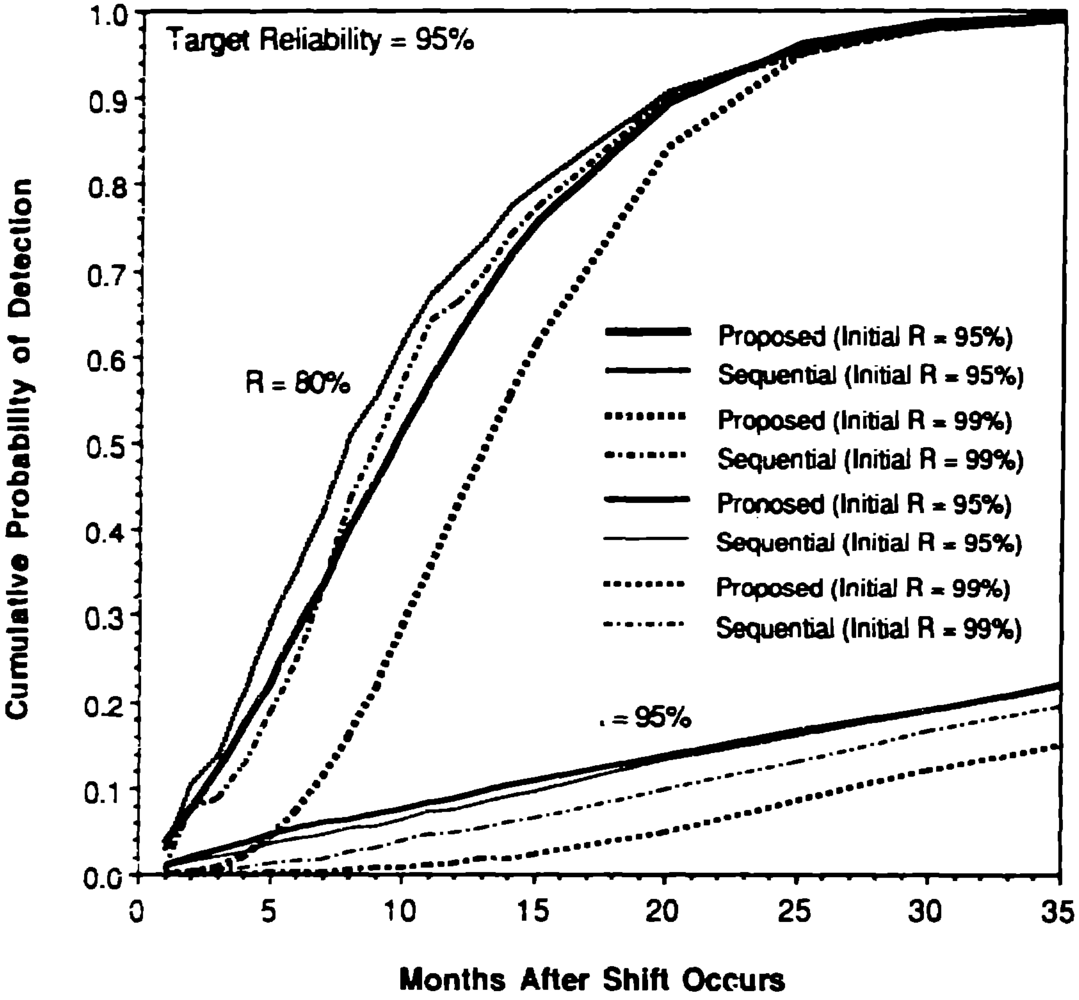


Figure 4

## Comparative Performance Of Both The Proposed Problem Diesel (4/25) and Base-Case Sequential Triggers For Detecting Two-Diesel Degradation To Reliability R (Initial Reliability = 99%)

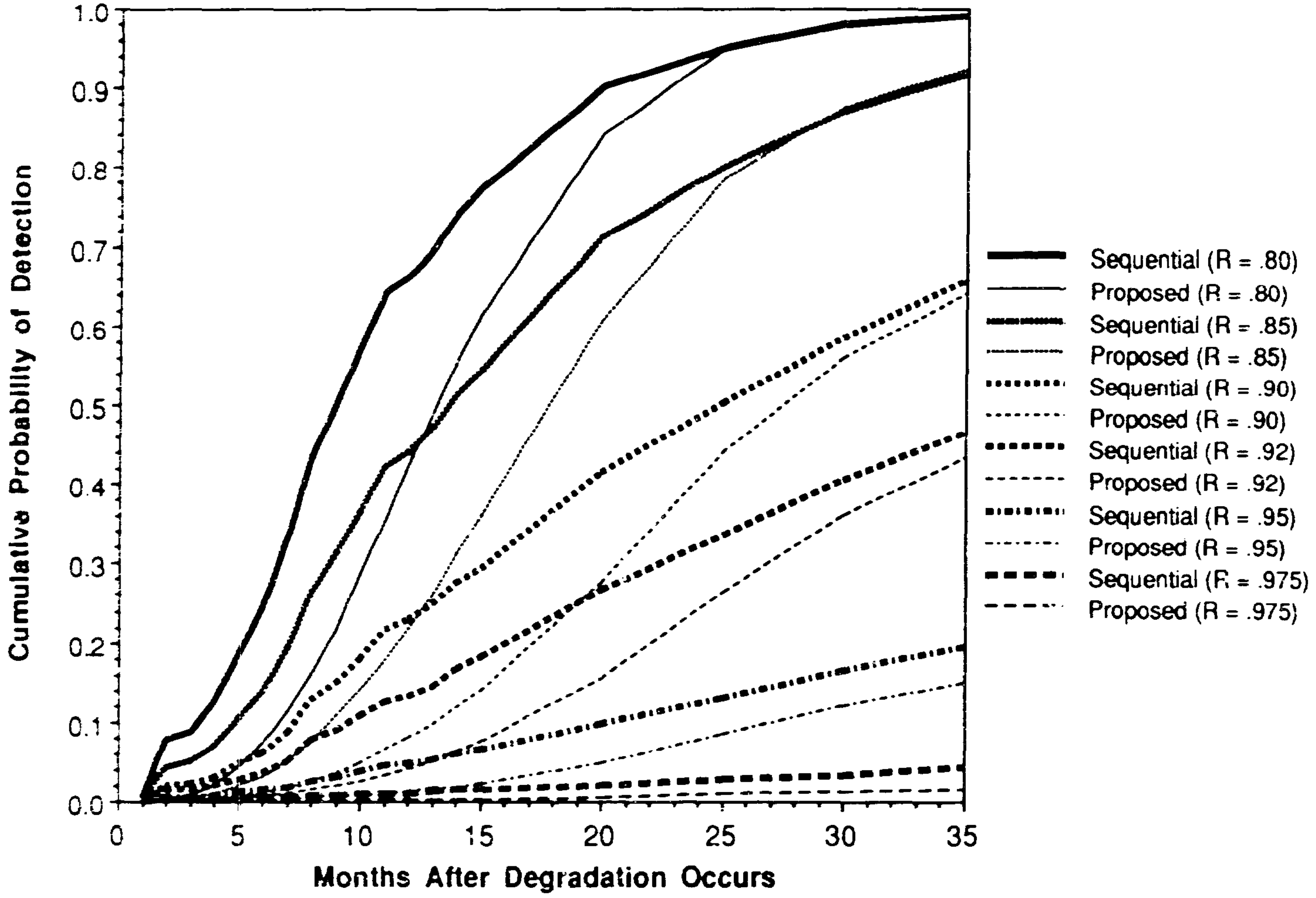


Figure 5

# Comparative Performance Of Both The Proposed Problem Diesel (4/25) and Wald Sequential Triggers For Detecting A Single-Diesel Shift To Reliability R At Month 20 (Initial Reliability = 97.5%)

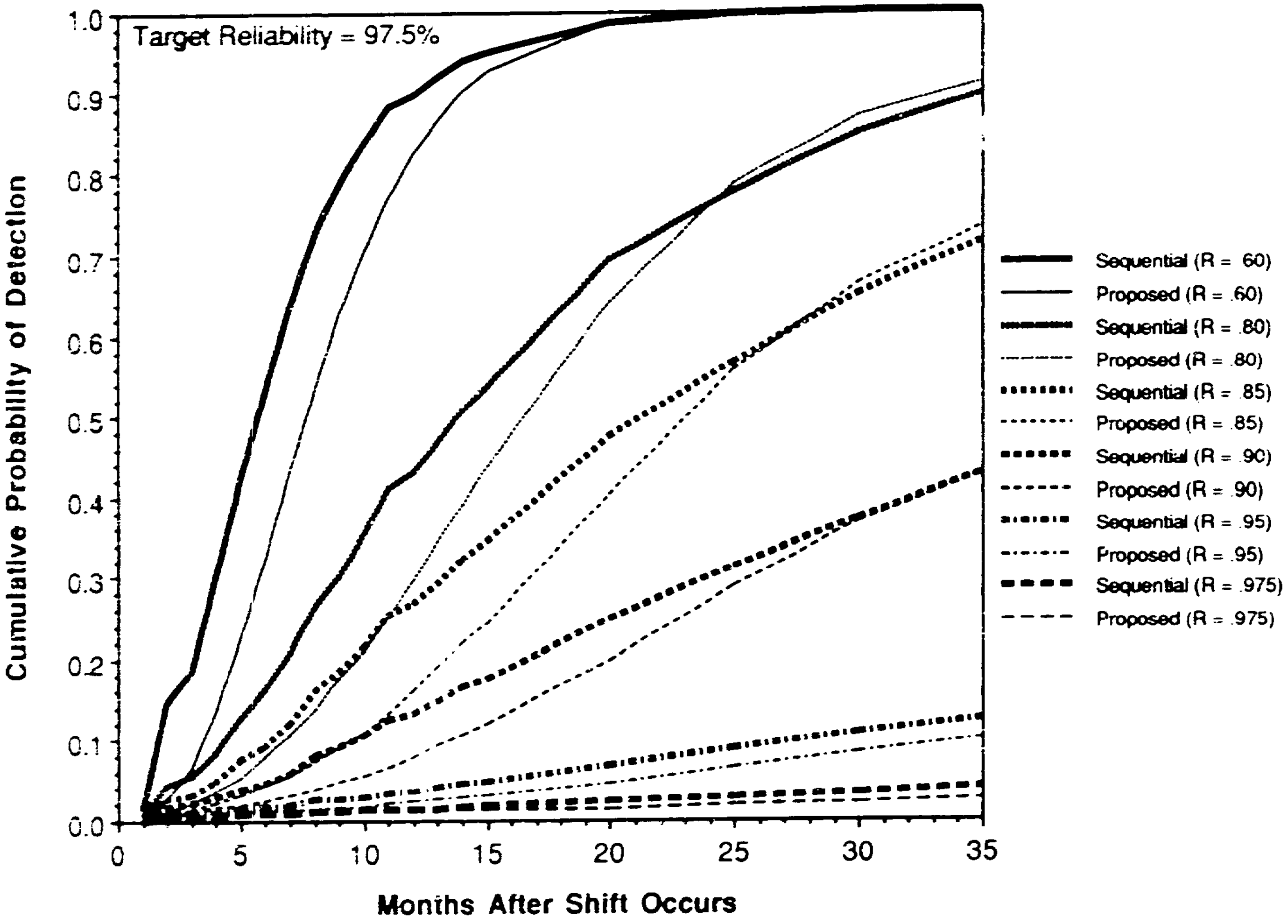


**Figure 6**

**Comparative Performance Of Both The Proposed Problem Diesel (4/25) and Wald Sequential Triggers For Detecting Two-Diesel Degradation To R = 80% (Initial Reliability = 99%)**

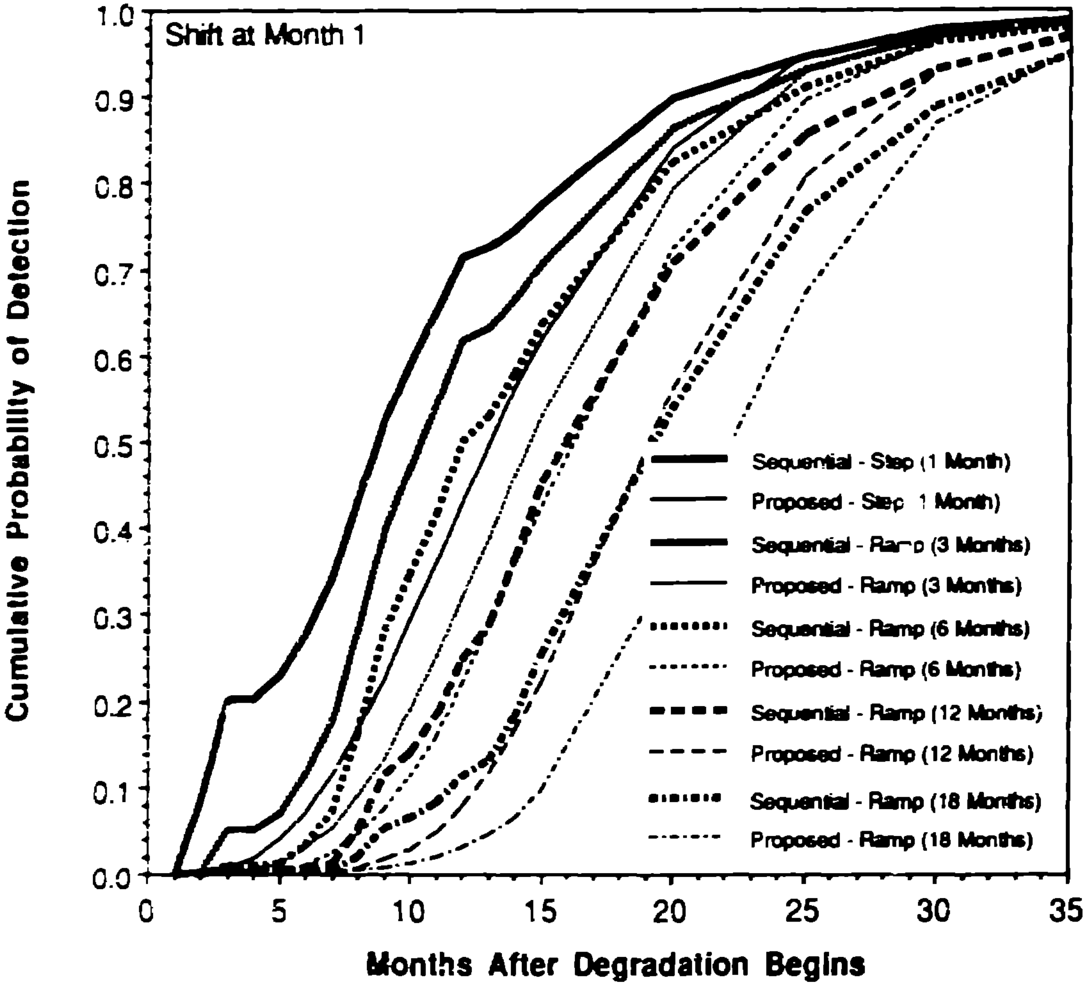


Figure 7

# Conditional Probability of Single-Diesel Degradation to Reliability RD Prior To Month k Given a Trigger Condition At Month k for Both Problem Diesel Trigger Procedures (EPS = 1.0E-2)

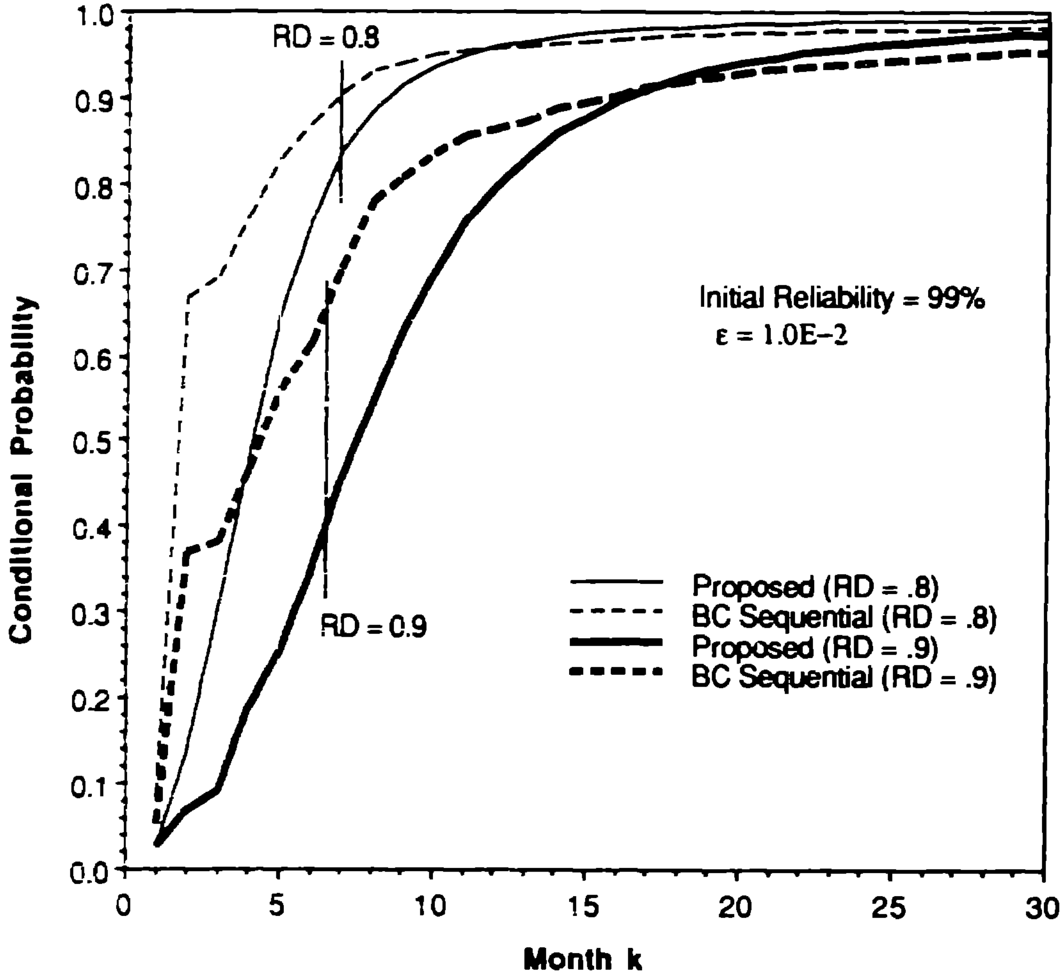
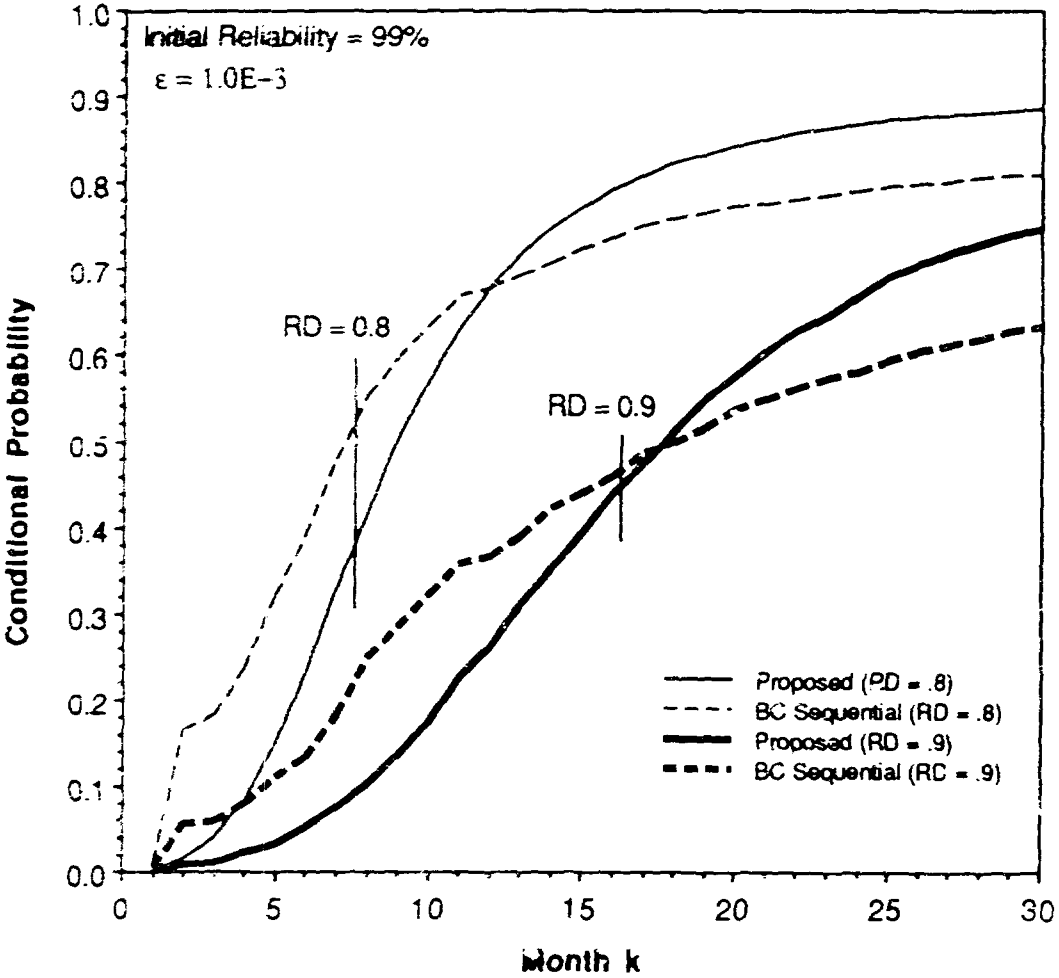


Figure 8

## Conditional Probability of Single-Diesel Degradation to Reliability RD Prior To Month k Given a Trigger Condition At Month k for Both Problem Diesel Trigger Procedures (EPS = 1.0E-3)



## Figure 9

## Conditional Probability of Single-Diesel Degradation to Reliability RD Prior To Month k Given a Trigger Condition At Month k for Both Problem Diesel Trigger Procedures (EPS = 1.0E-4)