

On Multibrane Solutions in Open String Field Theory

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Existence of open string field theory solutions describing configurations of multiple space-filling D-branes has been a subject of numerous speculations for quite some time. In this talk we present some new results giving further support to these ideas.

§1. Introduction

Since the original insight by Sen¹⁾ that the tachyon vacuum of open string field theory can be constructed in the so-called universal sector of the conformal field theory, and that it describes a state with no D-brane, it led naturally to the question whether there are other physically distinct solutions in this sector. In particular, whether there is a solution describing a state of multiple D-branes. The result of numerous numerical searches in level truncation was inconclusive if not negative, but there were good arguments based on vacuum string field theory which suggested otherwise.^{2),3)}

The starting point of the present work was a simple observation, that up to this date, there had been no computation of the energy for a class of analytic solutions of the open string field theory of the form^{***)}

$$\Psi = Fc \frac{KB}{1-F^2} cF, \quad (1.1)$$

where $F(K)$ is an arbitrary analytic function of K . In this form the solution was written down first by Okawa⁵⁾ generalizing the second author's original solution⁶⁾ corresponding to the choice $F(K) = e^{-K/2}$.

The outcome of the energy computation should be a functional that given an analytic function F produces a number. A very distinctive feature of this functional is that it has to be invariant under smooth deformations of F , such that $(1-F^2)^{-1}$ is also smooth, since arbitrary value F' can be obtained from F via a gauge transformation⁷⁾ $\Psi \rightarrow \Psi' = U^{-1}(Q + \Psi)U$, where

$$\begin{aligned} U &= 1 - FBcF + \frac{1-F^2}{1-F'^2} F' BcF', \\ U^{-1} &= 1 - F' BcF' + \frac{1-F'^2}{1-F^2} FBcF. \end{aligned} \quad (1.2)$$

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***) The notation used here is identical to that of many recent papers on the subject (see e.g. Ref. 4) for a lightning review of the subject).

Although this might be sufficient information for making educated guesses, let us instead proceed with a careful computation.

For the string field to be geometric (i.e. a superposition of wedge states with insertions, see Ref. 4) for further discussion) we demand that

$$F(K) = \int_0^\infty d\alpha f(\alpha)e^{-\alpha K}. \tag{1.3}$$

This imposes the aforementioned condition of analyticity in K . Very reasonable additional condition is called L_0 -safety which is equivalent to the statement that $F(z)$ be holomorphic for $\text{Re } z > 0$ and bounded by a polynomial in $|z|$ in the whole domain $\text{Re } z \geq 0$.

§2. Energy computation

The energy of a general solution of the form (1.1) can be most easily computed from the kinetic term. After a few simple algebraic manipulations one finds

$$\begin{aligned} \langle \Psi, Q\Psi \rangle = & \left\langle \frac{K}{G}, (1-G), \frac{K}{G}, KG \right\rangle - \left\langle K, (1-G), \frac{K}{G}, K \right\rangle \\ & - \left\langle \frac{K}{G}, (1-G), K, K \right\rangle + \left\langle K, (1-G), K, \frac{K}{G} \right\rangle, \end{aligned} \tag{2.1}$$

where

$$G = 1 - F^2 \tag{2.2}$$

and to simplify our notation we have introduced

$$\langle F_1, F_2, F_3, F_4 \rangle = \langle F_1(K)cF_2(K)cF_3(K)cF_4(K)cB \rangle \tag{2.3}$$

for general $F_i(K)$. Assuming that all F_i are L_0 -safe and geometric, we can compute this basic correlator by writing

$$F_i(K) = \int_0^\infty f_i(\alpha)e^{-\alpha K}, \tag{2.4}$$

and using a formula from Refs. 5), 6) and 8)

$$\begin{aligned} & \langle e^{-\alpha_1 K} c e^{-\alpha_2 K} c e^{-\alpha_3 K} c e^{-\alpha_4 K} c B \rangle \\ & = \langle c(\alpha_1)c(\alpha_1 + \alpha_2)c(\alpha_1 + \alpha_2 + \alpha_3)c(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)B \rangle_{C_s} \\ & = \frac{s^2}{4\pi^3} \left[\alpha_4 \sin \frac{2\pi\alpha_2}{s} - (\alpha_3 + \alpha_4) \sin \frac{2\pi(\alpha_2 + \alpha_3)}{s} \right. \\ & \quad + \alpha_2 \sin \frac{2\pi\alpha_4}{s} - (\alpha_2 + \alpha_3) \sin \frac{2\pi(\alpha_3 + \alpha_4)}{s} \\ & \quad \left. + \alpha_3 \sin \frac{2\pi(\alpha_2 + \alpha_3 + \alpha_4)}{s} + (\alpha_2 + \alpha_3 + \alpha_4) \sin \frac{2\pi\alpha_3}{s} \right], \end{aligned}$$

where we have denoted $s = \sum_{i=1}^4 \alpha_i$. To find the energy one needs the quadruple integral

$$\int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4 f_1(\alpha_1) f_2(\alpha_2) f_3(\alpha_3) f_4(\alpha_4) \times \langle e^{-\alpha_1 K} c e^{-\alpha_2 K} c e^{-\alpha_3 K} c e^{-\alpha_4 K} c B \rangle, \quad (2.5)$$

which in general might be quite hard to evaluate, especially since f_i will typically be distributions and not ordinary functions. It is thus desirable to reexpress the correlator (2.3) in terms of F_i only. To do that, we use a little trick. Inside the multiple integral over α_i we insert an identity in the form

$$1 = \int_0^\infty ds \delta \left(s - \sum_{i=1}^4 \alpha_i \right) = \int_0^\infty ds \int_{-i\infty}^{+i\infty} \frac{dz}{2\pi i} e^{sz} e^{-z \sum_{i=1}^4 \alpha_i}, \quad (2.6)$$

which allows us to treat s as independent of the other integration variables α_i . The integral over all α_i can be easily performed and we find

$$\begin{aligned} \langle F_1, F_2, F_3, F_4 \rangle = \int_0^\infty ds \int_{-i\infty}^{i\infty} \frac{dz}{2\pi i} \frac{s^2}{4\pi^3} e^{sz} \frac{1}{2i} \left[-F_1 \Delta F_2 F_3 F_4' + F_1 \Delta (F_2 F_3') F_4 \right. \\ \left. + F_1 \Delta (F_2 F_3) F_4' - F_1 F_2' F_3 \Delta F_4 + F_1 F_2 \Delta (F_3' F_4) + F_1 F_2' \Delta (F_3 F_4) \right. \\ \left. + \Delta F_1 F_2 F_3' F_4 + (s F_1 + F_1') F_2 \Delta F_3 F_4 \right], \quad (2.7) \end{aligned}$$

where for convenience we have omitted common argument z and also introduced an operator Δ defined as

$$(\Delta F)(z) = F \left(z - \frac{2\pi i}{s} \right) - F \left(z + \frac{2\pi i}{s} \right). \quad (2.8)$$

Using this general formula to evaluate (2.1) one finds long expression with 38 terms. To simplify this further we can close the integration contours by adding a half-circle at infinity in the $\text{Re } z < 0$ half-plane. If the falloff of the integrand is sufficiently fast, these extra contributions are actually vanishing. Most of the terms depend on z through the combination $z \pm \frac{2\pi i}{s}$. Having closed the contour, one can now split the integral into 38 individual integrals, apply change of variables and deform the contour back to its original position for each of the terms. With a judicious choice of these shifts which removes s dependence of the denominators the whole expression simplifies to^{*)}

$$\langle \Psi, Q\Psi \rangle = \frac{3}{\pi^2} \int_0^\infty ds \oint \frac{dz}{2\pi i} e^{sz} \left[z^2 s \frac{G'(z)}{G(z)} + (z\partial_z - s\partial_s) \left(\frac{izs^2}{8\pi} \frac{\Delta(zG)(z)}{G(z)} \right) \right]. \quad (2.9)$$

The second term is effectively a total derivative which under suitable conditions does not contribute, while the first term can be simplified by performing the s integral.

^{*)} We have performed the algebraic manipulations symbolically with Mathematica.

The final result for the energy $E = \frac{1}{6} \langle \Psi, Q\Psi \rangle$ is thus

$$\frac{E}{\text{Vol}} = \frac{1}{2\pi^2} \oint_C \frac{dz}{2\pi i} \frac{G'(z)}{G(z)}, \tag{2.10}$$

where C is a contour running upward the imaginary axis, bypassing possible singularity at the origin on the left, and finally closing at infinity in the $\text{Re } z < 0$ plane. The open string coupling constant has been set to unity and Vol denotes the D-brane volume.

§3. Energy from the closed string overlap

Ellwood has recently observed⁹⁾ that the value of Shapiro-Thorn-Hashimoto-Itzhaki-Gaiotto-Rastelli-Sen-Zwiebach invariant, or Ellwood invariant for short,

$$\langle I | c\bar{c}V_{\text{cl}}(i) | \Psi \rangle = \mathcal{A}_{\Psi}^{\text{disk}}(V_{\text{cl}}) - \mathcal{A}_0^{\text{disk}}(V_{\text{cl}}) \tag{3.1}$$

is equal to the difference of the closed string one-point function between the vacuum described by the string field Ψ and the reference perturbative vacuum $\psi = 0$. He found that for the tachyon vacuum solution $\mathcal{A}_{\Psi}^{\text{disk}}(V_{\text{cl}})$ vanishes, in agreement with the fact that in the absence of D-branes there are no disk amplitudes to be computed. Since then it has become a standard practise to evaluate this invariant every time a new analytic solution appears, as in Ref. 7). One could also consider the full boundary state constructed in Ref. 10) but for simplicity we restrict ourselves to the Ellwood invariant only.

Let us thus compute the invariant for a general solution of the form (1.1). By a simple modification of Ellwood's original computation or applying the scaling arguments of Ref. 7) one easily derives

$$\begin{aligned} \langle c\bar{c}V_{\text{cl}}(i)c\tilde{F}BcF^2 \rangle &= \int_0^\infty d\alpha \int_0^\infty d\beta \frac{2i}{\pi} \beta \tilde{f}(\alpha) f(\beta) \langle V_{\text{cl}}(i) \rangle_{UHP}^{\text{matter}} \\ &= \tilde{F}(0) \partial F^2(0) \mathcal{A}_0^{\text{disk}}(V_{\text{cl}}), \end{aligned} \tag{3.2}$$

where f and \tilde{f} are inverse Laplace transforms of F^2 and \tilde{F} respectively. For a solution of the equations of motion $\tilde{F} = K/(1 - F^2)$ and hence

$$\left\langle c\bar{c}V_{\text{cl}}(i) F c \frac{KB}{1 - F^2} cF \right\rangle = - \lim_{z \rightarrow 0} z \frac{G'(z)}{G(z)} \mathcal{A}_0^{\text{disk}}(V_{\text{cl}}), \tag{3.3}$$

where $G = 1 - F^2$ as in the previous section. We have used cyclicity of the bracket and the fact that F commutes with midpoint insertions. Interestingly, we see that for G holomorphic or meromorphic at zero, the Ellwood invariant and hence also the new background one-point function is given by an integer multiple of the one-point function in the perturbative vacuum.

The closed string one-point function for an appropriate choice of the graviton vertex operator can measure the energy of the configuration, and therefore the Ellwood invariant would measure the difference in energy between the new configuration

and that of the perturbative vacuum. We can thus write an alternative formula for the energy of the class of solutions (1.1) as

$$\frac{E}{\text{Vol}} = -\frac{1}{2\pi^2} \lim_{z \rightarrow 0} z \frac{G'(z)}{G(z)}, \quad (3.4)$$

where we fixed the overall coefficient by matching to the correct answer for the tachyon vacuum.

At first sight the results (2.10) and (3.4) look rather different. Note however that the function G is required to be holomorphic in the right half plane. If we assume it is meromorphic at zero, then G'/G can have at most first order pole and hence (3.4) can be rewritten as

$$\frac{E}{\text{Vol}} = -\frac{1}{2\pi^2} \oint_{C_0} \frac{dz}{2\pi i} \frac{G'(z)}{G(z)}, \quad (3.5)$$

where C_0 is now a small contour around the origin. The difference between (2.10) and (3.5) is given by a contour integral along the imaginary axis, bypassing possible singularity at zero on the right, and closing at infinity in the $\text{Re } z < 0$ plane. Assuming that the function G'/G happens to be holomorphic at infinity we can then shrink the new contour $C - C_0$ around the point at infinity and we get zero. This shows that under the above-mentioned assumptions both energy computations (2.10) and (3.4) give an identical answer.

§4. Discussion

We have shown that the class of string field solutions of the form

$$\Psi = Fc \frac{KB}{1 - F^2} cF \quad (4.1)$$

have their energy density equal to

$$\frac{E}{\text{Vol}} = \frac{1}{2\pi^2} \oint_C \frac{dz}{2\pi i} \frac{G'(z)}{G(z)} = -\frac{1}{2\pi^2} \lim_{z \rightarrow 0} z \frac{G'(z)}{G(z)} \quad (4.2)$$

under certain holomorphicity assumptions. For holomorphic functions both expressions give integer multiples of single D-brane tension and hence a very natural guess would be that some of these solutions describe configurations of multiple D-branes. For $G(z) \sim z^n$ for small z the energy comes out negative. Of course $n = 1$ is the tachyon vacuum, but what about $n \geq 2$? This looks like a negative number of branes. We thus expect the solution to be sick and one of the signs of this sickness is a term of negative scaling dimension in (1.1) with dimension less than -1 . This implies that at least some coefficients in the Fock basis will be divergent.

The really interesting question is whether solutions with positive number of branes can make some sense and it is a subject of an ongoing investigation.¹¹⁾ The function G has to behave as $G \sim z^{-n}$ and such a solution would describe a configuration of $n + 1$ D-branes. The main problem is that such a G is not a well-defined L_0 -safe string field and it is not yet clear whether this presents a substantial problem also for Ψ . Happily, the negative scaling dimension terms, which ruled out the

negative tension configurations, are not present in this case. One possible example of multiple D-brane solutions would be given by

$$G(z) = \left(\frac{z+1}{z} \right)^n, \quad (4.3)$$

but we have not yet verified whether the corresponding solution Ψ has a well-behaved level expansion. Additionally, we would like to see enhancement of cohomology around such solutions as well.

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