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2016 J. Phys. B: At. Mol. Opt. Phys. 49 170502

(http://iopscience.iop.org/0953-4075/49/17/170502)

IOP Publishing

Journal of Physics B: Atomic, Molecular and Optical Physics

J. Phys. B: At. Mol. Opt. Phys. **49** (2016) 170502 (3pp)

doi:10.1088/0953-4075/49/17/170502

Viewpoint

On multidimensional solitons and their legacy in contemporary Atomic, Molecular and Optical physics

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(Some figures may appear in colour only in the online journal)

This viewpoint relates to an article by B A Malomed, D Mihalache, F Wise, and L Torner (2005 *J. Opt. B: Quantum Semiclass. Opt.* **7** R53–R72) and was published as part of a series of viewpoints celebrating 50 of the most influential papers published in the *Journal of Physics* series, which is celebrating its 50th anniversary.

This review article (RA) was a brief but comprehensive survey of the general area of *multidimensional solitons*, i.e., two- and three-dimensional (2D and 3D) modes self-trapped as a result of the competition between the linear effects of diffraction and dispersion, which tend to stretch any wave packet in spatial and temporal directions, and nonlinear self-compression of the wave field. The RA has produced an appreciable impact, having been cited 532 times (Google Scholar, as of 12 April 2016).

Multidimensional solitons draw continuously renewed interest in many branches of physics, finding especially important realizations in nonlinear photonics (optics and plasmonics) and Bose–Einstein condensates (BECs); in particular, spatiotemporal optical solitons are also known as ‘light bullets’. The solitons are classified as *fundamental* ones, which carry no topological structure, and various topological modes, including 2D and 3D solitons with embedded vorticity, and more sophisticated 3D states, such as *hopfions*, i.e., vortex tori with intrinsic twist, which carry two independent topological numbers.

Unlike 1D solitons, which are normally stable, their multidimensional counterparts are vulnerable to severe instabilities. Indeed, the ubiquitous cubic self-attractive nonlinearity, which readily creates solitons, simultaneously gives rise to the critical and supercritical *collapse*, i.e., spontaneous formation of a singularity after a finite propagation distance or time, in the 2D and 3D geometries, respectively. The collapse destabilizes fundamental solitons, while their vortex solitons are subject to a still stronger splitting instability against perturbations breaking the axial symmetry of the vortices. Accordingly, a challenging problem is search for physically relevant settings that admit stabilization of the solitons, the settings being categorized according to the underlying stabilization mechanisms. Several generic mechanisms have been identified in the RA and developed in subsequent works: (i) the use of quadratic (second-harmonic-generating) or saturable nonlinearities, which do not lead to the collapse, and thus make fundamental solitons automatically stable, but failing to stabilize vortices; (ii) effective trapping potentials (in particular, spatially periodic ones, such as photonic lattices in optics, or optical lattices in BEC), which may stabilize 2D and 3D solitons of all types (lattice potentials create stable vortex solitons in the form of multipeak complexes, with the vorticity represented by phase shifts between adjacent peaks); (iii) competing nonlinearities, such as combinations of attractive cubic and repulsive

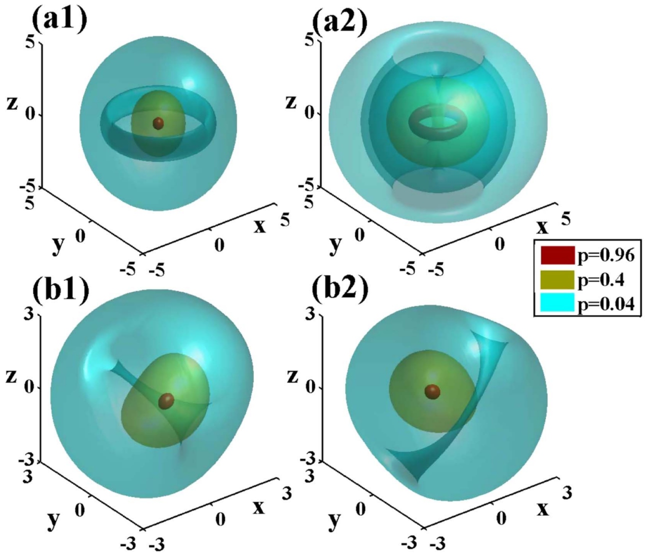

Figure 1. Examples of metastable three-dimensional solitons produced by the two-component BEC model with spin-orbit coupling, as per [12]: (a1, a2) density (p) profiles of the fundamental and vortex components; (b1, b2) the same in a soliton with the fundamental and vortex terms mixed in each component.

quintic terms, which secure partial stabilization of vortices; (iv) *management* techniques, which impose periodic switch of the nonlinearity between attraction and repulsion, making it possible to stabilize 2D fundamental solitons. Following the review provided in the RA, these techniques, as well as their combinations, have been developed in a large number of works, revealing many possibilities for the creation of stable solitons of different types.

An essential peculiarity of this research area, which was stressed in the RA, and remains obvious presently, is disbalance between a very large number of theoretical predictions and few experimental results. Nevertheless, in the course of 10 years since the appearance of the RA, several essential experimental findings have been published. These include, in particular, a soliton in the BEC of ^{85}Rb atoms with aspect ratio 2.5 of the trapping potential, which makes the soliton's shape close to isotropic [1]; fundamental [2] and vortex [3] 'optical bullets' created in arrays of optical fibers, which may be considered as semi-discrete media; the creation of (2+1)D spatial fundamental solitons in bulk optical media with a cubic-quintic [4] competing nonlinearity; 2D exciton-polariton *gap solitons* (supported by a lattice potential) in a microcavity [5]; direct observation of filamentation of ultrashort laser pulses in a Kerr medium in the case of negative group-velocity dispersion (which is necessary for the formation of 'bullets') [6], and the observation of a characteristic structure of the self-compressing 'bullet', composed of a high-density core and a surrounding ring pattern [7]. The creation of truly stable multidimensional vortex solitons in continuous media has not been reported yet (solitary vortices, whose limited stabilization is supported by nonlinear loss, were observed very recently [8]).

The theoretical work in the area has been developing in many directions since the publication of the RA, being, to a large extent, stimulated by results summarized in it. In particular, completely new settings allowing the creation of multidimensional solitons have been elaborated. One of them is the use of D-dimensional media with *repulsive* nonlinearity, whose local strength grows from the center to periphery, as a function of distance r, at any rate faster than r^D. This setting supports a variety of robust 2D and 3D solitons, including quite sophisticated ones, such as *soliton gyroscopes* and *hopfions* [9]. This direction is related to a still broader area of studies of solitons in media with effective potentials induced by spatial modulation of the nonlinearity [10]. Further, recent considerations of BEC with linear *spin–orbit coupling* between its components give rise to objects which were assumed impossible: *stable solitons* in free 2D [11] and 3D [12] space, supported by the attractive cubic nonlinearity, without the help of any trapping potential. They are built of mixed fundamental and vortical components, see the figure 1. These findings help to understand the profound difference between the stabilization in 2D and 3D settings: in the former case, the system creates stable solitons as *ground states*, while in 3D a ground state does not exist in the presence of the supercritical collapse, the solitons being metastable modes.

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