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Abstract

Separation of Duty (SoD) is widely considered to be a fundamental principle in computer security. A Static SoD (SSoD) policy states that in order to have all permissions necessary to complete a sensitive task, the cooperation of at least a certain number of users is required. In Role-Based Access Control (RBAC), Statically Mutually Exclusive Roles (SMER) constraints are used to enforce SSoD policies. In this paper, we pose and answer fundamental questions related to the use of SMER constraints to enforce SSoD policies. We show that directly enforcing SSoD policies is intractable (**coNP**-complete), while checking whether an RBAC state satisfies a set of SMER constraints is efficient. Also, we show that verifying whether a given set of SMER constraints enforces an SSoD policy is intractable (**coNP**-complete) and discuss why this intractability result should not lead us to conclude that SMER constraints are not an appropriate mechanism for enforcing SSoD policies. We also show how to generate SMER constraints that are as accurate as possible for enforcing an SSoD policy.

1 Introduction

Separation of Duty (SoD) is widely considered to be a fundamental principle in computer security [1, 2, 12]. In its simplest form, the principle states that if two steps are needed to perform a sensitive task, then two different users should each perform one of the two steps. More generally, when n steps are needed to perform a sensitive task, an SoD policy requires the cooperation of at least k (for some $k \leq n$) different users to complete the task.

Consider the following example of buying and paying for goods. The steps to perform such a task (taken from [2]) are: (1) ordering the goods and recording the details of the order; (2) recording the arrival of the invoice and verifying that the details on the invoice match the details of the order; (3) verifying that the goods have been received, and the features of the goods match the details on the invoice; and (4) authorizing the payment to the supplier against the invoice. We would want to ensure that the payment is not released on an order that was never placed and that the received goods match those in the order and those in the invoice. A policy that requires a different user to perform each step may be too restrictive. It may be permissible, for instance, that the user who places the order also records the arrival of the invoice. One may require that (a) at least three users are required to perform all four steps, and (b) two different users are required to perform steps (1) and (4) (i.e., no single user can order a good and authorize payment for it).

An SoD policy may be enforced either statically or dynamically. In dynamic enforcement, the system maintains a history of each task instance (e.g., a particular order). The history includes information on who

performed each step. Before a user performs a step on the instance, the system checks to ensure that the SoD policy is not violated. This is referred to as Dynamic SoD in the literature [13, 10].[1] In Dynamic SoD, a user may be able to perform any particular step in a task instance; however, the user cannot also perform other steps in that instance.

In static enforcement, Static SoD (SSoD) policies are used. Each SSoD policy states that no $k-1$ users together have all permissions to complete a sensitive task. Such an SSoD policy can be enforced by carefully assigning permissions to users, without maintaining a history for every task instance. It may seem that if an SSoD policy is satisfied, then the corresponding SoD policy is also satisfied. However, care must be taken to ensure this. Consider the example described above. Suppose that initially a user Bob has the permission to order goods. After placing an order, Bob's order permission is revoked and then Bob is assigned to have the permission to authorize payments. Now Bob can authorize a payment against the order he placed earlier. The SoD policy is violated even though Bob never has the order permission and payment permission at the same time. Such situations can be avoided, for example, by requiring that a user is not participating in any active task instance while being assigned a permission, or by treating such task instances specially (e.g., by maintaining a history for them).

Separation of Duty has been studied extensively in Role-Based Access Control (RBAC) [6, 7, 14]. As Ferraiolo et al. [6] state, "one of RBAC's great advantages is that SoD rules can be implemented in a natural and efficient way." A purpose of this paper is to examine this statement in detail. In RBAC, permissions are associated with roles, and users are granted membership in appropriate roles, thereby acquiring the roles' permissions. RBAC uses mutual exclusion constraints to implement SoD policies. The most common kind of mutual exclusion constraint is Statically Mutually Exclusive Roles (SMER). For example, a SMER constraint could be "no user is allowed to be a member of both r_1 and r_2". More generally, a SMER constraint requires that no user is a member of t or more roles in a set of m roles $\{r_1, r_2, \cdots, r_m\}$. SMER constraints are used in most RBAC models, including the RBAC96 models by Sandhu et al. [14] and the proposed NIST standard for RBAC [7]. Literature in RBAC also studies dynamic mutually exclusive roles (DMER) constraints. With such a constraint, a user is prevented from activating mutually exclusive roles simultaneously in a session. SMER and DMER constraints are the only types of constraints included in the proposed NIST standard for RBAC [7]. The rationale provided in that work is that such constraints are the only ones prevalent in commercial RBAC products.

As we discuss in Section 2, DMER constraints are not suitable for enforcing SoD policies, either statically or dynamically. On the other hand, SMER constraints enforce SoD policies statically. In this paper, we examine the use of SMER constraints to enforce SoD policies.

SSoD policies are *objectives* that need to be achieved. They exist independent of whether RBAC is used to manage the access permissions. Each SSoD policy specifies the minimum number of users that can perform a sensitive task. On the other hand, SMER constraints are *mechanisms* introduced to achieve SSoD policies. These constraints are specific to RBAC. Each constraint limits the role memberships any user may have.

In the literature, this distinction between objectives and mechanisms is sometimes not clearly made. This is evident in the way these constraints are referred to in the literature. SMER constraints are often called Static SoD constraints, and DMER are called Dynamic SoD constraints.

When we make a clear distinction between objectives (SSoD policies) and mechanisms (SMER constraints), several interesting problems arise. For example, the *verification* problem is whether a set of SMER constraints indeed enforces an SSoD policy, and the *generation* problem is how do we generate a set of con-

[1] Nash and Poland [10] refer to this as object SoD and consider it as one kind of Dynamic SoD.

straints that is adequate to enforce an SSoD policy. Although the use of SMER constraints to support SoD has been studied for over a decade, surprisingly these problems have not been examined in the literature as such, to the best of our knowledge.

1.1 Contributions and organization

Our contributions in this paper are as follows.

- We provide precise definitions for SSoD policies and SMER constraints, and for the verification and generation problems.
- We show that directly enforcing SSoD policies in RBAC is intractable ($\mathbf{coNP}$-complete), while enforcing SMER constraints is efficient.
- We show that the verification problem is intractable ($\mathbf{coNP}$-complete), even for a basic subcase of the problem, but reduces naturally to the satisfiability problem (SAT) [3], for which there exist algorithms that have been proven to work well in practice [3]. We discuss the implications of these results.
- We define what it means for a set of SMER constraints to precisely enforce an SSoD policy, characterize the policies for which such constraints exist, and show how they are generated. For other SSoD policies, we present an efficient algorithm that generates sets of SMER constraints that minimally enforce the policies.

The results reported here are fundamental to understand the effectiveness of using SMER constraints to enforce SoD in RBAC. The verification and generation algorithms are also of practical significance in RBAC systems that use SMER constraints to enforce SSoD policies.

The remainder of the paper is organized as follows. We discuss related work in the next section. In Section 3, we give definitions of SSoD policies and SMER constraints, as well as the computational complexities for enforcing them. In Section 4, we study the verification problem. In Section 5, we study the generation problem. We conclude with Section 6. Proofs not included in the main body are in Appendix A.

2 Related Work

To our knowledge, in the literature the notion of SoD first appeared in Saltzer and Schroeder [12] under the name "separation of privilege." They introduced this as one of the eight design principles for the protection of information in computer systems. They credited Roger Needham with making the following observation in 1973: a protection mechanism that requires two keys to unlock it is more robust and flexible than one that requires only a single key. No single accident, deception, or breach of trust is sufficient to compromise the protected information.

Clark and Wilson's commercial security policy for integrity [2] identified SoD along with well-formed transactions as two major mechanisms of fraud and error control. The use of well-formed transactions ensures that information within the computer system is internally consistent. Separation of duty ensures that the objects in the physical world are consistent with the information about these objects in the computer system.

Sandhu [13] presented a history-based mechanism for dynamically enforcing SoD policies. Nash and Poland [10] emphasized the difference between dynamic and static enforcement of SoD policies. In the former, a user may perform any step in a sensitive task provided that the user does not also perform another step on that data item. In the latter, users are constrained a-priori from performing certain steps.

In one of the earliest paper on RBAC, Ferraiolo and Kuhn [4] used the terms Static and Dynamic SoD to refer to static and dynamic enforcement of SoD. In a subsequent paper, Ferraiolo et al. [5] defined Static SoD as: "A user is authorized as a member of a role only if that role is not mutually exclusive with any of the other roles for which the user already possesses membership". Observe that this is the requirement of SMER constraints. Similarly, Dynamic SoD was defined as forbidding a user from activating roles that are mutually exclusive. We call these DMER constraints. We believe that the terminology in [5] is confusing as it blurs the distinction between objectives and mechanisms. The same terminology is later used by several authors and is adopted in the NIST proposed standard for RBAC [7].

DMER constraints are introduced in [5] under the name DSoD constraints. This may be because they are the "dynamic" version of SMER constraints, which are referred to as SSoD constraints in [5]. However, as we now discuss, DMER constraints do not seem to enforce SoD policies. A DMER constraint prevents a user from simultaneously activating mutually exclusive roles in a session. In RBAC, each session has only one user. Thus, a sensitive task cannot be finished in one session; different sessions are required. Consider the example discussed in Section 1. Suppose that the order permission and the payment permission are assigned to two roles that are mutually exclusive according to a DMER constraint. Bob can start a session, activating the role having the order permission, create an order, end the session, start another session, activating the role having the payment permission, and authorize a payment against the order. This violates the SoD policy.

Kuhn [9] discussed mutual exclusion of roles for separation of duty and proposes a safety condition: that no one user should possess the privilege to execute every step of a task, thereby being able to execute the task. We observe that our definition for safety in Section 3.1 is a generalization of Kuhn's definition [9]: setting k to 2 gives us Kuhn's definition. Kuhn [9] did not discuss either the verification problem or the generation problem.

3 Static Separation of Duty and Mutually Exclusive Roles

In this section, we give precise definitions for Static Separation of Duty policies, RBAC, and SMER constraints. *Users* (often referred to as *subjects* in the literature) and *permissions* are at the core of any access control system. The state of the access control system specifies the set of permissions each user has. In this paper, we treat permissions as if they were opaque, i.e., we do not consider the internal structure of permissions. We also assume that permissions are not related, e.g., the possession of one or more permissions does not imply the possession of another permission.

3.1 Static Separation of Duty (SSoD) policies

Definition 1 (SSoD policies) A k-n SSoD (k-out-of-n Static Separation of Duty) policy is expressed as

$$\mathsf{ssod}\langle\{p_1, \ldots, p_n\}, k\rangle$$

where each p_i is a permission and n and k are integers such that $1 < k \leq n$. This policy means that there should not exist a set of fewer than k users that together have all these permissions. In other words, at least k users are required to perform a task that requires all the permissions in $\{p_1, \ldots, p_n\}$.

Intuitively, the permissions in a k-n SSoD policy are the permissions needed to carry out a sensitive task, and the policy guarantees that at least k users are needed to successfully execute it. The specification of an SSoD policy involves identifying a sensitive task, the permissions needed to complete it, and the minimum number of collaborating users authorized to complete it.

We assume a basic level of familiarity with RBAC; readers are referred to [7, 14] for an introduction to RBAC. We assume that there are three countably infinite sets: $\mathcal{U}$ (the set of all possible users), $\mathcal{R}$ (the set of all possible roles), and $\mathcal{P}$ (the set of all possible permissions).

Definition 2 (RBAC States) An RBAC state γ is a 3-tuple $\langle UA, PA, RH \rangle$, in which the user assignment relation $UA \subset \mathcal{U} \times \mathcal{R}$ associates users with roles, the permission assignment relation $PA \subset \mathcal{R} \times \mathcal{P}$ associates roles with permissions, and the role hierarchy relation $RH \subset \mathcal{R} \times \mathcal{R}$ is a partial order among roles in $\mathcal{R}$. When $(r_1, r_2) \in RH$, we say that r_1 is senior to r_2, which means that every user who is a member of r_1 is also a member of r_2, and every permission that is associated with r_2 is also associated with r_1.

An RBAC state $\gamma = \langle UA, PA, RH \rangle$ determines the set of roles a user is a member of and the set of permissions a user possesses. Formally, γ determines two functions, $\mathsf{roles}_\gamma : \mathcal{U} \rightarrow 2^{\mathcal{R}}$ and $\mathsf{perms}_\gamma : \mathcal{U} \rightarrow 2^{\mathcal{P}}$, where $2^{\mathcal{R}}$ is the powerset of $\mathcal{R}$. The two functions are defined as follows:

$$\begin{array}{lcl} \mathsf{roles}_\gamma[u] & = & \{\ r \in \mathcal{R} \ \mid\ \exists r_1 \in \mathcal{R}\, [\, (u, r_1) \in UA \ \wedge\ (r_1, r) \in RH\,]\ \} \\ \mathsf{perms}_\gamma[u] & = & \{\ p \in \mathcal{P} \ \mid\ \exists r_1, r_2 \in \mathcal{R}\, [\, (u, r_1) \in UA \ \wedge\ (r_1, r_2) \in RH \ \wedge\ (r_2, p) \in PA\,]\ \} \end{array}$$

Definition 3 (SSoD Safety and the SC-SSoD problem) We say that an RBAC state γ is *safe* with respect to an SSoD policy $\mathsf{ssod}\langle \{p_1, \ldots, p_n\}, k \rangle$ if in state γ no $k-1$ users together have all the permissions in the policy. More precisely,

$$\forall u_1 \cdots u_{k-1} \in \mathcal{U}\ \left(\left(\bigcup_{i=1}^{k-1} \mathsf{perms}_\gamma[u_i] \right) \not\supseteq \{p_1, \ldots, p_n\} \right).$$

An RBAC state γ is *safe* with respect to a set E of SSoD policies if it is safe with respect to every policy in the set, and we write it as $safe_E(\gamma)$. SC-SSoD (the Safety Checking problem for SSoD policies) is defined as follows: Given an RBAC state γ and a set E of SSoD policies, determine if $safe_E(\gamma)$ is true.

Observe that if no $k-1$ users together have all the permissions in the policy, then no set of fewer than k users together have all the permissions.

Example 1 Consider the task of ordering and paying for goods discussed in Section 1. We have a permission corresponding to each step in the task; these permissions are p_{order}, $p_{invoice}$, p_{goods}, and $p_{payment}$. We have the following set of SSoD policies:

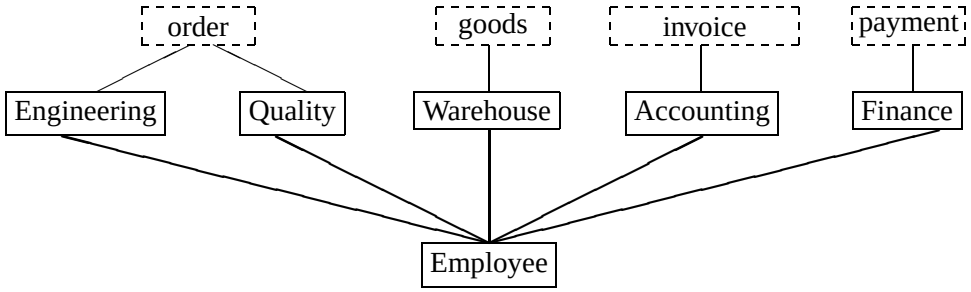
$$\begin{array}{lcl} E_1 & = & \{e_1, e_2\} \\ e_1 & = & \mathsf{ssod}\langle \{p_{order}, p_{invoice}, p_{goods}, p_{payment}\}, 3 \rangle \\ e_2 & = & \mathsf{ssod}\langle \{p_{order}, p_{payment}\}, 2 \rangle \end{array}$$

Consider the following RBAC state $\gamma_1 = \langle UA_1, PA_1, RH_1 \rangle$, where

$$UA_1 = \{\ (\mathsf{Alice},\ \mathsf{Warehouse}), (\mathsf{Alice},\ \mathsf{Finance}),\ (\mathsf{Bob},\ \mathsf{Accounting}), (\mathsf{Bob},\ \mathsf{Quality})\ \},$$

and PA_1 and RH_1 are given in Figure 1. The state γ_1 is not safe with respect to e_1, a 3-4 SSoD policy, as the 2 users Alice and Bob together possess all 4 permissions in e_1.

Given a set E of SSoD policies, suppose an RBAC system starts at a state that is safe with respect to E. Each time one is about to make a change to the system that may affect the safety, one checks whether the RBAC state resulted from the proposed change is safe and makes the change only then. Such a change may be adding a new user-role assignment to UA, adding a new role-permission assignment to PA, or adding a new pair to RH. This approach to ensuring that an RBAC system is safe requires solving SC-SSoD, which turns out to be intractable.



$$
\begin{aligned}
RH_1 &= \{\, (\mathsf{Engineering},\ \mathsf{Employee}),\ (\mathsf{Quality},\ \mathsf{Employee}),\ (\mathsf{Warehouse},\ \mathsf{Employee}),\\
&\quad\ (\mathsf{Accounting},\ \mathsf{Employee}),\ (\mathsf{Finance},\ \mathsf{Employee})\ \}.\\
PA_1 &= \{\, (\mathsf{Engineering},\ p_{order}),\ (\mathsf{Quality},\ p_{order}),\ (\mathsf{Warehouse}, p_{goods}),\\
&\quad\ (\mathsf{Accounting},\ p_{invoice}),\ (\mathsf{Finance},\ p_{payment})\ \}.
\end{aligned}
$$

Figure 1: A sample role hierarchy and permission assignment. Roles are shown in solid boxes, and permissions in dashed boxes. A line segment represents either a role-role relationship, or the assignment of a permission to a role.

Theorem 1 SC-SSoD *is* **coNP**-*complete.*

See Appendix A for the proof. (The proofs of all theorems and lemmas not found in the main body are in Appendix A.) The proof of **coNP**-hardness is done by reducing the set covering problem to the complement of SC-SSoD. In that reduction, each permission is assigned to one role and the role hierarchy relation is empty; thus the problem remains **coNP**-complete even when we restrict to the case of flat RBAC (i.e., RBAC without role hierarchy). The fact that SC-SSoD is intractable suggests that enforcing SSoD policies directly can be computationally expensive.

With the following reasoning, we observe that even if we have to check whether an SSoD policy is violated only when adding a new user-to-role assignment, the check can be inefficient. Given an SSoD policy $e = \mathsf{ssod}\langle\{p_1, \ldots, p_n\}, k\rangle$ and an RBAC state γ that is safe with respect to e, suppose we want to check whether the state γ' resulted from adding a new user-role assignment (u, r) is safe with respect to e. Let $i = |\mathsf{perms}_{\gamma'}[u] \cap \{p_1, \ldots, p_n\}|$ be the number of permissions in e that u would have in γ', then we are left with checking whether γ' is safe with respect to a $(k-1)$-$(n-i)$ SSoD policy, which remains **coNP**-complete by the above theorem.

Efficient algorithms for SC-SSoD exist when all the SSoD policies in E have small k. For example, when checking whether γ is safe with respect to a 2-n SSoD policy, one only needs to compute the permissions of every single user and check whether it is a superset of the permissions in the policy. This has worst-case time complexity $O(N_u(N_u + N_r + N_p))$, where N_u is the number of users in γ, N_r the number of roles, and N_p the number of permissions.

3.2 Statically Mutually Exclusive Role (SMER) constraints

In RBAC, constraints such as mutually exclusive roles are introduced to enforce SSoD policies. In the most basic form, two roles may be declared to be mutually exclusive in the sense that no user is allowed to be a member of both roles. Below we present a generalized form of such constraints.

Definition 4 (SMER Constraints) A t-m SMER (t-out-of-m Statically Mutually Exclusive Roles) constraint is expressed as

$$\mathsf{smer}\langle\{r_1, \ldots, r_m\}, t\rangle$$

where each r_i is a role, and m and t are integers such that $1 < t \leq m$. This constraint forbids any user from being a member of t or more roles in $\{r_1, \ldots, r_m\}$.

A t-m SMER constraint is said to be *canonical* of *cardinality* t when $t = m$.

Definition 5 (Satisfying SMER constraints and the SC-SMER problem) We say that an RBAC state γ *satisfies* a SMER constraint $\mathsf{smer}\langle\{r_1, \ldots, r_m\}, t\rangle$ when

$$\forall u \in \mathcal{U} \quad (\mid \mathsf{roles}_\gamma[u] \cap \{r_1, \ldots, r_m\} \mid < t)\,.$$

Otherwise, we say that γ *violates* the SMER constraint. An RBAC state *satisfies* a set C of SMER constraints if it satisfies every constraint in the set, and we write it as $satisfies_C(\gamma)$. SC-SMER (the Satisfaction Checking problem for SMER constraints) is defined as follows: Given an RBAC state γ and a set C of SMER constraints, determine whether γ satisfies C.

Observe that each SMER constraint restricts only the role memberships of *a single* user, in contrary to a k-n SSoD policy, which restricts the permissions possessed by a set of $k-1$ users. Because of this, there is an efficient algorithm to check whether an RBAC state γ satisfies a set E of SMER constraints.

Theorem 2 SC-SMER *is in* **P**.

Proof. The algorithm is as follows. For each t-m SMER constraint in C and for each user in γ, one first computes the set of all roles the user is a member of, then counts how many roles in this set also appear in the set of roles in the SMER constraint, and finally compare this number with t. This algorithm has a time complexity of $O(N_u N_r M)$, where N is the number of users in γ, N_r the number of roles in γ, and M is the number of constraints. ∎

Further observe that when a user-role assignment is about to be added to UA, one only needs to check the role memberships of that user, which can be done in time $O(N_r M)$.

4 The Enforcement Verification problem

The facts that SC-SSoD is intractable and that an efficient algorithm exists for SC-SMER provide a justification for using SMER constraints to enforce SSoD policies. This justification is new to the best of our knowledge, as the computational complexity of SC-SSoD was not studied in the literature.

When using SMER constraints to enforce SSoD policies, a natural question to ask is whether a set of SMER constraints is adequate to enforce a set of SSoD policies. Clearly, the answer to this question also depends on the permission assignment PA and the role hierarchy RH. For instance, if all permissions in an SSoD policy are assigned to one role, then no set of SMER constraints enforces that policy.

Definition 6 (Enforcement and the EV **problem)** Given $PA \subset \mathcal{P} \times \mathcal{R}$, $RH \subset \mathcal{R} \times \mathcal{R}$, a set E of SSoD policies, and a set C of SMER constraints. We say C *enforces* E (under PA and RH) when

$$\forall UA \subset \mathcal{U} \times \mathcal{R}\ [satisfies_C(\langle PA, RH, UA\rangle) \Rightarrow safe_E(\langle PA, RH, UA\rangle)]$$

EV (the Enforcement Verification problem) is defined as follows: Given PA, RH, a set E of SSoD policies, and a set C of SMER constraints, determine whether C enforces E (under PA and RH).

Example 2 Continuing from Example 1, we consider the following set of SMER constraints

$$\begin{array}{lcl} C_1 & = & \{c_1, c_2, c_3\} \\ c_1 & = & \mathsf{smer}\langle\{\mathsf{Warehouse}, \mathsf{Accounting}, \mathsf{Finance}\},\ 2\rangle \\ c_2 & = & \mathsf{smer}\langle\{\mathsf{Engineering}, \mathsf{Finance}\},\ 2\rangle \\ c_3 & = & \mathsf{smer}\langle\{\mathsf{Quality}, \mathsf{Finance}\},\ 2\rangle \end{array}$$

The constraint c_1 ensures that three users are required to have role memberships in Warehouse, Accounting, and Finance, which are needed to have the permissions p_{goods}, $p_{invoice}$, and $p_{payment}$. This ensures safety with respect to the SSoD policy e_1. The constraints c_2 and c_3 together ensure the safety with respect to e_2. Thus C_1 enforces E_1 under PA_1 and RH_1.

In Example 1, we observed that the state γ_1 is not safe with respect to E_1; therefore, it does not satisfy C_1. In particular, γ_1 violates the constraint c_1 because Alice is assigned to both Warehouse and Accounting.

We now establish an upper bound on the computational complexity of EV.

Lemma 3 EV *is in* **coNP**.

4.1 Simplifying the EV problem

We show that every set of SMER constraints can be equivalently represented using a set of canonical (t-t) SMER constraints.

Definition 7 (SMER Equivalence) Let C_1 and C_2 be two sets of SMER constraints. We say that C_1 and C_2 are *equivalent* when for every RBAC state γ, γ satisfies C_1 if and only if γ satisfies C_2.

Clearly, if C_1 and C_2 are equivalent, then C_1 enforces E under PA and RH if and only if C_2 enforces E under PA and RH; thus one can replace C_1 in an EV problem instance with C_2 and vice versa.

Lemma 4 *For every t-m SMER constraint c, there exists a set C' of canonical SMER constraints of cardinality t such that C' and $\{c\}$ are equivalent.*

Proof. Given a t-m SMER constraint $c = \langle\{r_1, \ldots, r_m\}, t\rangle$, where $m > t$. Let C' be

$$\{\ \mathsf{smer}\langle R, t\rangle\ \mid\ R \subset \{r_1, \ldots, r_m\} \wedge |R| = t\ \}.$$

That is, C' is the set of all constraints $\mathsf{smer}\langle R, t\rangle$ such that R is a size-t subset of $\{r_1, \ldots, r_m\}$. It is easy to see that violating any constraint in C' implies violating the constraint c and violating the constraint c implies violating some constraint in C'. Therefore, C' and $\{c\}$ are equivalent. ∎

It follows from Lemma 4 that for every set C of SMER constraints, there exists a set C' of canonical SMER constraints such that C and C' are equivalent. Furthermore, given an instance of EV in which the set E contains more than one SSoD policy, one can verify these policies one by one. Without loss of generality, we assume that E is a singleton set, i.e., $E = \{e\}$ consists of one policy. This enables us to limit our attention to the following special case of EV.

Definition 8 CEV (the Canonical Enforcement Verification problem) is defined as follows: Given PA, RH, a singleton set $\{e\}$ of SSoD policies and a set C of canonical SMER constraints, determine whether C enforces $\{e\}$.

An algorithm solving CEV can be used to solve EV, as any EV instance can be translated into a set of CEV instances. However, the resulting CEV instance may have an exponential blowup in size, as one needs to generate $\binom{m}{t}$ SMER constraints for each t-m SMER constraint. On the other hand, if an RBAC system uses only canonical constraints to start with, then such blowup does not occur. Also, in the case that $t = 2$, we have a CEV instance the size of which is quadratic in m.

4.2 Algorithms and complexity for CEV

It is easier to think about the complement of CEV, denoted by $\overline{\text{CEV}}$: If C does not enforce $\{\mathsf{ssod}\langle\{p_1, \cdots, p_n\}, k\rangle\}$, then there exists a user-to-role assignment for $k-1$ users such that all the SMER constraints in C are satisfied but these $k-1$ users together possess all permissions $\{p_1, \cdots, p_n\}$. It turns out that this problem is closely related to SAT, the satisfiability problem of propositional formulas in conjunctive normal form. See Appendix C for an introduction to SAT.

Theorem 5 $\overline{\text{CEV}}$ *reduces to* SAT.

This reduction means that we can use algorithms for SAT to solve CEV. Given a CEV instance, the answer is yes if and only if the corresponding SAT instance is not satisfiable.

We now show that CEV is **coNP**-hard by showing that a special case of it is **coNP**-complete. The special case we consider is whether a set of 2-2 SMER constraints satisfies a 2-n SSoD policy. Recall that a 2-2 SMER constraint specifies two roles are mutually exclusive, i.e., no user can be a member of both roles. This is the most common kind of constraints considered in the literature. A 2-n SSoD specifies that no single user is allowed to possess all of n given permissions. This is the simplest and most common kind of SSoD policy. This special case is thus arguably the simplest verification problem.

Theorem 6 *Determining whether a set of* 2-2 *SMER constraints enforces a* 2-n *SSoD policy is* **coNP**-*complete.*

That this problem is **coNP**-hard is shown by reducing MONOTONE-3-2-SAT (which is shown to be **NP**-complete in Theorem 20 in Appendix C) to the complement of this problem.

Corollary 7 EV *and* CEV *are* **coNP**-*complete.*

Proof. Follows directly from Lemma 3 and Theorem 6. ∎

4.3 Efficiency of verification in practice

The fact that even the most basic form of EV is intractable is surprising. Observe that enforcing SSoD policies directly by solving SC-SSOD is efficient for 2-n SSoD policies. These results cast doubts on the effectiveness of the approach of using SMER constraints to enforce SSoD policies, which has been adopted in the literature without being questioned for years. However, complexity class is only part of the story, and we now make some observations in favor of this approach.

When using SMER constraints to enforce SSoD policies, EV, which can be computationally expensive, only needs to be performed when either a new role-role relationship is added to the role hierarchy or a permission in an SSoD policy is assigned to some role. When a user is assigned to a role, only constraint checking (SC-SMER) needs to be performed, which is quite efficient. On the other hand, when enforcing

SSoD policies directly, the expensive safety checking (SC-SSoD) needs to be performed every time a user is assigned to a role of which the user was not already a member. Because user-to-role assignment is the most dynamic relation, enforcing SSoD policies directly is overall more expensive than using SMER constraints.

In the proof of Theorem 6, we use a reduction from MONOTONE-3-2-SAT to $\overline{\text{CEV}}$. In the reduction we generate a 2-n SSoD policy with n being unbounded. When a sensitive task involves only a small number of permissions, then CEV can be done efficiently.

Even though CEV is intractable (**coNP**-complete), it means only that there exist difficult problem instances that take exponential amount of time in the worst case using existing algorithms. SAT has been studied extensively for several decades (see e.g. [3]). Many clever algorithms exist that can answer most instances efficiently. Many problems, including database, planning, computer-aided design, machine vision and automated reasoning, are reduced to SAT and solved using SAT algorithms. This often results in better performance than solving those problems directly. The fact that CEV naturally reduces to SAT means that one can benefit from the extensive research on SAT to provide practical enforcement checking.

The complexity of SC-SSoD is calculated in the number of users plus the number of roles and the complexity of CEV is calculated in the number of roles only. (In both cases, one needs to consider only the permissions in the SSoD policies, rather than all permissions in the RBAC state.) Given that most RBAC systems have many more users than roles, enforcement verification is likely to be more efficient in practice.

Finally, although checking whether an arbitrary set of SMER constraints enforces a set of SSoD policies may be expensive, SMER constraints may be generated from a set of SSoD policies and need not be verified.

5 Generating SMER Constraints

Section 4 considers verifying that SMER constraints in RBAC enforce the desired SSoD policies. In this section we study the problem of generating a set of SMER constraints that are adequate for enforcing SSoD policies. We examine the following questions: How do we define a notion of precision in enforcing SSoD policies, as there are often multiple sets of constraints that enforce the same set of SSoD policies? How do we compare the "degree of restriction" of different sets of SMER constraints? What kinds of SMER constraints are needed in expressing SSoD policies, e.g., do 3-3 SMER constraints add additional expressive power over 2-2 SMER constraints?

5.1 Enforceability of SSoD policies

Definition 9 (Enforceable SSoD configurations) We define *an SSoD configuration* to be a 3-tuple $\langle PA, RH, E\rangle$, where E is a set of SSoD policies. An SSoD configuration is *enforceable* if there exists a set C of SMER constraints such that C enforces E under PA and RH.

Lemma 8 *An SSoD configuration $\langle PA, RH, E\rangle$ is **not** enforceable if and only if there exists an SSoD policy $\mathsf{ssod}\langle\{p_1, \cdots, p_n\}, k\rangle$ in E such that $k-1$ roles together have all the permissions in $\{p_1, \cdots, p_n\}$.*

Theorem 9 *Determining whether an SSoD configuration is enforceable is* **coNP***-complete.*

Similar to SC-SSoD, efficient algorithms exist when all the SSoD policies in the configuration have small k.

5.2 RSSoD requirements

As SMER constraints are about role memberships and SSoD policies are about permissions, the first step of the generation process is to translate a policy on permissions to requirements on roles, using information in PA and RH. We now define such role-level SSoD requirements.

Definition 10 (RSSoD requirements) A k-n RSSoD (k-out-of-n Role-based Static Separation of Duty) requirement is expressed as

$$\mathsf{rssod}\langle\{r_1, \ldots, r_n\}, k\rangle \tag{1}$$

where each r_i is a role and n and k are integers such that $1 < k \leq n$. The meaning is that there should not exist a set of fewer than k users that together have memberships in all the n roles in the requirement. We also say k users are required to *cover* the set of n roles.

We say that an RBAC state γ is *safe* with respect to the above RSSoD requirement when

$$\forall u_1 \cdots u_{k-1} \in \mathcal{U} \left(\left(\bigcup_{i=1}^{k-1} \mathsf{roles}_\gamma[u_i] \right) \not\supseteq \{r_1, \ldots, r_n\} \right).$$

An RBAC state γ is *safe* with respect to a set D of RSSoD requirements if it is safe with respect to every requirement in D, and we write it as $safe_D(\gamma)$.

Given an SSoD configuration $\langle PA, RH, E\rangle$, we say that it is *equivalent* to a set D of RSSoD requirements if

$$\forall UA \subset \mathcal{U} \times \mathcal{R} \ [safe_E(\langle UA, PA, RH\rangle) \Leftrightarrow safe_D(\langle UA, PA, RH\rangle)]$$

where $\Leftrightarrow$ means logical equivalence.

Example 3 The SSoD configuration given in Figure 1 is equivalent to the following set of RSSoD requirements.

$$\begin{array}{lcl} D_1 & = & \{d_1, d_2, d_3, d_4\} \\ d_1 & = & \mathsf{rssod}\langle\{\ \mathsf{Engineering},\ \mathsf{Warehouse},\ \mathsf{Accounting},\ \mathsf{Finance}\},3\rangle \\ d_2 & = & \mathsf{rssod}\langle\{\ \mathsf{Quality},\ \mathsf{Warehouse},\ \mathsf{Accounting},\ \mathsf{Finance}\},3\rangle \\ d_3 & = & \mathsf{rssod}\langle\{\ \mathsf{Engineering},\ \mathsf{Finance}\},2\rangle \\ d_4 & = & \mathsf{rssod}\langle\{\ \mathsf{Quality},\ \mathsf{Finance}\},2\rangle \end{array}$$

In Appendix B we discuss the generation of RSSoD requirements that are equivalent to SSoD configurations. A special case is when we are given an SSoD configuration $\langle PA, RH, \{e = \mathsf{ssod}\langle\{p_1, \ldots, p_n\}, k\rangle\}\rangle$, and each permission p_i is assigned to exactly one role r_i in PA and RH. Then the configuration is equivalent to the singleton set of RSSoD requirement $\{d = \mathsf{ssod}\langle\{r_1, \ldots, r_n\}, k\rangle\}$.

In the rest of this section, we discuss the generation of a set of SMER constraints to enforce one RSSoD requirement.

5.3 Precise enforcement of RSSoD requirements

From the proof of Lemma 8, it is clear that any enforceable SSoD configuration can be enforced using only 2-2 SMER constraints. This shows the power of 2-2 SMER constraints: they are sufficient to enforce any enforceable SSoD policy. However this might be at a great cost in terms of flexibility.

Ideally, one would like to generate SMER constraints that "precisely capture" the restrictions inherent to the RSSoD requirements. We now seek to formalize this.

Definition 11 Let D be a set of RSSoD requirements and C be a set of SMER constraints, we say that C *enforces* D when

$$\forall \text{ RBAC state } \gamma \ [satisfies_C(\gamma) \Rightarrow safe_D(\gamma)]$$

We say that C *is necessary to enforce* D when

$$\forall \text{ RBAC state } \gamma \ [safe_D(\gamma) \wedge live_D(\gamma) \Rightarrow satisfies_C(\gamma)]$$

where $live_D(\gamma)$ means that for every role r appearing in D, there exists a user who is a member of r.
We say C *precisely enforces* D if C enforces D and is necessary to enforce D. Sometimes we abuse the terminology slightly to say a constraint c enforces an RSSoD requirement d.

We now give two cases where precise enforcement can be achieved.

Lemma 10 *Given a k-k RSSoD requirement* $d = \mathsf{rssod}\langle\{r_1, \cdots, r_k\}, k\rangle$, *the constraint* $c = \mathsf{smer}\langle\{r_1, \cdots, r_k\}, 2\rangle$ *precisely enforces* d.

Lemma 11 *Given a 2-n RSSoD requirement* $d = \mathsf{rssod}\langle\{r_1, \cdots, r_n\}, 2\rangle$, *the constraint* $c = \mathsf{smer}\langle\{r_1, \cdots, r_n\}, n\rangle$ *precisely enforces the configuration.*

In fact, as we prove in Lemma 19 (in Section 5.5 after results needed for the proof have been developed), for every k and n such that $2 < k < n$, there exists no set of SMER constraints that precisely enforces a k-n RSSoD requirement. That is, the two special cases in Lemmas 10 and 11 are the only cases where precise enforcement can be achieved. As precise enforcement is not achievable in many cases, we give methods to generate "good" sets of SMER constraints that are as precise as possible.

5.4 Expressive power of different t-m SMER constraints

Before discussing the generation of "good" sets of SMER constraints, we look at the expressive power of t-m SMER constraints using different values of t and m. We would like to answer questions such as: Does an RBAC system that supports 3-3 SMER constraints have more expressive power than an RBAC system that supports only 2-2 SMER constraints? Answers to such questions will help developers of RBAC systems to decide which kinds of constraints to support.

From Lemma 4 we know that t-m SMER constraints, where $m > t$, can be equivalently represented using t-t SMER constraints; thus non-canonical constraints do not add additional expressive power in terms of enforcing SSoD policies. From Lemma 8, we know that 2-2 SMER constraints are sufficient for enforcing (albeit not precisely) any enforceable SSoD configuration. We now show that 2-2 SMER constraints (or 2-n SMER constraints which can be equivalently expressed using 2-2 SMER constraints) are required in the sense that they cannot be replaced with other k-n SMER (where $k \geq 3$) constraints.

Lemma 12 *There exist RSSoD requirements that cannot be enforced without using* 2-n *SMER constraints.*

Proof. A t-t RSSoD requirement can be enforced only by using 2-n SMER constraints, as these are the only type of constraints that prevent two roles from being assigned to a single user. ∎

Although 2-2 SMER constraints are sufficient to enforce all enforceable SSoD configurations, other constraints are needed to enforce some SSoD configurations more precisely.

Lemma 13 *For any $n > 2$, there exists an RSSoD requirement that can be precisely enforced using a canonical constraint of cardinality n but cannot be precisely enforced using any set of t-m SMER constraints with $t < n$.*

This lemma suggests that if one wants to enforce an arbitrary RSSoD requirement as precisely as possible, then one needs to support n-n SMER constraints for arbitrary n.

5.5 Generating "good" sets of SMER constraints

As we show in this section (Lemma 19), SSoD policies cannot always be precisely enforced. Thus it is desirable to compare different sets of SMER constraints and determine which set "more precisely" enforces a set of SSoD policies.

Definition 12 Let C_1 and C_2 be two sets of SMER constraints. We say that C_1 is *at least as restrictive as* C_2 (denoted by $C_1 \trianglerighteq C_2$) if

$$\forall \text{ RBAC state } \gamma \ \left[\ satisfies_{C_1}(\gamma) \Rightarrow satisfies_{C_2}(\gamma) \ \right].$$

The $\trianglerighteq$ relation among all sets of SMER constraints is a partial order. When $C_1 \trianglerighteq C_2$ but not $C_2 \trianglerighteq C_1$, we say that C_1 is *more restrictive than* C_2 (denoted by $C_1 \triangleright C_2$). Observe that by definition, C_1 and C_2 are *equivalent* (Definition 7) if and only if $C_1 \trianglerighteq C_2$ and $C_2 \trianglerighteq C_1$.

When neither $C_1 \trianglerighteq C_2$ nor $C_2 \trianglerighteq C_1$, we say C_1 and C_2 are *incomparable*.

When both C and C' enforce a set D of RSSoD requirements, there are four cases: (1) $C \triangleright C'$; (2) $C' \triangleright C$ (3); C and C' are equivalent; and (4) C and C' are incomparable. In case (1), C' is preferable to C for enforcing D as it is less restrictive (and thus more precise). Similarly, in case (2), C is preferable to C'. In case (3), either C or C' can be used; the choice does not matter. In case (4), the decision to choose C over C' (or C' over C) depends on other policy considerations.

Our philosophy for dealing with the generation problem is to generate all the sets of SMER constraints that are minimal for enforcing D (for any such set, no other set is more preferable than it) and leave the decision to choose which one to use to the system administrator.

Definition 13 Given a set D of RSSoD requirements, we say that a set C of SMER constraints is *minimal* for enforcing D if C enforces D and there does not exist a different set C' of SMER constraints such that C' also enforces D and $C \triangleright C'$ (C is more restrictive than C').

Lemma 14 *If a set C of SMER constraints precisely enforces a set D of RSSoD requirement, then for any C' that also enforces D, $C' \trianglerighteq C$, i.e., C' is at least as restrictive as C.*

Lemma 15 *Let C be a set of SMER constraints that precisely enforces a set D of RSSoD requirement. C is minimal for enforcing D. Furthermore, if C_1 is also minimal for enforcing D, then C and C_1 are equivalent.*

Proof. Given any C' that also enforces D, it follows from Lemma 14 that $C' \trianglerighteq C$, thus it cannot be $C \triangleright C'$ (which implies that $\neg(C' \trianglerighteq C)$).

Given any C_1 that is also minimal for enforcing D, it follows from Lemma 14 that $C_1 \trianglerighteq C$. By Definition 13, it cannot be that $C_1 \triangleright C$; thus C_1 and C must be equivalent. ∎

Lemma 16 *Given a set D of RSSoD requirements, if both C_1 and C_2 are minimal for enforcing D and C_1 and C_2 are incomparable, then there exists no set C of SMER constraints that precisely enforces D.*

Proof. By Contradiction. If a set C that precisely enforces D exists, then from Lemma 15, C is equivalent to C_1 and to C_2. This contradicts the fact that C_1 and C_2 are incomparable. ∎

We now present a simple algorithm to generate all singleton sets of SMER constraints that are minimal for enforcing one RSSoD requirement.

The SMER-Gen Algorithm
Input: RSSoD requirement rssod$\langle R, k\rangle$
Output: a set S of minimal SMER constraints

```
1 let n = |R|, S = ∅
2 if k = 2
3   output smer⟨R, n⟩
4 else
5   for all j from 2 to ⌊(n−1)/(k−1)⌋ + 1
6     let m = (k − 1)(j − 1) + 1
7     for each size-m subset R′ of R
8       output smer⟨R′, j⟩
9 end
```

Example 4 The above algorithm, when taking rssod $\langle\{\mathsf{Engineering},\ \mathsf{Warehouse},\ \mathsf{Accounting},\ \mathsf{Finance}\},3\rangle$ as input, generates the following sets of SMER constraints.

$$\begin{array}{rcl} C_2 & = & \{c_4, c_5, c_6, c_7\} \\ c_4 & = & \mathsf{smer}\langle\{\mathsf{Warehouse}, \mathsf{Accounting}, \mathsf{Finance}\},\ 2\rangle \\ c_5 & = & \mathsf{smer}\langle\{\mathsf{Engineering}, \mathsf{Accounting}, \mathsf{Finance}\},\ 2\rangle \\ c_6 & = & \mathsf{smer}\langle\{\mathsf{Engineering}, \mathsf{Warehouse}, \mathsf{Finance}\},\ 2\rangle \\ c_7 & = & \mathsf{smer}\langle\{\mathsf{Engineering}, \mathsf{Warehouse}, \mathsf{Accounting}\},\ 2\rangle \end{array}$$

Any SMER constraint from C_2 is sufficient to satisfy the RSSoD requirement. The constraints are all minimal and incomparable. Each of them left a different role unconstrained. If one's internal policy is that Warehouse may be left unconstrained, then one may pick c_5 as the desirable constraint to use.

The correctness of this algorithm is justified by the following two lemmas.

Lemma 17 *Given an RSSoD requirement d, each SMER constraint generated by SMER-Gen(d) is minimal for enforcing d.*

Lemma 18 *Given an RSSoD requirement d, each SMER constraint that is minimal for enforcing d is generated by SMER-Gen(d).*

Lemma 19 *Given a k-n RSSoD requirement where $2 < k < n$, there exists no set of SMER constraints that precisely enforces it.*

Our algorithm does not generate all sets of SMER constraints that are minimal to enforce an RSSoD requirement. Constraint sets of cardinality greater than 1 may exist that are minimal for enforcing the requirement. Our algorithm generates all possible minimal singletons in the form of k-m SMER constraints. It is up to the system administrator to choose the most appropriate constraint from those candidates.

6 Conclusions and Future Work

We have posed and answered several fundamental questions related to the use of SMER constraints to enforce SSoD policies, while making a clear distinction between objectives and mechanisms. We have shown that directly enforcing SSoD policies is intractable ($\mathbf{coNP}$-complete), while enforcing SMER constraints is efficient. We have also shown that verifying whether a set of SMER constraints enforces a set of SSoD policies is intractable ($\mathbf{coNP}$-complete), even for a basic subcase of the problem, but reduces naturally to the satisfiability problem (SAT), for which there exist algorithms that have been proven to work well in practice [3]. We have discussed why these intractability results should not lead us to conclude that SMER constraints are not an appropriate mechanism for enforcing SSoD policies.

We have defined minimal and precise enforcement. We have also characterized the kinds of policies for which precise enforcement is achievable and shown what constraints precisely enforce such policies. We have also presented an algorithm that generates all singleton SMER constraint sets that minimally enforce an RSSoD requirement.

An interesting problem that remains is whether the generation algorithm can be improved to consider preexisting SMER constraints and to consider a set of SSoD policies as a whole rather than individually. Other constraints have also been proposed for RBAC, e.g., cardinality constraints and constraints on permission assignment [14]. It would be interesting to examine using SMER constraints together with these constraints to enforce SSoD policies.

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A Proofs

Proof of Theorem 1.

Proof. Consider the complement of SC-SSoD, denoted by $\overline{\text{SC-SSoD}}$. If an RBAC state γ is not safe wrt. E, then there exists a k-n SSoD policy in E and $k-1$ users such that in γ these $k-1$ users together have the n permissions in the SSoD policy. It suffices to show that $\overline{\text{SC-SSoD}}$ is $\mathbf{NP}$-complete.

We first show that $\overline{\text{SC-SSoD}}$ is in $\mathbf{NP}$. If one correctly guesses the k-n SSoD policy being violated and the $k-1$ users that together have all the n permissions in the policy, verifying that the guess is correct can be done in polynomial time: compute the union of the $k-1$ users' permissions and check whether it is a superset of the set of permissions in the SSoD policy.

We now show that $\overline{\text{SC-SSoD}}$ is $\mathbf{NP}$-hard by reducing the set covering problem (page 201 of [11]) to it. In the set covering problem, the inputs are a finite set $\mathcal{S}$, a family $F = \{S_1, \ldots, S_\ell\}$ of subsets of $\mathcal{S}$, and a budget B. The goal is to determine whether there exist B sets in F whose union is $\mathcal{S}$. This problem is $\mathbf{NP}$-complete [8, 11].

The reduction is straightforward. Given $\mathcal{S}$, F, and B, construct an SSoD policy e as follows: Let each element in $\mathcal{S}$ map to a permission in the policy, let k be $B+1$ and let n be the size of $\mathcal{S}$. We have constructed a k-n SSoD policy. Construct an RBAC state γ as follows. For each corresponding permission in $\mathcal{S}$, create

a role to which the permission is assigned. For each different subset S_i ($1 \leq i \leq \ell$) in F, create a user u_i to which all roles in S_i are assigned. The resulting RBAC state γ is not safe with respect to $\{e\}$ if and only if B sets in F cover $\mathcal{S}$. ∎

Proof of Lemma 3

Proof. Consider the complement of EV, denoted by $\overline{\text{EV}}$, it suffices to show that $\overline{\text{EV}}$ is in **NP**. To show this, we need to show that given PA, RH, C, E, if C does not enforce E under PA and RH, then a short (polynomial in the input size) evidence exists such that it can be verified in polynomial time.

If a set C of t-m SMER constraints does not enforce a set E of k-n SSoD policies under PA and RH, then there exists a counter-example, i.e., a user-role assignment UA such that $satisfies_C(\langle UA, PA, RH\rangle)$ is true but $safe_E(\langle UA, PA, RH\rangle)$ is false. That is, there exists a k-n SSoD policy in E that is violated by $k-1$ users. If such an UA exists, then a subset of the UA that consists of just the $k-1$ users is also a counter-example. Thus, the smallest counter-example has size linear in the size of the input. Once the counter-example is guessed, its correctness can be verified in time polynomial in the size of the input. This shows that $\overline{\text{EV}}$ is in **NP**. ∎

Proof of Theorem 5

Proof. An instance of the $\overline{\text{CEV}}$ problem is given by PA, RH, a set C of canonical constraints, and a k-n SSoD policy e. We need to map such an instance to a SAT instance such that the SAT instance is satisfiable if and only if C does not enforce $\{e\}$. In other words, if the SAT instance is satisfiable, then we can find a user assignment relation UA such that the constraints in C are satisfied but the state $\langle UA, PA, RH\rangle$ is not safe with respect to e.

We first give such a mapping for a subcase of $\overline{\text{CEV}}$ where e is a 2-n SSoD policy, i.e., no single user has all n permissions in e. When constructing a SAT instance from such a $\overline{\text{CEV}}$ instance, our goal is to find a user-to-role assignment for one single user such that this user has all permissions in e without violating any constraint in C.

The SAT instance is constructed as follows. For each role r appearing in PA, RH, C, create a propositional variable v_r. Intuitively, if v_r is true, then the user is a member of the role v_r. Construct the set of clauses for the SAT instance as follows.

- For each permission p in e, let $r'_1, r'_2, \cdots, r'_\ell$ be all the roles that are associated with the permission p, add the clause
$$v_{r'_1} \vee v_{r'_2} \vee \cdots \vee v_{r'_\ell}$$
This clause means that, to have the permission p, the user must be a member of one of the roles that are associated with the permission p.
- For each constraint $c \in C$, let $c = \mathsf{ssod}\langle\{r_1, r_2, \cdots, r_t\}, t\rangle$; for each i from 1 to $k-1$, add to the instance the following clause
$$\neg v_{r_1} \vee \neg v_{r_2} \vee \cdots \vee \neg v_{r_t}$$
This clause means that, to satisfy the constraint, there must be at least one role in $\{r_1, r_2, \cdots, r_t\}$ of which the user is not a member.
- For each role hierarchy relationship $(r_1, r_2) \in RH$, add to the instance the following clause
$$\neg v_{r_1} \vee v_{r_2}$$

This clause means that if a user is a member of r_1, then it must also be a member of r_2.

If the SAT instance is satisfiable, let I be a truth assignment that makes the instance true, then assign the user to be a member of every role r such that v_r is true in I. The user has all permissions in e without violating any constraint in C; therefore, C does not enforce e. On the other hand, if C does not enforce e, then there exists a UA such that in the RBAC state $\langle UA, PA, RH\rangle$ there exists a user who has all permissions in e, then the role memberships of the user in this state give rise to a truth assignment that make the SAT instance true.

We now give the mapping for any instance of $\overline{\text{CEV}}$. Given a k-n SSoD policy e where $k > 2$, we need to consider role memberships of $k-1$ different users at the same time. Our goal is to find a user assignment relation such that $k-1$ together have all permissions in e, yet none of the $k-1$ user's role memberships violate constraints in C.

Given PA, RH, let C be a set of canonical constraints and e be a k-n SSoD policy. Construct a SAT instance as follows. For each role appearing in PA, RH, C, create $k-1$ proposition variables. The propositional variables created for a role r is denoted $v_r^1, v_r^2, \cdots, v_r^{k-1}$. Intuitively, v_r^i is true when the i^{th} user is a member of the role r. Then construct the set of clauses for the SAT instance as follows.

- For each permission p in e, let $r'_1, r'_2, \cdots, r'_\ell$ be all the roles that are associated with the permission p, add the clause
$$v_{r'_1}^1 \vee \cdots \vee v_{r'_1}^{k-1} \ \vee \ v_{r'_2}^1 \vee \cdots \vee v_{r'_2}^{k-1} \ \vee \cdots \vee \ v_{r'_\ell}^1 \vee \cdots \vee v_{r'_\ell}^{k-1}$$
To have the permission p, at least one of the $k-1$ users must be a member of one of these roles.

- For each constraint $c \in C$, let $c = \mathsf{ssod}\langle\{r_1, r_2, \cdots, r_t\}, t\rangle$; for each i from 1 to $k-1$, add the following clause
$$\neg v_{r_1}^i \vee \neg v_{r_2}^i \vee \cdots \vee \neg v_{r_t}^i$$
To satisfy the constraint, for every user, there must exist a role in $\{r_1, r_2, \cdots, r_t\}$ of which the user is not a member.

- For each role hierarchy relationship $(r_1, r_2) \in RH$, and for each i from 1 to $k-1$, add the following clause
$$\neg v_{r_1}^i \vee v_{r_2}^i$$
This clause means that if a user is a member of r_1, then it must also be a member of r_2.

The SAT instance is satisfiable if and only if C does not enforce $\{e\}$. ∎

Proof of Theorem 6.

Proof. That this problem is in **coNP** follows from Lemma 3.

We prove that this problem is **coNP**-hard by reducing MONOTONE-3-2-SAT to the complement of this problem. We define MONOTONE-3-2-SAT as being a special case of monotone-CNF-SAT where each negative clause is only composed of two literals. We show in Appendix C that MONOTONE-3-2-SAT is **NP**-complete.

Given a MONOTONE-3-2-SAT problem composed of positive clauses of the form $(v_{i_1} \vee v_{i_2} \vee v_{i_3})$, $1 \le i \le n$, and negative clauses $(\neg v_{j_1} \vee \neg v_{j_2})$, $1 \le j \le m$, we do the following reduction: (1) Each propositional variable v is mapped to a role $r(v)$. (2) Each positive clause $(v_{i_1} \vee v_{i_2} \vee v_{i_3})$ is mapped to a

permission p_i assigned to the three roles $r(v_{i_1})$, $r(v_{i_2})$, and $r(v_{i_3})$. (3) Each negative clause $(\neg v_{j_1} \vee \neg v_{j_2})$ is mapped to a 2-2 SMER constraint $\mathsf{smer}\langle\{r(v_{j_1}), r(v_{j_2})\}, 2\rangle$.

As can be seen, the MONOTONE-3-2-SAT instance is satisfiable if and only if the 2-n SSoD policy $\mathsf{ssod}\langle\{p_1, p_2, ..., p_n\}, 2)\rangle$ can be violated while satisfying all the 2-2 SMER constraints. ∎

Proof of Lemma 8

Proof. If such a situation exists, then no matter what set of SMER constraints we use, one can always assign $k-1$ different users to the $k-1$ roles without violating any SMER constraint, resulting in an unsafe state. On the other hand, if such a situation does not exist, one can use 2-2 SMER constraints to declare every pair of roles in PA and RH to be mutually exclusive, this forbids any user from being assigned to two roles. Clearly, any state satisfying these constraints is safe. ∎

Proof of Theorem 9

Proof. We consider the complement of the problem, that is, to determine whether an SSoD configuration is *not* enforceable. This problem is clearly in $\mathbf{NP}$: from Lemma 8 given a solution set of $k-1$ roles we compute the set of permissions assigned to those roles, and check whether it is a superset of the set of permissions in the SSoD policy. We show that the problem is $\mathbf{NP}$-hard by reducing the set covering problem, which is $\mathbf{NP}$-complete, to it. In the set covering problem, the inputs are a finite set $\mathcal{S}$, a family $F = \{S_1, \ldots, S_\ell\}$ of subsets of $\mathcal{S}$, and a budget B. The goal is to determine whether there exists B sets in F whose union is $\mathcal{S}$.

Given an instance of the set covering problem: $\mathcal{S}$, F, and B, construct an SSoD policy e as follows. Let each element in $\mathcal{S}$ map to a permission in the policy, let k be $B+1$, and let n be the size of $\mathcal{S}$. We have constructed a k-n SSoD policy. For each different subset S_i ($1 \leq i \leq \ell$) in F, create a role r_i and assign to it all permissions corresponding to the elements in S_i. B sets in F cover $\mathcal{S}$ if and only if the SSoD configuration is not enforceable. ∎

Proof of Lemma 10

Proof. The requirement d means that k users are required to cover all k roles. The constraint c means that no user is allowed to be a member of 2 roles in the set. We first show that c enforces d. If no user is a member of 2 roles from the set of k roles, then clearly k users are needed to cover the k roles. We then show that c is necessary. Given an RBAC state γ that violates c, we show that $live_{\{d\}}(\gamma)$ and $safe_{\{d\}}(\gamma)$ cannot both be true. As γ violates c, there exists a user who has memberships in 2 roles from the set of k roles. If $live_{\{d\}}(\gamma)$ is true, then for every role other than the 2 roles there exists a user who is a member of it. Thus $k-1$ users cover the k roles, and $safe_{\{d\}}(\gamma)$ is false. ∎

Proof of Lemma 11

Proof. The requirement d means that 2 users are required to cover all n roles. The constraint c means that no user is allowed to be a member of all the roles in the set. We first show that c enforces e. If no user is a member of all n roles from the set, then clearly, at least 2 users are required to cover all the n roles. We then show that c is necessary. Given an RBAC state γ that violates c, there is one user who has all roles from the set of n roles. Thus $safe_{\{d\}}(\gamma)$ must be false. ∎

Proof of Lemma 13

Proof. Consider the 2-n RSSoD requirement $d = \mathsf{rssod}\langle\{r_1, \cdots, r_n\}, 2\rangle$ (at least 2 users are required to cover the n roles). The n-n SMER constraint $c = \mathsf{smer}\langle\{r_1, \cdots, r_n\}, n\rangle$ (no single user is allowed to a member of all n roles) precisely enforces the configuration, as was shown in lemma 10.

We now show that no set of SMER constraints with $t < n$ precisely enforces d. Assume, for the sake of contradiction, that there exists such a set. Then there exists a set C of canonical constraints of cardinalities less than n that also precisely enforces d. At least one constraint, c, in C must be such that all roles in the constraint are in $\{r_1, \cdots, r_n\}$; otherwise, one could assign one user to have all roles in $\{r_1, \cdots, r_n\}$ without violating any constraint in C. Because c is a canonical constraint of cardinality $t < n$, the set S of roles in c is a strict subset of $\{r_1, \cdots, r_n\}$. This constraint c is not necessary for implementing the SSoD configuration, as an RBAC state in which a user is assigned to be a member of all roles in S is safe with respect to the requirement d, as long as the member is not a member of some role in $\{r_1, \cdots, r_n\} - S$. ∎

Proof of Lemma 14

Proof. We need to show that: (A) for every RBAC state γ, $satisfies_{C'}(\gamma)$ implies $satisfies_C(\gamma)$. We show that this is equivalent to proving (B) for every RBAC state γ' such that $live_D(\gamma')$, $satisfies_{C'}(\gamma')$ implies $satisfies_C(\gamma')$. (A) clearly implies the (B). We now show that (B) implies (A). Suppose, for the sake of contradiction, that (B) is true but (A) is not, then there exists a state γ such that $satisfies_{C'}(\gamma)$ is true, but $satisfies_C(\gamma)$ is not. If such a state γ exists, then there exists a state γ_1 such that the role hierarchy relation in γ_1 is empty, and $satisfies_{C'}(\gamma_1)$ is true, but $satisfies_C(\gamma_1)$ is false. The state γ_1 can be constructed by computing all role memberships in γ and assign these role memberships using UA. We now construct a state γ_2 by adding to γ_1 the following user-to-role assignments: for each role r mentioned in D assigns a new user (one who is not a member of any role in γ_1) to be a member of r. (Different users are used for different roles, so each new user is a member of exactly one role.) We denote the resulting state γ_2. Clearly, $live_D(\gamma_2)$ is true. Furthermore, $satisfies_{C'}(\gamma)$ if and only if $satisfies_{C'}(\gamma_2)$, and $satisfies_C(\gamma)$ if and only if $satisfies_C(\gamma_2)$. Therefore, $satisfies_{C'}(\gamma_2)$ is true but $satisfies_C(\gamma_2)$ is not. This is in contradiction with (B) is true.

If C' enforces D, then for every γ such that $live_D(\gamma)$ is true, $satisfies_{C'}(\gamma)$ implies $safe_D(\gamma)$, which further implies $satisfies_C(\gamma)$. Thus C' is at least as restrictive as C. ∎

Proof of Lemma 17

Proof. Given a j-m SMER constraint c generated by the algorithm for an k-n RSSoD requirement d, let R_c be the set of roles in c, and R_d be the set of roles in d.

By Lemma 4 we know that c can be equivalently expressed as a set C of j-j SMER constraints. Assume, for the sake of contradiction, that the solution is not minimal. Then there exists a set C' of SMER constraints that also enforces d and C' is less restrictive than C, i.e., $(C \trianglerighteq C') \wedge \neg(C' \trianglerighteq C)$.

Because $\neg(C' \trianglerighteq C)$, there exists a state γ such that $satisfies_{C'}(\gamma)$ is true but $satisfies_C(\gamma)$ is not. This means that at least one j-j SMER constraint $c_v \in C$ is violated by γ. Let R_v be the set of j roles in c_v; there exists a user in γ who is a member of the j roles in R_v. As γ satisfies C', having one user being a member of all roles in R_v does not violate C'.

We now construct another state γ' such that $satisfies_{C'}(\gamma')$ is true, but $safe_{\{d\}}(\gamma')$ is not. This contradicts that C' enforces d.

In order to construct γ', we first construct another state γ_1 as follows:

- Use $k-2$ users to cover the roles in $R_c - R_v$ with each user being a member of at most $j-1$ roles in $R_c - R_v$. This is possible for the following reasons. First, $|R_c - R_v| = m - j$. Second, from the algorithm, $m = (k-1)(j-1)+1$. Thus $k-2$ users can cover $(k-2)(j-1)$ roles, where $(k-2)(j-1) = m-j$.
- Assign one of the $k-2$ users to be a member of all roles in $R_d - R_c$.

Observe that $satisfies_C(\gamma_1)$ is true, as γ_1 satisfies c. This is because c does not place any restriction on role memberships in roles in $R_d - R_c$ and each user in γ_1 has at most $j-1$ roles in R_c. Because $C \trianglerighteq C'$, $satisfies_{C'}(\gamma_1)$ is also true.

We now construct γ' by adding to γ_1 a new user and assigning the user to be a member of all roles in R_v. The state γ' has $k-1$ users; together they have memberships in all roles in R_d. Thus $safe_{\{d\}}(\gamma')$ is false. The state γ' also satisfies C', as the role memberships of the $k-2$ users in γ_1 do not violate C' and neither does the new user who is a member of all roles in R_v. ∎

Proof of Lemma 18

Proof. Given an k-n RSSoD requirement d, we show that any j'-m' SMER c that is not generated by the algorithm is not minimal in enforcing d. We show this using case-by-case analysis.

Case 1: $j' > \left\lfloor \frac{n-1}{k-1} \right\rfloor + 1$. Then $j' \geq \left\lfloor \frac{n-1}{k-1} \right\rfloor + 2$. By assigning to $k-1$ users $j'-1$ roles each, we are able to cover $(k-1)(j'-1)$ roles. Observe that for every pair of positive integers x, y, $\lfloor \frac{y}{x} \rfloor \geq \frac{y-(x-1)}{x}$. Thus, $(k-1)(j'-1) \geq (k-1)(\left\lfloor \frac{n-1}{k-1} \right\rfloor + 1) \geq (k-1)(\frac{n-k+1}{k-1} + 1) = n$. Therefore, c does not enforce d.

Case 2: $j' \leq \left\lfloor \frac{n-1}{k-1} \right\rfloor + 1$ and $R_c \not\subseteq R_d$, where R_c is the set of roles in c and R_d is the set of roles in d. Consider the SMER constraint $c' = \mathsf{smer}\langle j', R_c \cap R_d \rangle$. Clearly, $c \rhd c'$, and c' enforces d if and only if c enforces d. Therefore c is not minimal.

Case 3: $j' \leq \left\lfloor \frac{n-1}{k-1} \right\rfloor + 1$, $R_c \subseteq R_d$, and $m' = m$, where $m = (k-1)(j'-1)+1$. This is not possible, as such a constraint c will be generated by the algorithm.

Case 4: $j' \leq \left\lfloor \frac{n-1}{k-1} \right\rfloor + 1$, $R_c \subseteq R_d$, and $m' > m$. As the algorithm generates a j'-m SMER constraint for each size-m subset of R_d, there exist a constraint c' generated by the algorithm with a set of roles R' such that $R' \subset R_c$. This implies $c' \rhd c$, therefore c' is not minimal.

Case 5: $j' \leq \left\lfloor \frac{n-1}{k-1} \right\rfloor + 1$, $R_c \subseteq R_d$, and $m' < m$. We have $m' \leq (k-1)(j'-1)$. By assigning to $k-1$ users at most $j'-1$ roles each, we are able to cover all m' roles in c without violating c. By further assigning to one user to be a member of all roles in $R_d - R_c$, the state is not safe with respect to d while satisfying c. Thus c does not enforce d. ∎

Proof of Lemma 19

Proof. It suffices to prove that when $2 < k < n$, the algorithm generates at least two SMER constraints, as then, we know that each such constraint is minimal (Lemma 17) and therefore there exists no set of SMER constraints that precisely enforces the RSSoD requirement (Lemma 16).

To show that the algorithm generates at least two SMER constraints when $2 < k < n$, we observe that either (1) $\left\lfloor \frac{n-1}{k-1} \right\rfloor > 1$, or (2) $\left\lfloor \frac{n-1}{k-1} \right\rfloor = 1$. If (1) is true, then j takes on at least the two values 2 and 3 in

the outer loop, and therefore the inner loop (lines 7–8) is executed at least twice for different values of j, thereby generating two different SMER constraints.

If (2) is true, then the outer loop (line 5) is executed only once. For that execution of the outer loop, $m = k < n$. Thus there exist more than one size-m subsets of R. Therefore, the inner loop (lines 7–8) executes at least twice for two different size-m subsets R' of R, thereby giving two different SMER constraints. ∎

B Generating RSSoD Requirements

As we mention in Section 5.2, there is an intermediate step between defining SSoD policies and designing SMER constraints that implement them. The step is the mapping of SSoD policies to RSSoD requirements. SSoD policies are expressed in terms of permissions; a set of RSSoD requirements expresses an SSoD policy in terms of roles. Definition 10 defines an RSSoD requirement. In this section, we discuss the problem of generating a set of RSSoD requirements from an SSoD policy and observe that the algorithm can be inefficient for arbitrary policies, but should be efficient for typical SSoD policies.

Following is our algorithm to generate a set of RSSoD requirements from an SSoD policy. In line 10, the algorithm returns an error if the SSoD policy cannot be satisfied for the given role structure.

Input: SSoD policy $\mathsf{ssod}\langle P, k\rangle$
Output: set S of RSSoD requirements or error

```
1  let n = |P|, S = R = ∅
2  for each p_i ∈ P
3    let R_i = set of roles for which p_i is authorized
4  for each r_1 ∈ R_1
5    add r_1 to R
6    for each r_2 ∈ R_2
7      if r_2 ∉ R add r_2 to R
           ⋮
8          for each r_n ∈ R_n
9            if r_n ∉ R add r_n to R
10           if |R| < k return error: SSoD cannot be satisfied
11           else
12             copy R to R'
13             add rssod⟨R', k⟩ to S
14           if r_n was added to R at this level of nesting
15             remove r_n from R
           ⋮
16     if r_2 was added to R at this level of nesting
17       remove r_2 from R
18   remove r_1 from R
19 return S
```

In line 3, the algorithm adds to R_i the roles to which the permission p_i is directly assigned and the roles that inherit p_i from R_i. Therefore, the lines 2–3 run with worst-case time-complexity $O(nN_r)$, where N_r is

the number of roles in the role-hierarchy RH and n is the number of permissions in the SSoD policy. The entire algorithm runs with worst-case time-complexity $O({N_r}^n)$. Thus, if SSoD policies contain only small sets of permissions, then the algorithm is reasonably efficient. If n can be any number of permissions, then the algorithm is exponential in the size of the state.

The algorithm is efficient for "shallow" role-hierarchies. For instance, if the role-hierarchy is flat, that is, for any two roles $r_1, r_2 \in \mathcal{R}$, we have $(r_1, r_2) \notin RH$, and if a permission is assigned to at most one role, then the lines 2–3 dominate the lines 4–16 in running time, and the algorithm runs with worst-case time complexity $O(nN_r)$.

C SAT

A boolean literal or variable is one that can take on a value from $\{0, 1\}$. A boolean expression is one of the following: (a) a boolean variable, (b) $\neg\phi$, (c) $\phi_1 \vee \phi_2$, or (d) $\phi_1 \wedge \phi_2$, where ϕ, ϕ_1 and ϕ_2 are boolean expressions. (c) is called a disjunction, and (d) is called a conjunction. A truth assignment to a boolean expression is the assignment of either 0 or 1 to each variable in the expression. A boolean expression is in Conjunctive Normal Form (CNF), if it can be written as $\bigwedge_{i=1}^{n} C_i$ where $n \geq 1$ and each C_i is called a clause, a clause is either a literal or a disjunction of literals, and a literal is either a boolean variable or its complement. A boolean expression is satisfiable if there exists a truth assignment to it such that it evaluates to 1. The SAT problem is the problem of determining whether an expression in CNF is satisfiable. The complement of the satisfiability problem is the validity problem: whether for any truth assignment, the expression evaluates to 1.

The 3-SAT problem is SAT with each clause has exactly 3 literals. It is well-known that SAT and MONOTONE-SAT are **NP**-complete, and their complements are **coNP**-complete.

Monotone 3-2-SAT is NP-complete

MONOTONE-3-SAT is 3-SAT with each clause containing either only positive literals or only negative literals; it is known to be **NP**-complete [8]. We use MONOTONE-3-2-SAT to denote SAT with each clause containing either 3 positive literals or 2 negative literals.

Theorem 20 MONOTONE-3-2-SAT *is* **NP**-*complete.*

Proof. MONOTONE-3-2-SAT is clearly in **NP**. We show that it is **NP**-hard by reducing 3-SAT to MONOTONE-3-2-SAT.

Let $(\ell_1 \vee \ell_2 \vee \ell_3)$ be a clause. Case (1): all three literals are positive. No change needs to be made. Case (2): one is negative. Wlog, assume that ℓ_3 is negative. This clause can be equivalently represented using a positive clause $(\ell_1 \vee \ell_2 \vee w)$ and a negative clause $(\neg w \vee \ell_3)$, where w is a newly introduced propositional variable. This technique turns one literal from negative to positive by introducing a new propositional variable and a new length-2 negative clause. Case (3): two are negative. Apply the above technique twice. Case (4): three are negative. Apply the above technique three times.

∎

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