

## On Nano Generalized Pre Regular Closed Sets in Nano Topological Spaces

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**Abstract:** In this paper a class of sets called Nano generalized pre regular closed sets is introduced and its properties are studied. Further the notation of Nano pre regular  $T_{1/2}$  space and Nano generalized pre regular continuity are discussed.

**Keywords:** Nano Generalized Pre Regular Closed sets, Nano Pre Regular  $T_{1/2}$  spaces, Ngpr continuous, Ngpr irresolute.

### I. Introduction

In 1970, Levine [6] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. Later on N. Palaniappan [7] studied the concept of regular generalized closed set in a topological space. Maki et al [1] introduced the concepts of generalized pre closed sets and pre generalized closed sets in an analogous manner. In 1977, Y. Gnanambal [10] have introduce the concept of generalized pre regular closed sets in topological spaces. In 2011, Sharmistha Bhattacharya [9] have introduced the notation of generalized regular closed sets in topological space. The notation of Nano topology was introduced by Lellis Thivagar [4], which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined Nano closed sets, Nano interior and Nano closure. Moreover in this paper we defined Nano pre regular  $T_{1/2}$  spaces as the spaces in which every Nano generalized pre regular closed sets is Nano pre closed. We also introduce the notation of Nano generalized pre regular continuity and study its properties.

### II. Preliminaries

**Definition 2.1** [5] Let  $U$  be a non-empty finite set of objects called the universe and  $R$  be an equivalence relation on  $U$  named as the indiscernibility relation. Then  $U$  is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair  $(U, R)$  is said to be the **approximation space**. Let  $X \subseteq U$ .

1. The lower approximation of  $X$  with respect to  $R$  is the set of all objects, which can be for certain classified as  $X$  with respect to  $R$  and its is denoted by  $L_R(X)$ .

$$L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$$

where  $R(x)$  denotes the equivalence class determined by  $x \in U$ .

2. The upper approximation of  $X$  with respect to  $R$  is the set of all objects, which can be possibly classified as  $X$  with respect to  $R$  and it is denoted by  $U_R(X)$ .

$$U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}.$$

3. The boundary region of  $X$  with respect to  $R$  is the set of all objects, which can be possibly classified neither as  $X$  nor as not  $X$  with respect to  $R$  and it is denoted by  $B_R(X)$ .

$$B_R(X) = U_R(X) - L_R(X).$$

**Property 2.2** [5] If  $(U, R)$  is an approximation space and  $X, Y \subseteq U$ , then

1.  $L_R(X) \subseteq X \subseteq U_R(X)$ .
2.  $L_R(\emptyset) = U_R(X) = \emptyset$  &  $L_R(U) = U_R(U) = U$ .
3.  $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$ .
4.  $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$ .
5.  $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$ .

6.  $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$ .
7.  $L_R(X) \subseteq L_R(Y) \ \& \ U_R(X) \subseteq U_R(Y)$  whenever  $X \subseteq Y$ .
8.  $U_R(X^c) = [L_R(X)]^c \ \& \ L_R(X^c) = [U_R(X)]^c$ .
9.  $U_R U_R(X) = L_R U_R(X) = U_R(X)$ .
10.  $L_R L_R(X) = U_R L_R(X) = L_R(X)$ .

**Definition 2.3** [4] Let  $U$  be the universe,  $R$  be an equivalence relation on  $U$  and  $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$  where  $X \subseteq U$ . Then by property  $\tau_R(X)$  satisfies the following axioms:

1.  $U$  and  $\phi \in \tau_R(X)$
2. The union of the elements of any subcollection of  $\tau_R(X)$  is in  $\tau_R(X)$
3. The intersection of the elements of any finite subcollection of  $\tau_R(X)$  is in  $\tau_R(X)$ .

Then  $\tau_R(X)$  is a topology on  $U$  called the **Nano topology** on  $U$  with respect to  $X$ .  $(U, \tau_R(X))$  as the Nano topological space. The elements of  $\tau_R(X)$  are called as Nano-open sets and complement of Nano open sets is called Nano closed.

**Definition 2.4** [4] If  $(U, \tau_R(X))$  is a Nano topological space with respect to  $X$  where  $X \subseteq U$  and if  $A \subseteq U$ , then the **Nano interior** of  $A$  is defined as the union of all Nano-open subsets contained in  $A$  and it is denoted by  $NInt(A)$ . That is,  $NInt(A)$  is the largest Nano-open subset of  $A$ . The **Nano closure** of  $A$  is defined as the intersection of all Nano closed sets containing  $A$  and it is denoted by  $NCl(A)$ .  $NCl(A)$  is the smallest Nano closed set containing  $A$ .

**Definition 2.5** [8] Let  $(U, \tau_R(X))$  be a Nano topological space and  $A \subseteq U$ . Then  $A$  is said to be

1. Nano Pre-open if  $A \subseteq NInt(NCl(A))$ .
2. Nano Pre-closed if  $NCl(NInt(A)) \subseteq A$ .
3. Nano Regular open if  $A \subseteq NInt(NCl(A))$ .
4. Nano Regular closed if  $NCl(NInt(A)) \subseteq A$ .

**Definition 2.6** [3] Let  $(U, \tau_R(X))$  be a Nano topological space. A subset  $A$  of  $(U, \tau_R(X))$  is called **Nano generalized closed set** (briefly Ng-closed) if  $NCl(A) \subseteq V$  where  $A \subseteq V$  and  $V$  is Nano open.

**Definition 2.7** [2] If  $(U, \tau_R(X))$  is a Nano topological space with respect to  $X$  where  $X \subseteq U$  and if  $A \subseteq U$ , then the Nano pre interior of  $A$  is defined as the union of all Nano pre open subsets of  $A$  contained in  $A$  and it is denoted by  $NpInt(A)$ .  $NpInt(A)$  is the largest Nano pre open subset of  $A$ . The Nano pre closure of  $A$  is defined as the intersection of all Nano pre closed sets containing  $A$  and it is denoted by  $Npcl(A)$ . That is,  $Npcl(A)$  is the smallest Nano pre closed set containing  $A$ .

**Definition 2.8** [2] A subset  $A$  of  $(U, \tau_R(X))$  is called **Nano generalized pre closed set** (briefly Ngp-closed) if  $Npcl(A) \subseteq V$  whenever  $A \subseteq V$  and  $V$  is Nano open in  $(U, \tau_R(X))$ .

**Definition 2.9** [5] Let  $(U, \tau_R(X))$  and  $(V, \tau_R(Y))$  be a Nano topological spaces. Then a mapping  $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$  is Nano continuous on  $U$  if the inverse image of every Nano – open set in  $V$  is Nano – open in  $U$ .

### III. Nano Generalized Pre Regular Closed Sets And Nano Pre Regular $T_{1/2}$ Spaces

Throughout this paper  $(U, \tau_R(X))$  is a Nano topological space with respect to  $X$  where  $X \subseteq U$ ,  $R$  is an equivalence relation on  $U$ . Then  $U/R$  denotes the family of equivalence classes of  $U$  by  $R$ . In this section, we define and study forms of Nano generalized closed sets and Nano pre regular  $T_{1/2}$  spaces.

**Definition 3.1** A subset  $A$  of  $(U, \tau_R(X))$  is called Nano Generalized pre regular closed set (briefly Ngpr – closed) if  $Npcl(A) \subseteq A$  whenever  $A \subseteq V$  and  $V$  is Nano regular open in  $U$ .

**Definition 3.2** A set  $A \subseteq U$  is called Nano generalized pre regular open if and only if its complement is Nano generalized pre regular closed.

**Example 3.3** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a\}, \{c\}, \{b,d\}\}$  and  $X = \{a,b\}$ . Then  $\tau_R(X) = \{U, \Phi, \{a\}, \{a,b,d\}, \{b,d\}\}$  which are open sets. The Nano generalized pre closed sets =  $\{U, \Phi, \{b\}, \{c\}, \{d\}, \{a, c\}, \{b, c\},$

$\{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}$ . The Nano regular closed sets =  $\{U, \Phi, \{b, c, d\}, \{a, c\}, \{c\}\}$ . The Nano generalized pre regular closed sets =  $\{U, \Phi, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{b, d\}, \{a, d\}, \{a, b\}, \{d\}, \{b\}, \{a\}\}$ .

**Remark 3.4**  $Npcl(U - A) = U - Np \text{int}(A)$

**Theorem 3.5** Every Nano regular generalized closed set is Nano generalized pre regular closed.

**Proof.** Let  $A \subseteq U$  be a nano regular generalized closed. Let  $A \subseteq V$  and  $V$  is nano regular open. Then  $Ncl(A) \subseteq V$  since  $A$  is Nrg - closed. Since every Nano closed set is Nano pre closed,  $Npcl(A) \subseteq Ncl(A)$ . Therefore,  $Npcl(A) \subseteq V$ . Hence  $A$  is Ngpr - closed.

**Theorem 3.6** Let  $A$  be Ngpr - closed in  $(U, \tau_R(X))$ . Then  $Npcl(A) - A$  does not contain any non - empty Nano regular closed set.

**Proof.** Let  $F$  be a Nano regular closed set such that  $F \subseteq Npcl(A) - A$ . Then  $F \subseteq U - A$  implies  $A \subseteq U - F$ .  $A$  is Ngpr - closed and  $U - F$  is Nano regular open. Therefore  $Npcl \subseteq U - F$ . It is  $F \subseteq U - Npcl(A)$ . Hence  $F \subseteq Npcl(A) \cap (U - Npcl(A)) = \phi$ . This shows  $F = \phi$ .

**Theorem 3.7** If  $A$  is Ngpr - closed and  $A \subseteq B \subseteq Npcl(A)$  then  $B$  is Ngpr - closed.

**Proof.** Let  $B \subseteq V$  where  $V$  is Nano regular open. Then  $A \subseteq B$  implies  $A \subseteq V$ . Since  $A$  is Ngpr - closed  $Npcl(A) \subseteq V$ .  $B \subseteq Npcl(A)$  implies  $Npcl(B) \subseteq Npcl(A)$ . Thus  $Npcl(B) \subseteq U$  and shows that  $B$  is Ngpr - closed.

**Theorem 3.8**  $A \subseteq U$  is Ngpr - open if and only if  $F \subseteq Np \text{int}(A)$  whenever  $F$  is Nano regular closed and  $F \subseteq A$ .

**Proof.** *Necessity:* Let  $A$  be Ngpr-open. Let  $F$  be Nano regular closed and  $F \subseteq A$ . Then  $U - A \subseteq U - F$  where  $U - F$  is Nano regular open. Ngpr - closedness of  $U - F$  implies  $Npcl(U - A) \subseteq U - F$ . By remark 3.4,  $U - Np \text{int}(A) \subseteq U - F$ . It is  $F \subseteq Npin(A)$ . *Sufficiency:* Suppose  $F$  is Nano regular closed and  $F \subseteq A$  implies  $F \subseteq Np \text{int}(A)$ . Let  $U - A \subseteq V$  where  $V$  is Nano regular open. Then  $U - V \subseteq A$  where  $U - V$  is Nano regular closed. By hypothesis,  $U - V \subseteq Np \text{int}(A)$ . It is  $U - Np \text{int}(A) \subseteq V$ . By remark 3.4,  $Npcl(U - A) \subseteq V$ . This implies  $U - A$  is Ngpr-closed and  $A$  is Ngpr-open.

**Proposition 3.9** Let  $Npo(U)$  be Nano closed under finite intersections. If  $A$  is Ngpr open and  $B$  is Ngpr open then  $A \cap B$  is Ngpr-open.

**Proof.** Let  $U - (A \cap B) = (U - A) \cup (U - B) \subseteq F$ . Where  $F$  is Nano regular open. Then  $U - A \subseteq F$  and  $U - B \subseteq F$ . Since  $A$  and  $B$  are Ngpr - open,  $Npcl(U - A) \subseteq F$  and  $Npcl(U - B) \subseteq F$ . By hypothesis,  $Npcl((U - A) \cup (U - B)) \subseteq Npcl(U - A) \cup Npcl(U - B) \subseteq F$ . It is  $Npcl(U - (A \cap B)) \subseteq F$ . This shows that  $A \cap B$  is Ngpr - open.

**Theorem 3.10**

1.  $NPO(\tau_R(X)) \subset NGPRO(\tau_R(X))$ .
  2. A space  $(U, \tau_R(X))$  is Nano pre regular  $T_{1/2}$  if and only if  $NPO(\tau_R(X)) = NGPRO(\tau_R(X))$ .
- Proof.**
1. Let  $A$  Nano open. Then  $U - A$  is Nano pre closed and so Ngpr - closed. This implies  $A$  is Ngpr - open. Hence  $NPO(\tau_R(X)) \subset NGPRO(\tau_R(X))$ .
  2. *Necessity :* Let  $(U, \tau_R(X))$  be Nano pre regular  $T_{1/2}$ . Let  $NGPRO(\tau_R(X))$ . Then  $U - A$  is Ngpr - closed. By hypothesis  $U - A$  is Nano pre closed and this  $A \in NPO(\tau_R(X))$ . Hence  $NGPRO(\tau_R(X)) = NPO(\tau_R(X))$ . *Sufficiency :* Let  $NPO(\tau_R(X)) = NGPRO(\tau_R(X))$ . Let  $A$  be Ngpr - closed. Then  $U - A$  is Ngpr - open. Hence  $U - A \in NPO(\tau_R(X))$ . Thus  $A$  is Nano pre closed there by implying  $(U, \tau_R(X))$  is nano pre regular  $T_{1/2}$ .

#### IV. NGPR Continuous And NGPR Irresolute

Throughout this paper  $U$  and  $V$  are nonempty, finite universe.  $X \subseteq U$ ,  $Y \subseteq V$  &  $Z \subseteq W$ ;  $U/R$ ,  $V/R'$  &  $W/R''$  denotes the family of equivalence classes by equivalence relations  $R, R', R''$  respectively on  $U, V$  &  $W$ .  $(U, \tau_R(X))$ ,  $(V, \tau_{R'}(Y))$ ,  $(W, \tau_{R''}(Z))$  are the nano topological spaces with respect to  $X, Y, Z$  respectively.

**Definition: 4.1** A function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is called Ngpr continuous if  $f^{-1}(V)$  is Ngpr closed in  $(U, \tau_R(X))$  for every Nano closed set of  $(V, \tau_{R'}(Y))$ .

**Example: 4.2** Let  $U = \{a, b, c, d\}$ ,  $\tau_R(X) = \{U, \Phi, \{a\}, \{a,b,d\}, \{b,d\}\}$  and  $V = \{a, b, c\}$ ,  $\tau_{R'}(Y) = \{U, \phi, \{a\}, \{b,c\}\}$ . Define  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  by  $f(a) = a, f(c)=b$  and  $f(d) = c$ . Since every subset of  $(U, \tau_R(X))$  is Ngpr - closed,  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is Ngpr - Continuous.

**Definition: 4.3** A function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is called Ngpr irresolute if  $f^{-1}(V)$  is Ngpr closed in  $(U, \tau_R(X))$  for every Ngpr closed set  $V$  of  $(V, \tau_{R'}(Y))$ .

**Example: 4.4** Let  $U = \{a, b, c, d\}$ ,  $\tau_R(X) = \{U, \Phi, \{a\}, \{a,b,d\}, \{b,d\}\}$  and  $V = \{a, b, c\}$ ,  $\tau_{R'}(Y) = \{U, \phi, \{a\}, \{a,b,d\}, \{b,d\}\}$ . Define  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  by  $f(a) = b, f(b) = c$  and  $f(c) = d$  and  $f(d) = a$ . Then the inverse image of every Ngpr closed set is Ngpr closed under  $f$ . Hence  $f$  is Ngpr - irresolute.

**Theorem: 4.5** Let  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  be Nano regular irresolute and Nano pre closed. Then for every Ngpr - closed set of  $A$  of  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ .  $f(A)$  is Ngpr closed in  $(V, \tau_{R'}(Y))$ .

Proof. Let  $A$  be Ngpr - closed in  $(U, \tau_R(X))$ . Let  $f(A) \subseteq V'$  where  $V'$  is Nano regular open in  $V$ . Then  $A \subseteq f^{-1}(V')$ . Since  $f$  is Nano regular irresolute and  $A$  is Ngpr - closed,  $Npcl(A) \subseteq f^{-1}(V')$ . That is,  $f(Npcl(A)) \subseteq V'$ . Now  $Npcl(f(A)) \subseteq Npcl(f(Npcl(A))) = f(Npcl(A)) \subseteq V'$ . Since  $f$  is Nano pre closed. Hence  $f(A)$  is Ngpr - closed in  $(V, \tau_{R'}(Y))$ .

**Theorem: 4.6** Let  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  and  $g : (V, \tau_{R'}(Y)) \rightarrow (W, \tau_{R''}(Z))$  be any two functions. Then

1.  $g \circ f$  is Ngpr - Continuous, if  $g$  is Nano Continuous and  $f$  is Ngpr - continuous.
2.  $g \circ f$  is Ngpr - irresolute, if  $g$  is Nano irresolute and  $f$  is Ngpr - irresolute.
3.  $g \circ f$  is Ngpr Continuous, if  $g$  is Ngpr continuous and  $f$  is Ngpr irresolute.

Proof. 1. Let  $V'$  be closed in  $(W, \tau_{R''}(Z))$ . Then  $g^{-1}(V')$  is closed in  $(V, \tau_{R'}(Y))$ . Since  $g$  is Nano continuous. Ngpr continuous of  $f$  implies  $f^{-1}[g^{-1}(V')]$  is Ngpr - closed in  $(U, \tau_R(X))$ . That is  $(g \circ f)^{-1}(V')$  is Ngpr - closed in  $(U, \tau_R(X))$ . Hence  $(g \circ f)$  is Ngpr - continuous. 2. Let  $V'$  be Nano closed in  $(W, \tau_{R''}(Z))$ . Since  $g$  is Ngpr - irresolute,  $g^{-1}(V')$  is Ngpr closed in  $(V, \tau_{R'}(Y))$ . As  $f$  is Ngpr - irresolute  $f^{-1}[g^{-1}(V')] = (g \circ f)^{-1}(V')$  is Ngpr closed in  $(U, \tau_R(X))$ . Therefore  $g \circ f$  is Ngpr - irresolute. 3. Let  $V'$  be closed in  $(W, \tau_{R''}(Z))$ . Since  $g$  is Ngpr - continuous.  $g^{-1}(V')$  is ngpr closed in  $(V, \tau_{R'}(Y))$ . As  $f$  is Ngpr irresolute  $f^{-1}[g^{-1}(V')] = (g \circ f)^{-1}(V')$  is Ngpr - closed in  $(U, \tau_R(X))$ .  $g \circ f$  is Ngpr Continuous.

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