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On Necessary and Sufficient Conditions for L^p -Estimates of Riesz Transforms Associated to Elliptic Operators on \mathbb{R}^n and Related Estimates

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Contents

Acknowledgements	ix
Introduction	xi
Notation	xvii
Chapter 1. Beyond Calderón-Zygmund operators	1
Chapter 2. Basic L^2 theory for elliptic operators	9
2.1. Definition	9
2.2. Holomorphic functional calculus on L^2	9
2.3. L^2 off-diagonal estimates	10
2.4. Square root	12
2.5. The conservation property	12
Chapter 3. L^p theory for the semigroup	15
3.1. Hypercontractivity and uniform boundedness	15
3.2. $W^{1,p}$ elliptic estimates and hypercontractivity	17
3.3. Gradient estimates	19
3.4. Summary	21
3.5. Sharpness issues	22
3.6. Analytic extension	22
Chapter 4. L^p theory for square roots	25
4.1. Riesz transforms on L^p	25
4.2. Reverse inequalities	32
4.3. Invertibility	36
4.4. Applications	37
4.5. Riesz transforms and Hodge decomposition	38
Chapter 5. Riesz transforms and functional calculi	41
5.1. Blunck & Kunstmann's theorem	41
5.2. Hardy-Littlewood-Sobolev estimates	42
5.3. The Hardy-Littlewood-Sobolev-Kato diagram	44
5.4. More on the Kato diagram	47
Chapter 6. Square function estimates	51
6.1. Necessary and sufficient conditions for boundedness of vertical square functions	51
6.2. On inequalities of Stein and Fefferman for non-tangential square functions	60

Chapter 7. Miscellani	65
7.1. Local theory	65
7.2. Higher order operators and systems	66
Appendix A. Calderón-Zygmund decomposition for Sobolev functions	69
Appendix. Bibliography	73