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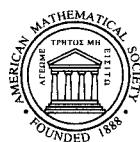
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## On Necessary and Sufficient Conditions for $L^p$ -Estimates of Riesz Transforms Associated to Elliptic Operators on $\mathbb{R}^n$ and Related Estimates

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