Bipolar Neutrosophic Refined Sets and Their Applications in Medical Diagnosis

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ABSTRACT

This paper proposes concept of bipolar neutrosophic refined set and its some operations. Firstly, a score certainty and accuracy function to compare the bipolar neutrosophic refined information is defined. Secondly, to aggregate the bipolar neutrosophic refined information, a bipolar neutrosophic refined weighted average operator and a bipolar neutrosophic refined weighted geometric operator is developed. Thirdly, to select the most desirable one(s) in a decision making problem, based on operators and the score, certainty and accuracy functions, a bipolar neutrosophic refined multiple criteria decision-making approach, in which the evaluation values of alternatives on the attributes take the form of bipolar neutrosophic refined numbers, is proposed. Finally, an algorithm is introduced and a numerical example is employed to illustrate the effectiveness and feasibility of this method in medical diagnosis.

Keywords: Neutrosophic set, bipolar neutrosophic set, bipolar neutrosophic refined set, average operator, geometric operator, score, certainty and accuracy function, multi-criteria decision making.

1   INTRODUCTION

In decision making problems involve data which may contain uncertainties and these data may be dealt with using a wide range of existing theories such as probability theory, fuzzy set theory [24], intuitionistic fuzzy set theory [1], fuzzy multi set theory [15], intuitionistic fuzzy multi set theory [13,14], bipolar fuzzy set theory [2,9] and so on. Then, Smaradanche [17] proposed a completely new set theory, which is called neutrosophic set theory, for dealing with uncertainties. Recently there has been a rapid growth of interest in these theory in [6, 8,10,11,12,18,20,23,26].

Neutrosophic set is a type of important mathematical structure for dealing with uncertainties and there are several types of fuzzy set extensions in neutrosophic set theory, for example, interval valued neutrosophic sets [19], neutrosophic refined-sets(multi-sets) [7,16], etc. The theory of neutrosophic refined-set has applied widely by many researchers as it can be seen in [3,4,21,24]. Bipolar neutrosophic set [5] is another extension of neutrosophic set whose membership functions is different from the above extensions. In 2015, Deli et al. [5] initiated an extension of neutrosophic set named bipolar neutrosophic set. They introduced the concept of bipolar neutrosophic sets which is an extension of the fuzzy sets, bipolar fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets.

In this paper, we combine the concept of a bipolar neutrosophic set [5] and neutrosophic multisets [7,16]. The remaining part of this paper is organized as follows. In section 2, we recall some concepts about fuzzy set theory, fuzzy multiset, bipolar fuzzy set, bipolar fuzzy multiset, neutrosophic set, neutrosophic multiset and bipolar neutrosophic set. In section 3, we give definition of bipolar neutrosophic refined set and its some operations with properties. In section 4, we present an application in medical diagnosis via bipolar neutrosophic refined set theory to illustrate the effectiveness and feasibility of this method. In section 5, we conclude this paper.

2   PRELIMINARIES

In this section, we give the basic definitions and results fuzzy set theory [24], intuitionistic fuzzy set theory [1], fuzzy multi set theory [15], intuitionistic fuzzy multi set theory [13,14], bipolar fuzzy set theory [2,9], bipolar fuzzy multiset [20], neutrosophic set [17], bipolar neutrosophic set [5] and neutrosophic multisets [7,16]. that are useful for subsequent discussions.

Definition 2.1.[24] Let U be a universe. Then a fuzzy set X over U is defined by

$$X = \left\{ \left(\mu_X(x) / x \right) : x \in U \right\}$$

where, $\mu_X(x)$ is called membership function of X and defined by $\mu_X(x) : U \to [0,1]$. For each $x \in U$, the value $\mu_X(x)$ represents the degree of x belonging to the fuzzy set X.

Definition 2.2. [10] A bipolar fuzzy set $\tilde{B}$ in U is an object having the form

$$\tilde{B} = \left\{ \left(x, \mu_{\tilde{B}}^+(x), \mu_{\tilde{B}}^-(x) \right) : x \in U \right\}$$

where, $\mu_{\tilde{B}}^+(x) : U \to [0,1]$, $\mu_{\tilde{B}}^-(x) : U \to [-1,0]$, $\mu_{\tilde{B}}^+(x)$ denotes for positive information and $\mu_{\tilde{B}}^-(x)$ denotes for negative information.

Definition 2.3. [15] Let k be a positive integer. A multi-fuzzy set $\tilde{A}$ in U is a set of ordered sequences

$$\tilde{A} = \left\{ x / \left(\mu_1(x), \mu_2(x), ..., \mu_i(x), ..., \mu_k(x) \right) : x \in U \right\}$$

where, $\mu_i U \to [0,1]$, $i = 1, 2, ..., k$.

Definition 2.4. [20] Let k be a positive integer. A bipolar multi-fuzzy set $\tilde{M}$ in U is a set of ordered sequences having the form

$$\tilde{M} = \left\{ x / \left(\left(\mu_1^+(x), \mu_1^-(x) \right), \left(\mu_2^+(x), \mu_2^-(x) \right), ..., \left(\mu_i^+(x), \mu_i^-(x) \right), ..., \left(\mu_k^+(x), \mu_k^-(x) \right) \right) : x \in U \right\}$$

where, $\mu_i^+(x) : U \to [0,1], \mu_i^-(x) : U \to [-1,0]$, $i = 1, 2, ..., k$, every μ_i^+ denotes for positive information and μ_i^- denotes for negative information.

Definition 2.5. [1] Let U be a universe. An intuitionistic fuzzy set I on U can be defined as follows:

$$I = \left\{ \left\langle x, \mu_I(x), v_I(x) \right\rangle : x \in U \right\}$$

where, $\mu_I(x) : U \rightarrow [0,1]$ and $\nu_I(x) : U \rightarrow [0,1]$ such that $0 \leq \mu_I(x) + \nu_I(x) \leq 1$ for any $x \in U$.

Definition 2.6. [17] Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic set (N-set) A in U is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A, and a falsity- membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real element of $[0,1]$. It can be written as

$$A = \left\{ \left\langle u, T_A(x), I_A(x), F_A(x) \right\rangle : x \in E, T_A(x), I_A(x), F_A(x) \in [0,1] \right\}$$

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.7. [4,7] Let E be a universe. A neutrosophic muti(refined)-set(NM-set) on E can be defined as follows:

$$A_{NM} = \left\{ \left\langle x, \left(T_A^1(x), T_A^2(x), ..., T_A^P(x)\right), \left(I_A^1(x), I_A^2(x), ..., I_A^P(x)\right), \left(F_A^1(x), F_A^2(x), ..., F_A^P(x)\right) \right\rangle : x \in X \right\}$$

where

$$T_A^1(x), T_A^2(x), ..., T_A^P(x), I_A^1(x), I_A^2(x), ..., I_A^P(x), F_A^1(x), F_A^2(x), ..., F_A^P(x) : E \rightarrow [0,1]$$

such that

$$0 \leq T_A^i(x) + I_A^i(x) + F_A^i(x) \leq 3 \text{ for } i = 1, 2, 3, ..., P \text{ for any } x \in X.$$

Here, $\left(T_A^1(x), T_A^2(x), ..., T_A^P(x)\right)$, $\left(I_A^1(x), I_A^2(x), ..., I_A^P(x)\right)$ and $\left(F_A^1(x), F_A^2(x), ..., F_A^P(x)\right)$ is the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element x, respectively. Also, P is called the dimension of NM-set A.

Definition 2.8. [5] A bipolar neutrosophic set (BN-set) A in X is defined as an object of the form

$$A_{BN} = \left\{ \left\langle x, T^+(x), I^+(x), F^+(x), T^-(x), I^-(x), F^-(x) \right\rangle : x \in X \right\},$$

where $T^+, I^+, F^+ : X \rightarrow [1,0]$ and $T^-, I^-, F^- : X \rightarrow [-1,0]$.

The positive membership degree $T^+(x)$, $I^+(x)$, $F^+(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ corresponding to a bipolar neutrosophic set A and the negative membership degree $T^-(x)$, $I^-(x)$, $F^-(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ to some implicit counter-property corresponding to a BN-set A.

For two BN-set

$$A_{BN} = \left\{ \left\langle x, T_A^+(x), I_A^+(x), F_A^+(x), T_A^-(x), I_A^-(x), F_A^-(x) \right\rangle : x \in X \right\}$$

and

$$B_{BN} = \left\{ \left\langle x, T_B^+(x), I_B^+(x), F_B^+(x), T_B^-(x), I_B^-(x), F_B^-(x) \right\rangle : x \in X \right\}$$

Then,

1. $A_{BN} \subseteq B_{BN}$ if and only if

$$T_A^+(x) \le T_B^+(x), \qquad T_A^-(x) \ge T_B^-(x),$$

$$I_A^+(x) \ge I_B^+(x), \qquad I_A^-(x) \le I_B^-(x),$$

$$F_A^+(x) \ge F_B^+(x), \qquad F_A^-(x) \le F_B^-(x) \text{ for all } x \in X.$$

2. $A_{BN} = B_{BN}$ if and only if

$$T_A^+(x) = T_B^+(x), \qquad T_A^-(x) = T_B^-(x),$$

$$I_A^+(x) = I_B^+(x), \qquad I_A^-(x) = I_B^-(x),$$

$$F_A^+(x) = F_B^+(x), \qquad F_A^-(x) = F_B^-(x) \text{ for any } x \in X.$$

3. $A_{BN}^{\ c}$ if and only if

$$A_{BN}^{\ c} = \left\{ \left\langle x, F_A^+(x), 1 - I_A^+(x), T_A^+(x), F_A^-(x), 1 - I_A^-(x), T_A^-(x) \right\rangle : x \in X \right\}$$

4. $A_{BN} \cap B_{BN}$ if and only if

$$A_{BN} \cap B_{BN} = \left\{ \left\langle x, T_A^+(x) \wedge T_B^+(x), I_A^+(x) \vee I_B^+(x), F_A^+(x) \vee F_B^+(x), T_A^-(x) \wedge T_B^-(x), I_A^-(x) \vee I_B^-(x), \right. \right.$$
$$\left. \left. F_A^-(x) \vee F_B^-(x) \right\rangle : x \in X \right\}$$

5. $A_{BN} \cup B_{BN}$ if and only if

$$A_{BN} \cup B_{BN} = \left\{ \left\langle x, T_A^+(x) \vee T_B^+(x), I_A^+(x) \vee I_B^+(x), F_A^+(x) \wedge F_B^+(x), T_A^-(x) \vee T_B^-(x), I_A^-(x) \wedge I_B^-(x), \right. \right.$$
$$\left. \left. F_A^-(x) \wedge F_B^-(x) \right\rangle : x \in X \right\}$$

3. BIPOLAR NEUTROSOPHIC REFINED SETS

In the following, some definition and operations on bipolar neutrosophic set and neutrosophic multisets defined in [5] and [7,16], respectively, we extend this definition to bipolar neutrosophic refined set.

Definition 3.1. Let E be a universe. A bipolar neutrosophic refined set (BNR-set) A on E can be defined as follows:

$$A = \left\{ \left\langle x, \left(T_A^{1+}(x), T_A^{2+}(x), \ldots, T_A^{P+}(x), T_A^{1-}(x), T_A^{2-}(x), \ldots, T_A^{P-}(x) \right), \right. \right.$$

$$\left(I_A^{1+}(x), I_A^{2+}(x), \ldots, I_A^{P+}(x), I_A^{1-}(x), I_A^{2-}(x), \ldots, I_A^{P-}(x) \right),$$

$$\left. \left. \left(F_A^{1+}(x), F_A^{2+}(x), \ldots, F_A^{P+}(x), F_A^{1-}(x), F_A^{2-}(x), \ldots, F_A^{P-}(x) \right) \right\rangle : x \in X \right\}$$

where,

$$T_A^{1+}(x), T_A^{2+}(x), \ldots, T_A^{P+}(x), T_A^{1-}(x), T_A^{2-}(x), \ldots, T_A^{P-}(x) : E \rightarrow [0,1],$$

$$I_A^{1+}(x), I_A^{2+}(x), \ldots, I_A^{P+}(x), I_A^{1-}(x), I_A^{2-}(x), \ldots, I_A^{P-}(x) : E \rightarrow [0,1],$$

and

$$F_A^{1+}(x), F_A^{2+}(x), \ldots, F_A^{P+}(x), F_A^{1-}(x), F_A^{2-}(x), \ldots, F_A^{P-}(x) : E \rightarrow [0,1]$$

such that

$$0 \leq T_A^{i}(x) + I_A^{i}(x) + F_A^{i}(x) \leq 3 \ (i = 1, 2, 3, \ldots, P) \text{ and } T_A^{1+}(x) \leq T_A^{2+}(x) \leq \ldots \leq T_A^{P+}(x) \text{ for any } x \in E.$$

$$\left(T_A^{1+}(x), T_A^{2+}(x), \ldots, T_A^{P+}(x), T_A^{1-}(x), T_A^{2-}(x), \ldots, T_A^{P-}(x) \right),$$

$$\left(I_A^{1+}(x), I_A^{2+}(x), \ldots, I_A^{P+}(x), I_A^{1-}(x), I_A^{2-}(x), \ldots, I_A^{P-}(x) \right),$$

$$\left(F_A^{1+}(x), F_A^{2+}(x), \ldots, F_A^{P+}(x), F_A^{1-}(x), F_A^{2-}(x), \ldots, F_A^{P-}(x) \right)$$

is the truth membership sequence, indeterminacy membership sequence and falsity membership sequence of the element x, respectively. Also, P is called the dimension of BNR-set A.

The set of all bipolar neutrosophic refined sets on E is denoted by BNRS(E).

Definition 3.2. Let $A, B \in BNRS(E)$. Then,

1. A is said to be BNR subset of B is denoted by $A \tilde{\subseteq} B$ if

$$T_A^{i+}(x) \leq T_B^{i+}(x), I_A^{i+}(x) \geq I_B^{i+}(x), F_A^{i+}(x) \geq F_B^{i+}(x),$$

$$T_A^{i-}(x) \geq T_B^{i-}(x), I_A^{i-}(x) \leq I_B^{i-}(x), F_A^{i-}(x) \leq F_B^{i-}(x).$$

2. A is said to be BNR equal of B is denoted by $A = B$ if $T_A^{i+}(x) = T_B^{i+}(x), I_A^{i+}(x) = I_B^{i+}(x),$

$$F_A^{i+}(x) = F_B^{i+}(x), T_A^{i-}(x) = T_B^{i-}(x), I_A^{i-}(x) = I_B^{i-}(x), F_A^{i-}(x) = F_B^{i-}(x).$$

3. The complement of A denoted by $A^{\tilde{c}}$ and is defined by

$$A^{\tilde{c}} = \left\{ \left\langle x, \left(F_A^{1+}(x), F_A^{2+}(x), \ldots, F_A^{P+}(x), F_A^{1-}(x), F_A^{2-}(x), \ldots, F_A^{P-}(x) \right) \right. \right.$$

$$\left(1 - I_A^{1+}\left(x \right), 1 - I_A^{2+}\left(x \right), ..., 1 - I_A^{P+}\left(x \right), 1 - I_A^{1-}\left(x \right), 1 - I_A^{2-}\left(x \right), ..., 1 - I_A^{P-}\left(x \right) \right),$$

$$\left(T_A^{1+}\left(x \right), T_A^{2+}\left(x \right), ..., T_A^{P+}\left(x \right), T_A^{1-}\left(x \right), T_A^{2-}\left(x \right), ..., T_A^{P-}\left(x \right) \right) \Big\rangle : x \in X \}$$

Definition 3.3. Let $A, B \in BNRS\left(E \right)$. Then,

1. The union of A and B is denoted by $A \,\tilde{\cup}\, B = C$ and is defined by

$$C = \{ \Big\langle x, \left(T_C^{1+}\left(x \right), T_C^{2+}\left(x \right), ..., T_C^{P+}\left(x \right), T_C^{1-}\left(x \right), T_C^{2-}\left(x \right), ..., T_C^{P-}\left(x \right) \right),$$

$$\left(I_C^{1+}\left(x \right), I_C^{2+}\left(x \right), ..., I_C^{P+}\left(x \right), I_C^{1-}\left(x \right), I_C^{2-}\left(x \right), ..., I_C^{P-}\left(x \right) \right),$$

$$\left(F_C^{1+}\left(x \right), F_C^{2+}\left(x \right), ..., F_C^{P+}\left(x \right), F_C^{1-}\left(x \right), F_C^{2-}\left(x \right), ..., F_C^{P-}\left(x \right) \right) \Big\rangle : x \in E \}$$

where

$$T_C^{1+}\left(x \right) = \max \left\{ T_A^{1+}\left(x \right), T_B^{1+}\left(x \right) \right\}, \quad T_C^{1-}\left(x \right) = \min \left\{ T_A^{1-}\left(x \right), T_B^{1-}\left(x \right) \right\},$$

$$I_C^{1+}\left(x \right) = \min \left\{ I_A^{1+}\left(x \right), I_B^{1+}\left(x \right) \right\}, \quad I_C^{1-}\left(x \right) = \max \left\{ I_A^{1-}\left(x \right), I_B^{1-}\left(x \right) \right\},$$

$$F_C^{1+}\left(x \right) = \min \left\{ F_A^{1+}\left(x \right), F_B^{1+}\left(x \right) \right\}, \quad F_C^{1-}\left(x \right) = \max \left\{ F_A^{1-}\left(x \right), F_B^{1-}\left(x \right) \right\}$$

$\forall x \in E$ and $\left(i = 1, 2, 3, ..., P \right)$.

2. The intersection of A and B is denoted by $A \,\tilde{\cap}\, B = D$ and is defined by

$$D = \{ \Big\langle x, \left(T_D^{1+}\left(x \right), T_D^{2+}\left(x \right), ..., T_D^{P+}\left(x \right), T_D^{1-}\left(x \right), T_D^{2-}\left(x \right), ..., T_D^{P-}\left(x \right) \right),$$

$$\left(I_D^{1+}\left(x \right), I_D^{2+}\left(x \right), ..., I_D^{P+}\left(x \right), I_D^{1-}\left(x \right), I_D^{2-}\left(x \right), ..., I_D^{P-}\left(x \right) \right),$$

$$\left(F_D^{1+}\left(x \right), F_D^{2+}\left(x \right), ..., F_D^{P+}\left(x \right), F_D^{1-}\left(x \right), F_D^{2-}\left(x \right), ..., F_D^{P-}\left(x \right) \right) \Big\rangle : x \in E \}$$

where

$$T_D^{1+}\left(x \right) = \min \left\{ T_A^{1+}\left(x \right), T_B^{1+}\left(x \right) \right\}, \quad T_D^{1-}\left(x \right) = \max \left\{ T_A^{1-}\left(x \right), T_B^{1-}\left(x \right) \right\},$$

$$I_D^{1+}\left(x \right) = \max \left\{ I_A^{1+}\left(x \right), I_B^{1+}\left(x \right) \right\}, \quad I_D^{1-}\left(x \right) = \min \left\{ I_A^{1-}\left(x \right), I_B^{1-}\left(x \right) \right\},$$

$$F_D^{1+}\left(x \right) = \max \left\{ F_A^{1+}\left(x \right), F_B^{1+}\left(x \right) \right\}, \quad F_D^{1-}\left(x \right) = \min \left\{ F_A^{1-}\left(x \right), F_B^{1-}\left(x \right) \right\}$$

$\forall x \in E$ and $\left(i = 1, 2, 3, ..., P \right)$.

Proposition 3.4. Let $A, B, C \in BNRS\left(E \right)$. Then

1. $A \,\tilde{\cup}\, B = B \,\tilde{\cup}\, A$ and $A \,\tilde{\cap}\, B = B \,\tilde{\cap}\, A$

2. $A \,\tilde{\cup}\, \left(B \,\tilde{\cup}\, C \right) = \left(A \,\tilde{\cup}\, B \right) \,\tilde{\cup}\, C$ and $A \,\tilde{\cap}\, \left(B \,\tilde{\cap}\, C \right) = \left(A \,\tilde{\cap}\, B \right) \,\tilde{\cap}\, C$

Proof: The proofs can be easily made.

Theorem 3.5. Let $A, B \in BNRS(E)$. Then, De Morgan's law is valid.

1. $\left(A \tilde{\cup} B \right)^{\tilde{c}} = A^{\tilde{c}} \tilde{\cap} B^{\tilde{c}}$

2. $\left(A \tilde{\cap} B \right)^{\tilde{c}} = A^{\tilde{c}} \tilde{\cup} B^{\tilde{c}}$

Proof: $A, B \in BNRS(E)$ is given. From Definition 3.2 and Definition 3.3.

Definition 3.6. Let E is a real Euclidean space E^n. Then, a BNR-set A is convex if and only if

$$T_A^{i+} \left(ax + (1-a) y \right) \geq T_A^{i+} (x) \wedge T_A^{i+} (y), \qquad T_A^{i-} \left(ax + (1-a) y \right) \leq T_A^{i-} (x) \vee T_A^{i-} (y),$$

$$I_A^{i+} \left(ax + (1-a) y \right) \leq I_A^{i+} (x) \vee I_A^{i+} (y), \qquad I_A^{i-} \left(ax + (1-a) y \right) \geq I_A^{i-} (x) \wedge I_A^{i-} (y),$$

$$F_A^{i+} \left(ax + (1-a) y \right) \geq F_A^{i+} (x) \vee F_A^{i+} (y), \qquad F_A^{i-} \left(ax + (1-a) y \right) \geq F_A^{i-} (x) \wedge F_A^{i-} (y).$$

for every $x, y \in E$, $a \in I$ and $i = 1, 2, 3, ..., P$

Definition 3.7. Let E is a real Euclidean space E^n. Then, a BNR-set A is strongly convex if and only if

$$T_A^{i+} \left(ax + (1-a) y \right) > T_A^{i+} (x) \wedge T_A^{i+} (y), \qquad T_A^{i-} \left(ax + (1-a) y \right) < T_A^{i-} (x) \vee T_A^{i-} (y),$$

$$I_A^{i+} \left(ax + (1-a) y \right) < I_A^{i+} (x) \vee I_A^{i+} (y), \qquad I_A^{i-} \left(ax + (1-a) y \right) > I_A^{i-} (x) \wedge I_A^{i-} (y),$$

$$F_A^{i+} \left(ax + (1-a) y \right) < F_A^{i+} (x) \vee F_A^{i+} (y), \qquad F_A^{i-} \left(ax + (1-a) y \right) > F_A^{i-} (x) \wedge F_A^{i-} (y).$$

for every $x, y \in E$, $a \in I$ and $i = 1, 2, 3, ..., P$

Theorem 3.8. Let $A, B \in BNRS(E)$. Then, $A \tilde{\cap} B$ is a convex (strongly convex) when both A and B are convex (strongly convex).

Proof: It is clear from Definition 3.6-3.7.

Definition 3.9. [16] Let $A, B \in BNRS(E)$. Then,

1. Hamming distance $d_{HD}(A, B)$ between A and B, defined by

$$d_{HD}(A, B) = \sum_{j=1}^{P} \sum_{i=1}^{n} \left(\left| T_A^{j+}(x_i) - T_B^{j+}(x_i) \right| + \left| I_A^{j+}(x_i) - I_B^{j+}(x_i) \right| + \left| F_A^{j+}(x_i) - F_B^{j+}(x_i) \right| + \right.$$

$$\left. \left| T_A^{j-}(x_i) - T_B^{j-}(x_i) \right| + \left| I_A^{j-}(x_i) - I_B^{j-}(x_i) \right| + \left| F_A^{j-}(x_i) - F_B^{j-}(x_i) \right| \right)$$

2. Normalized hamming distance $d_{NHD}(A, B)$ between A and B, defined by

$$d_{NHD}(A,B) = \frac{1}{6nP} \sum_{j=1}^{P} \sum_{i=1}^{n} \left(\left| T_A^{j+}(x_i) - T_B^{j+}(x_i) \right| + \left| I_A^{j+}(x_i) - I_B^{j+}(x_i) \right| + \left| F_A^{j+}(x_i) - F_B^{j+}(x_i) \right| + \right.$$

$$\left. \left| T_A^{j-}(x_i) - T_B^{j-}(x_i) \right| + \left| I_A^{j-}(x_i) - I_B^{j-}(x_i) \right| + \left| F_A^{j-}(x_i) - F_B^{j-}(x_i) \right| \right)$$

3. Euclidean distance $d_{ED}(A,B)$ between A and B, defined by

$$d_{ED}(A,B) = \sum_{j=1}^{P} \sum_{i=1}^{n} \sqrt{ \begin{array}{l} \left(T_A^{j+}(x_i) - T_B^{j+}(x_i) \right)^2 + \left(I_A^{j+}(x_i) - I_B^{j+}(x_i) \right)^2 + \left(F_A^{j+}(x_i) - F_B^{j+}(x_i) \right)^2 + \\ \left(T_A^{j-}(x_i) - T_B^{j-}(x_i) \right)^2 + \left(I_A^{j-}(x_i) - I_B^{j-}(x_i) \right)^2 + \left(F_A^{j-}(x_i) - F_B^{j-}(x_i) \right)^2 \end{array} }$$

4. Normalized euclidean distance $d_{NED}(A,B)$ between A and B, defined by

$$d_{NED}(A,B) = \frac{1}{6nP} \sum_{j=1}^{P} \sum_{i=1}^{n} \sqrt{ \begin{array}{l} \left(T_A^{j+}(x_i) - T_B^{j+}(x_i) \right)^2 + \left(I_A^{j+}(x_i) - I_B^{j+}(x_i) \right)^2 + \left(F_A^{j+}(x_i) - F_B^{j+}(x_i) \right)^2 + \\ \left(T_A^{j-}(x_i) - T_B^{j-}(x_i) \right)^2 + \left(I_A^{j-}(x_i) - I_B^{j-}(x_i) \right)^2 + \left(F_A^{j-}(x_i) - F_B^{j-}(x_i) \right)^2 \end{array} }$$

4. MEDICAL DIAGNOSIS VIA NRS THEORY

In the following, the example on intuitionistic fuzzy multiset and neutrosophic multiset given in [13,14] and [7], respectively, we extend this definition to BNRS.

Let $P = \{P_1, P_2, P_3, P_4\}$ be a set of patients, D={Viral Fever, Tuberculosis, Typhoid, Throat Disease} be a set of diseases and S={Temperature, Cough, Throat Pain, Headache, Body Pain} be a set of symptoms. Then, we have relation among Symptoms and Diseases as;

Table-I: BNRS R: The relation among Symptoms and Diseases

	Viral Fever	Tuberculosis
Temperature	$(0.3, 0.5, 0.6, -0.3, -0.5, -0.6)$	$(0.2, 0.1, 0.8, -0.1, -0.7, -0.9)$
Cough	$(0.4, 0.2, 0.6, -0.2, -0.4, -0.9)$	$(0.3, 0.2, 0.1, -0.9, -0.8, -0.7)$
Throat Pain	$(0.6, 0.2, 0.8, -0.7, -0.5, -0.8)$	$(0.3, 0.4, 0.6, -0.3, -0.4, -0.6)$
Headache	$(0.5, 0.8, 0.4, -0.6, -0.4, -0.2)$	$(0.7, 0.5, 0.4, -0.6, -0.3, -0.5)$
Body Pain	$(0.5, 0.4, 0.3, -0.3, -0.4, -0.7)$	$(0.8, 0.7, 0.6, -0.3, -0.2, -0.2)$

	Typhoid	Throat Disease
Temperature	$(0.2, 0.8, 0.7, -0.5, -0.4, -0.7)$	$(0.1, 0.8, 0.3, -0.1, -0.2, -0.3)$
Cough	$(0.6, 0.5, 0.4, -0.3, -0.5, -0.4)$	$(0.5, 0.8, 0.4, -0.5, -0.6, -0.3)$
Throat Pain	$(0.3, 0.8, 0.1, -0.3, -0.2, -0.1)$	$(0.8, 0.7, 0.9, -0.9, -0.7, -0.8)$
Headache	$(0.3, 0.2, 0.1, -0.8, -0.5, -0.4)$	$(0.8, 0.9, 0.2, -0.4, -0.7, -0.5)$
Body Pain	$(0.4, 0.5, 0.7, -0.2, -0.1, -0.4)$	$(0.3, 0.5, 0.4, -0.5, -0.6, -0.7)$

The results obtained different time intervals such as: 8:00 am 12:00 am and 4:00 pm in a day as Table-II;

Table-II: BNRS Q: The relation between Patient and Symptoms

	Temperature	Cough	Throat Pain
P_1	$(0.8, 0.9, 0.2, -0.4, -0.7, -0.5)$ $(0.3, 0.8, 0.1, -0.3, -0.2, -0.1)$ $(0.3, 0.2, 0.1, -0.9, -0.8, -0.7)$	$(0.3, 0.5, 0.6, -0.3, -0.5, -0.6)$ $(0.5, 0.8, 0.4, -0.6, -0.4, -0.2)$ $(0.8, 0.7, 0.6, -0.3, -0.2, -0.2)$	$(0.8, 0.7, 0.9, -0.9, -0.7, -0.8)$ $(0.4, 0.2, 0.6, -0.2, -0.4, -0.9)$ $(0.7, 0.5, 0.4, -0.6, -0.3, -0.5)$
P_2	$(0.6, 0.2, 0.8, -0.7, -0.5, -0.8)$ $(0.5, 0.8, 0.4, -0.5, -0.6, -0.3)$ $(0.2, 0.1, 0.8, -0.1, -0.7, -0.9)$	$(0.3, 0.4, 0.6, -0.3, -0.4, -0.6)$ $(0.5, 0.4, 0.3, -0.3, -0.4, -0.7)$ $(0.4, 0.5, 0.7, -0.2, -0.1, -0.4)$	$(0.3, 0.8, 0.1, -0.3, -0.2, -0.1)$ $(0.3, 0.2, 0.1, -0.9, -0.8, -0.7)$ $(0.2, 0.8, 0.7, -0.5, -0.4, -0.7)$
P_3	$(0.4, 0.5, 0.7, -0.2, -0.1, -0.4)$ $(0.3, 0.5, 0.4, -0.5, -0.6, -0.7)$ $(0.8, 0.9, 0.2, -0.4, -0.7, -0.5)$	$(0.8, 0.7, 0.6, -0.3, -0.2, -0.2)$ $(0.4, 0.2, 0.6, -0.2, -0.4, -0.9)$ $(0.6, 0.2, 0.8, -0.7, -0.5, -0.8)$	$(0.5, 0.8, 0.4, -0.5, -0.6, -0.3)$ $(0.3, 0.2, 0.1, -0.8, -0.5, -0.4)$ $(0.3, 0.8, 0.1, -0.3, -0.2, -0.1)$
P_4	$(0.5, 0.8, 0.4, -0.6, -0.4, -0.2)$ $(0.8, 0.7, 0.9, -0.9, -0.7, -0.8)$ $(0.2, 0.1, 0.8, -0.1, -0.7, -0.9)$	$(0.4, 0.5, 0.7, -0.2, -0.1, -0.4)$ $(0.1, 0.8, 0.3, -0.1, -0.2, -0.3)$ $(0.2, 0.8, 0.7, -0.5, -0.4, -0.7)$	$(0.3, 0.5, 0.4, -0.5, -0.6, -0.7)$ $(0.5, 0.4, 0.3, -0.3, -0.4, -0.7)$ $(0.3, 0.2, 0.1, -0.9, -0.8, -0.7)$

	Throat Pain	Headache
P_1	$(0.2, 0.1, 0.8, -0.1, -0.7, -0.9)$ $(0.4, 0.2, 0.6, -0.2, -0.4, -0.9)$ $(0.5, 0.8, 0.4, -0.5, -0.6, -0.3)$	$(0.4, 0.5, 0.7, -0.2, -0.1, -0.4)$ $(0.8, 0.7, 0.6, -0.3, -0.2, -0.2)$ $(0.3, 0.5, 0.6, -0.3, -0.5, -0.6)$
P_2	$(0.3, 0.2, 0.1, -0.9, -0.8, -0.7)$ $(0.3, 0.5, 0.4, -0.5, -0.6, -0.7)$ $(0.1, 0.8, 0.3, -0.1, -0.2, -0.3)$	$(0.7, 0.5, 0.4, -0.6, -0.3, -0.5)$ $(0.5, 0.4, 0.3, -0.3, -0.4, -0.7)$ $(0.2, 0.8, 0.7, -0.5, -0.4, -0.7)$
P_3	$(0.4, 0.5, 0.7, -0.2, -0.1, -0.4)$ $(0.3, 0.8, 0.1, -0.3, -0.2, -0.1)$ $(0.2, 0.8, 0.7, -0.5, -0.4, -0.7)$	$(0.3, 0.4, 0.6, -0.3, -0.4, -0.6)$ $(0.8, 0.7, 0.9, -0.9, -0.7, -0.8)$ $(0.1, 0.8, 0.3, -0.1, -0.2, -0.3)$
P_4	$(0.8, 0.7, 0.9, -0.9, -0.7, -0.8)$ $(0.3, 0.5, 0.6, -0.3, -0.5, -0.6)$ $(0.8, 0.7, 0.6, -0.3, -0.2, -0.2)$	$(0.4, 0.5, 0.7, -0.2, -0.1, -0.4)$ $(0.5, 0.8, 0.4, -0.5, -0.6, -0.3)$ $(0.3, 0.8, 0.1, -0.3, -0.2, -0.1)$

The normalized Hamming distance between Q and R is computed as;

Table-III: The Normalized Hamming Distance between Q and R

	Viral Fever	Tuberculosis	Typhoid	Throat Disease
P_1	0,251	0,300	0,260	0,253
P_2	0,224	0,262	0,213	0,291
P_3	0,232	0,299	0,239	0,310
P_4	0,254	0,263	0,268	0,290

The lowest distance from the Table-III gives the proper medical diagnosis. Patient P_1 suffers from Viral Fever, Patient P_2 suffers from Typhoid, Patient P_3 suffers from Viral Fever and Patient P_4 suffers from Viral Fever.

5   CONCLUSION

In this paper, we introduced definition of bipolar neutrosophic refined sets and investigated some of their basic properties which is generalize the fuzzy set, fuzzy multiset, bipolar fuzzy set, bipolar fuzzy multiset, intuitionistic fuzzy multiset, neutrosophic multisets and bipolar neutrosophic set. The theory can be applied to problems that contain uncertainty such as problems in economic systems, algebraic structure, topological structure, pattern recognition, game theory and so on.

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