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# On new inequalities for *h*-convex functions via Riemann-Liouville fractional integration

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**Abstract.** In this paper, some new inequalities of the Hermite-Hadamard type for *h*-convex functions via Riemann-Liouville fractional integral are given.

## 1. Introduction

Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be a convex function and let $a, b \in I$, with $a < b$. The following inequality;

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2} \tag{1}$$

is known in the literature as Hadamard's inequality. Both inequalities hold in the reversed direction if $f$ is concave.

In [16], Varošanec introduced the following class of functions.

**Definition 1.1.** Let $h : J \subset \mathbb{R} \to \mathbb{R}$ be a positive function. We say that $f : I \subset \mathbb{R} \to \mathbb{R}$ is $h-$convex function or that $f$ belongs to the class $SX(h, I)$, if $f$ is nonnegative and for all $x, y \in I$ and $\lambda \in (0, 1)$ we have

$$f(\lambda x + (1-\lambda)y) \leq h(\lambda)f(x) + h(1-\lambda)f(y). \tag{2}$$

If the inequality in (2) is reversed, then $f$ is said to be $h-$concave, i.e., $f \in SV(h, I)$.

Obviously, if $h(\lambda) = \lambda$, then all nonnegative convex functions belong to $SX(h, I)$ and all nonnegative concave functions belong to $SV(h, I)$; if $h(\lambda) = \frac{1}{\lambda}$, then $SX(h, I) = Q(I)$; if $h(\lambda) = 1$, then $SX(h, I) \supseteq P(I)$ and if $h(\lambda) = \lambda^s$, where $s \in (0, 1)$, then $SX(h, I) \supseteq K_s^2$. For some recent results for $h-$convex functions we refer to the interested reader to the papers [3], [4], [12], [13].

**Definition 1.2.** ([1]) A function $h : J \to \mathbb{R}$ is said to be a superadditive function if

$$h(x + y) \geq h(x) + h(y) \tag{3}$$

for all $x, y \in J$.

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In [12], Sarıkaya et al. proved the following Hadamard type inequalities for $h-$convex functions.

**Theorem 1.3.** *Let $f \in SX(h, I)$, $a, b \in I$ with $a < b$ and $f \in L_1[a, b]$. Then*

$$\frac{1}{2h\left(\frac{1}{2}\right)} f\left(\frac{a+b}{2}\right) \le \frac{1}{b-a}\int_a^b f(x)dx \le [f(a)+f(b)]\int_0^1 h(\alpha)d\alpha. \tag{4}$$

In [14], Sarıkaya et al. proved the following Hadamard type inequalities for fractional integrals as follows.

**Theorem 1.4.** *Let $f : [a, b] \to \mathbb{R}$ be positive function with $0 \le a < b$ and $f \in L_1[a, b]$. If $f$ is convex function on $[a, b]$, then the following inequalities for fractional integrals hold:*

$$f\left(\frac{a+b}{2}\right) \le \frac{\Gamma(\alpha+1)}{2(b-a)^\alpha}\left[J_{a^+}^\alpha(b) + J_{b^-}^\alpha(a)\right] \le \frac{f(a)+f(b)}{2} \tag{5}$$

*with $\alpha > 0$.*

Now we give some necessary definitions and mathematical preliminaries of fractional calculus theory which are used throughout this paper.

**Definition 1.5.** Let $f \in L_1[a, b]$. The Riemann-Liouville integrals $J_{a^+}^\alpha f$ and $J_{b^-}^\alpha f$ of order $\alpha > 0$ with $a \ge 0$ are defined by

$$J_{a^+}^\alpha f(x) = \frac{1}{\Gamma(\alpha)}\int_a^x (x-t)^{\alpha-1} f(t)dt, \quad x > a$$

and

$$J_{b^-}^\alpha f(x) = \frac{1}{\Gamma(\alpha)}\int_x^b (t-x)^{\alpha-1} f(t)dt, \quad x < b$$

respectively where $\Gamma(\alpha) = \int_0^\infty e^{-u}u^{\alpha-1}du$. Here is $J_{a^+}^0 f(x) = J_{b^-}^0 f(x) = f(x)$.

In the case of $\alpha = 1$, the fractional integral reduces to the classical integral.

For some recent results connected with fractional integral inequalities see [2, 5–8, 10, 14, 15].

In [14], Sarikaya *et al.* proved a variant of the identity that established by Dragomir and Agarwal in [9, Lemma 2.1] for fractional integrals as the following.

**Lemma 1.6.** *Let $f : [a, b] \to \mathbb{R}$ be a differentiable mapping on $(a, b)$ with $a < b$. If $f' \in L[a, b]$, then the following equality for fractional integrals holds:*

$$\frac{f(a)+f(b)}{2} - \frac{\Gamma(\alpha+1)}{2(b-a)^\alpha}\left[J_{a^+}^\alpha f(b) + J_{b^-}^\alpha f(a)\right] = \frac{b-a}{2}\int_0^1 \left[(1-t)^\alpha - t^\alpha\right] f'(ta+(1-t)b)dt.$$

The aim of this paper is to establish Hadamard type inequalities for $h-$convex functions via Riemann-Liouville fractional integral.

## 2. Main results

**Theorem 2.1.** *Let $f \in SX(h, I)$, $a, b \in I$ with $a < b$ and $f \in L_1[a, b]$. Then one has inequality for $h-$convex functions via fractional integrals*

$$\frac{\Gamma(\alpha)}{(b-a)^{\alpha}}\left[J_{a^+}^{\alpha}(b)+J_{b^-}^{\alpha}(a)\right] \leq [f(a)+f(b)]\int_0^1 t^{\alpha-1}\left[h(t)+h(1-t)\right]dt \leq \frac{2\left[f(a)+f(b)\right]}{(\alpha p-p+1)^{\frac{1}{p}}}\left(\int_0^1 (h(t))^q\,dt\right)^{\frac{1}{q}} \tag{6}$$

*where $p^{-1} + q^{-1} = 1$.*

*Proof.* Since $f \in SX(h, I)$, we have

$$f(tx + (1-t)y) \leq h(t)f(x) + h(1-t)f(y)$$

and

$$f((1-t)x + ty) \leq h(1-t)f(x) + h(t)f(y).$$

By adding these inequalities we get

$$f(tx+(1-t)y) + f((1-t)x+ty) \leq [h(t)+h(1-t)]\left[f(x)+f(y)\right]. \tag{7}$$

By using (7) with $x = a$ and $y = b$ we have

$$f(ta+(1-t)b) + f((1-t)a+tb) \leq [h(t)+h(1-t)]\left[f(a)+f(b)\right]. \tag{8}$$

Then multiplying both sides of (8) by $t^{\alpha-1}$ and integrating the resulting inequality with respect to $t$ over $[0, 1]$, we get

$$\int_0^1 t^{\alpha-1}\left[f(ta+(1-t)b)+f((1-t)a+tb)\right]dt \leq \int_0^1 t^{\alpha-1}\left[h(t)+h(1-t)\right]\left[f(a)+f(b)\right]dt, \tag{9}$$

and

$$\frac{\Gamma(\alpha)}{(b-a)^{\alpha}}\left[J_{a^+}^{\alpha}(b)+J_{b^-}^{\alpha}(a)\right] \leq [f(a)+f(b)]\int_0^1 t^{\alpha-1}\left[h(t)+h(1-t)\right]dt \tag{10}$$

and thus the first inequality is proved.

To obtain the second inequality in (6), by using Hölder inequality for the right hand side of (10), we obtain

$$\begin{aligned}
&\int_0^1 t^{\alpha-1}\left[h(t)+h(1-t)\right]dt \\
\leq\;& \left(\int_0^1 \left(t^{\alpha-1}\right)^p dt\right)^{\frac{1}{p}}\left(\int_0^1 (h(t)+h(1-t))^q\,dt\right)^{\frac{1}{q}} \\
=\;& \left(\left.\frac{t^{\alpha p-p+1}}{\alpha p-p+1}\right|_0^1\right)^{\frac{1}{p}}\left(\int_0^1 (h(t)+h(1-t))^q\,dt\right)^{\frac{1}{q}} \\
=\;& \left(\frac{1}{\alpha p-p+1}\right)^{\frac{1}{p}}\left(\int_0^1 (h(t)+h(1-t))^q\,dt\right)^{\frac{1}{q}}
\end{aligned}$$

Then using Minkowski inequality

$$\begin{aligned}
& \left(\frac{1}{\alpha p-p+1}\right)^{\frac{1}{p}}\left(\int_0^1 (h(t)+h(1-t))^q\, dt\right)^{\frac{1}{q}} \\
\leq\; & \left(\frac{1}{\alpha p-p+1}\right)^{\frac{1}{p}}\left[\left(\int_0^1 (h(t))^q\, dt\right)^{\frac{1}{q}}+\left(\int_0^1 (h(1-t))^q\, dt\right)^{\frac{1}{q}}\right] \\
=\; & \frac{2}{(\alpha p-p+1)^{\frac{1}{p}}}\left(\int_0^1 (h(t))^q\, dt\right)^{\frac{1}{q}}
\end{aligned}$$

where the proof is completed. □

**Remark 2.2.** If we choose $\alpha = 1$ in Theorem 1, we obtain

$$\frac{1}{b-a}\int_a^b f(x)\, dx \leq [f(a)+f(b)]\int_0^1 h(t)dt \leq [f(a)+f(b)]\left(\int_0^1 (h(t))^q\, dt\right)^{\frac{1}{q}}.$$

**Corollary 2.3.** *(1) If we choose $h(\lambda)=\lambda$ in Remark 2.2, we get*

$$\frac{1}{b-a}\int_a^b f(x)\, dx \leq \frac{f(a)+f(b)}{2} \leq \frac{f(a)+f(b)}{(q+1)^{\frac{1}{q}}}$$

*for ordinary convex functions.*

*(2) If we choose $h(\lambda)=1$ in Remark 2.2, we get*

$$\frac{2}{b-a}\int_a^b f(x)\, dx \leq 2(f(a)+f(b))$$

*for $P-$functions. This inequality is a refinement of right hand side of (1) for $P-$functions.*

*(3) If we choose $h(\lambda)=\lambda^s$ in Remark 2.2, we get*

$$\frac{1}{b-a}\int_0^1 f(x)\, dx \leq \frac{f(a)+f(b)}{s+1} \leq \frac{f(a)+f(b)}{(sq+1)^{\frac{1}{q}}}$$

*for $s-$convex functions in the second sense with $s \in (0,1]$.*

**Theorem 2.4.** *Let $f \in SX(h,I)$, $a,b \in I$ with $a<b$, $h$ be superadditive on $I$ and $f \in L_1[a,b]$, $h \in L_1[0,1]$. Then one has inequality for $h-$convex functions via fractional integrals*

$$\frac{\Gamma(\alpha)}{(b-a)^\alpha}\left[J_{a^+}^\alpha(b)+J_{b^-}^\alpha(a)\right] \leq \frac{h(1)}{\alpha}\left[f(a)+f(b)\right]. \tag{11}$$

*Proof.* Since $f \in SX(h,I)$ and $h$ is superadditive, by using (8), we have

$$\begin{aligned}
f(ta+(1-t)b)+f((1-t)a+tb) &\leq [h(t)+h(1-t)]\left[f(a)+f(b)\right] \\
&\leq h(1)\left[f(a)+f(b)\right].
\end{aligned} \tag{12}$$

Then multiplying both sides of (12) by $t^{\alpha-1}$ and integrating the resulting inequality with respect to $t$ over $[0,1]$, we get

$$\int_0^1 t^{\alpha-1}\left[f(ta+(1-t)b)+f((1-t)a+tb)\right]dt \leq \int_0^1 t^{\alpha-1}h(1)\left[f(a)+f(b)\right]dt,$$

and

$$\frac{\Gamma(\alpha)}{(b-a)^{\alpha}}\left[J_{a^+}^{\alpha}(b)+J_{b^-}^{\alpha}(a)\right]\leq h(1)\left[f(a)+f(b)\right]\int_0^1 t^{\alpha-1}dt.$$

This completes the proof. □

**Remark 2.5.** If we choose $\alpha=1$ in Theorem 2.4, then (11) reduce to special version of right hand side of (4).

**Theorem 2.6.** *Let $h:J\subset\mathbb{R}\to\mathbb{R}$ and $f:[a,b]\to\mathbb{R}$ be positive functions with $0\leq a<b$ and $h^q\in L_1[0,1]$, $f\in L_1[a,b]$. If $|f'|$ is an $h-$convex mapping on $[a,b]$, then the following inequality for fractional integrals holds,*

$$\begin{aligned}
&\left|\frac{f(a)+f(b)}{2}-\frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}\left[J_{a+}^{\alpha}f(b)+J_{b-}^{\alpha}f(a)\right]\right| \qquad (13)\\
\leq\ &\frac{(b-a)\left[|f'(a)|+|f'(b)|\right]}{2}\left[\left(\frac{2^{\alpha p+1}-1}{2^{\alpha p+1}(\alpha p+1)}\right)^{\frac{1}{p}}-\left(\frac{1}{2^{\alpha p+1}(\alpha p+1)}\right)^{\frac{1}{p}}\right]\\
&\times\left[\left(\int_0^{\frac{1}{2}}(h(t))^q dt\right)^{\frac{1}{q}}+\left(\int_{\frac{1}{2}}^1(h(t))^q dt\right)^{\frac{1}{q}}\right]
\end{aligned}$$

*where $\alpha>0$, $p>1$ and $p^{-1}+q^{-1}=1$.*

*Proof.* From Lemma 1.6 and using the properties of modulus, we have

$$\left|\frac{f(a)+f(b)}{2}-\frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}\left[J_{a+}^{\alpha}f(b)+J_{b-}^{\alpha}f(a)\right]\right|\leq\frac{b-a}{2}\int_0^1\left|(1-t)^{\alpha}-t^{\alpha}\right|\left|f'(ta+(1-t)b)\right|dt.$$

Since $|f'|$ is $h-$convex on $[a,b]$, we have

$$\begin{aligned}
&\left|\frac{f(a)+f(b)}{2}-\frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}\left[J_{a+}^{\alpha}(b)+J_{b-}^{\alpha}(a)\right]\right| \qquad (14)\\
\leq\ &\frac{b-a}{2}\left\{\int_0^{\frac{1}{2}}\left[(1-t)^{\alpha}-t^{\alpha}\right]\left[h(t)|f'(a)|+h(1-t)|f'(b)|\right]dt\right.\\
&\left.+\int_{\frac{1}{2}}^1\left[t^{\alpha}-(1-t)^{\alpha}\right]\left[h(t)|f'(a)|+h(1-t)|f'(b)|\right]dt\right\}\\
=\ &\frac{b-a}{2}\left\{|f'(a)|\int_0^{\frac{1}{2}}(1-t)^{\alpha}h(t)dt-|f'(a)|\int_0^{\frac{1}{2}}t^{\alpha}h(t)dt\right.\\
&+|f'(b)|\int_0^{\frac{1}{2}}(1-t)^{\alpha}h(1-t)dt-|f'(b)|\int_0^{\frac{1}{2}}t^{\alpha}h(1-t)dt\\
&+|f'(a)|\int_{\frac{1}{2}}^1 t^{\alpha}h(t)dt-|f'(a)|\int_{\frac{1}{2}}^1(1-t)^{\alpha}h(t)dt\\
&\left.+|f'(b)|\int_{\frac{1}{2}}^1 t^{\alpha}h(1-t)dt-|f'(b)|\int_{\frac{1}{2}}^1(1-t)^{\alpha}h(1-t)dt\right\}.
\end{aligned}$$

In the right hand side of above inequality by using Hölder inequality for $p^{-1}+q^{-1}=1$ and $p>1$, we get

$$\int_0^{\frac{1}{2}}(1-t)^{\alpha}h(t)dt=\int_{\frac{1}{2}}^1 t^{\alpha}h(1-t)dt\leq\left[\frac{2^{\alpha p+1}-1}{2^{\alpha p+1}(\alpha p+1)}\right]^{\frac{1}{p}}\left(\int_0^{\frac{1}{2}}[h(t)]^q dt\right)^{\frac{1}{q}},$$

$$\int_0^{\frac{1}{2}} (1-t)^\alpha h(1-t)\,dt = \int_{\frac{1}{2}}^1 t^\alpha h(t)\,dt \le \left[\frac{2^{\alpha p+1}-1}{2^{\alpha p+1}(\alpha p+1)}\right]^{\frac{1}{p}} \left(\int_{\frac{1}{2}}^1 [h(t)]^q\,dt\right)^{\frac{1}{q}},$$

$$\int_0^{\frac{1}{2}} t^\alpha h(t)\,dt = \int_{\frac{1}{2}}^1 (1-t)^\alpha h(1-t)\,dt \le \left[\frac{1}{2^{\alpha p+1}(\alpha p+1)}\right]^{\frac{1}{p}} \left(\int_0^{\frac{1}{2}} [h(t)]^q\,dt\right)^{\frac{1}{q}}$$

and

$$\int_0^{\frac{1}{2}} t^\alpha h(1-t)\,dt = \int_{\frac{1}{2}}^1 (1-t)^\alpha h(t)\,dt \le \left[\frac{1}{2^{\alpha p+1}(\alpha p+1)}\right]^{\frac{1}{p}} \left(\int_{\frac{1}{2}}^1 [h(t)]^q\,dt\right)^{\frac{1}{q}}.$$

Then using the above inequalities in the right hand side of (14), we get

$$\begin{aligned}
&\left|\frac{f(a)+f(b)}{2} - \frac{\Gamma(\alpha+1)}{2(b-a)^\alpha}\left[J_{a+}^\alpha(b) + J_{b-}^\alpha(a)\right]\right| \\
\le\ & \frac{b-a}{2}\left\{\left|f'(a)\right|\left\{\left(\left[\frac{2^{\alpha p+1}-1}{2^{\alpha p+1}(\alpha p+1)}\right]^{\frac{1}{p}} - \left[\frac{1}{2^{\alpha p+1}(\alpha p+1)}\right]^{\frac{1}{p}}\right)\left(\int_0^{\frac{1}{2}} [h(t)]^q\,dt\right)^{\frac{1}{q}}\right\}\right. \\
&+\left(\left[\frac{2^{\alpha p+1}-1}{2^{\alpha p+1}(\alpha p+1)}\right]^{\frac{1}{p}} - \left[\frac{1}{2^{\alpha p+1}(\alpha p+1)}\right]^{\frac{1}{p}}\right)\left(\int_{\frac{1}{2}}^1 [h(t)]^q\,dt\right)^{\frac{1}{q}}\Bigg\} \\
&+\left|f'(b)\right|\left\{\left(\left[\frac{2^{\alpha p+1}-1}{2^{\alpha p+1}(\alpha p+1)}\right]^{\frac{1}{p}} - \left[\frac{1}{2^{\alpha p+1}(\alpha p+1)}\right]^{\frac{1}{p}}\right)\left(\int_{\frac{1}{2}}^1 [h(t)]^q\,dt\right)^{\frac{1}{q}}\right. \\
&+\left(\left[\frac{2^{\alpha p+1}-1}{2^{\alpha p+1}(\alpha p+1)}\right]^{\frac{1}{p}} - \left[\frac{1}{2^{\alpha p+1}(\alpha p+1)}\right]^{\frac{1}{p}}\right)\left(\int_0^{\frac{1}{2}} [h(t)]^q\,dt\right)^{\frac{1}{q}}\Bigg\} \\
=\ & \frac{b-a}{2}\left\{\left|f'(a)\right|\left(\left[\frac{2^{\alpha p+1}-1}{2^{\alpha p+1}(\alpha p+1)}\right]^{\frac{1}{p}} - \left[\frac{1}{2^{\alpha p+1}(\alpha p+1)}\right]^{\frac{1}{p}}\right)\left[\left(\int_0^{\frac{1}{2}} [h(t)]^q\,dt\right)^{\frac{1}{q}} + \left(\int_{\frac{1}{2}}^1 [h(t)]^q\,dt\right)^{\frac{1}{q}}\right]\right. \\
&+\left|f'(b)\right|\left(\left[\frac{2^{\alpha p+1}-1}{2^{\alpha p+1}(\alpha p+1)}\right]^{\frac{1}{p}} - \left[\frac{1}{2^{\alpha p+1}(\alpha p+1)}\right]^{\frac{1}{p}}\right)\left[\left(\int_0^{\frac{1}{2}} [h(t)]^q\,dt\right)^{\frac{1}{q}} + \left(\int_{\frac{1}{2}}^1 [h(t)]^q\,dt\right)^{\frac{1}{q}}\right]\Bigg\} \\
=\ & \frac{(b-a)\left[\left|f'(a)\right| + \left|f'(b)\right|\right]}{2}\left[\left(\frac{2^{\alpha p+1}-1}{2^{\alpha p+1}(\alpha p+1)}\right)^{\frac{1}{p}} - \left(\frac{1}{2^{\alpha p+1}(\alpha p+1)}\right)^{\frac{1}{p}}\right] \\
&\times\left[\left(\int_0^{\frac{1}{2}} (h(t))^q\,dt\right)^{\frac{1}{q}} + \left(\int_{\frac{1}{2}}^1 (h(t))^q\,dt\right)^{\frac{1}{q}}\right]
\end{aligned}$$

which is the desired result. The proof is completed. □

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