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ON ONE EXTENSION THEOREM DEALING WITH WEIGHTED ORLICZ–SLOBODETSKII SPACE. ANALYSIS ON CUBE

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Abstract. Having given weight $\tilde{\rho} = \rho(\operatorname{dist}(x, \partial Q))$ defined on cube Q and Orlicz function R, we construct the weight $\omega_\rho(\cdot,\cdot)$ defined on $\partial Q \times \partial Q$ and extension operator $\operatorname{Ext}^L : Lip_d(\partial Q) \mapsto Lip(Q)$ from Lipschitz functions defined on ∂Q with certain restricted support to Lipschitz functions defined on Q, independent of ρ and R, in such a way that Ext^L extends to the bounded operator from certain subspace of weighted Orlicz-Slobodetskii space $Y^{R,R}_{\omega_\rho}(\partial Q)$ subordinated to the weight ω_ρ to Orlicz Sobolev space $W^{1,R}_\rho(Q)$. Result is new in the unweighted Orlicz setting for general function R as well as in the weighted L^p setting.

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REFERENCES

[1] H.-D. ALBER, *Materials with memory. Initial-boundary value problems for constitutive equations with internal variables*, Lecture Notes in Mathematics 1682. Springer, Berlin 1998.
[2] N. ARONSZAJN, *Boundary value of functions with finite Dirichlet integral*, Techn. Report 14, University of Kansas 1955.
[3] J. M. BALL, *Convexity conditions and existence theorems in nonlinear elasticity*, Arch. Rat. Mech. Anal. **63** (1977), 337–403.
[4] A. CORUSO, *Two properties of norms in Orlicz spaces*, Matematiche (Catania) **56** (1) (2001), 183–194.
[5] J. CHABROWSKI, *The Dirichlet Problem with L^2-Boundary Data for Elliptic Linear Equations*, Lecture Notes in Math., vol. 1482, Springer-Verlag, Berlin, 1991.
[6] A. CIANCHI, *Some results in the theory of Orlicz spaces and applications to variational problems*, Nonlinear analysis, function spaces and applications, Vol. 6 (Prague, 1998), Acad. Sci. Czech Rep., Prague 1999, 50–92.
[7] A. CIANCHI, *Orlicz-Sobolev boundary trace embeddings*, Math. Z. **266** (2) (2010), 431–449.
[8] A. CIANCHI, R. KERMAN, L. PICK, *Boundary trace inequalities and rearrangements*, J. Anal. Math. **105** (2008), 241–265.
[9] R. N. DHARA, A. KAŁAMAJSKA, *On one extension theorem dealing with weighted Orlicz-Slobodetskii space. Analysis on Lipschitz subgraph and Lipschitz domain*, preprint avaliable at: `http://www.mimuw.edu.pl/badania/preprinty/preprinty-imat/?LANG=en`.
[10] T. K. DONALDSON, N. S. TRUDINGER, *Orlicz-Sobolev spaces and imbedding theorems*, J. Functional Analysis **8** (1971), 52–75.
[11] A. FOUGÉRES, *Thesis* (in French), University of Besançon, 1972.
[12] A. FOUGÉRES, *Théorèmes de trace et de prolongement dans les espaces de Sobolev et Sobolev-Orlicz*, (in French) C. R. Acad. Sci. Paris Sér. A-B 274 (1972), A181–A184.
[13] E. GAGLIARDO, *Caratterizzazioni delle tracce sulla frontiera relative ad alcune classi di funzioni in n variabili*, (in Italian) Rend. Sem. Mat. Univ. Padova **27** 1957 284–305.
[14] J.-P. GOSSEZ, *Nonlinear elliptic boundary value problems for equations with rapidly (or slowly) increasing coefficients*, Trans. Amer. Math. Soc. **190** (1974), 163–205.

Paper MIA-18-04

[15] J.-P. GOSSEZ AND V. MUSTONEN, *Variational inequalities in Orlicz-Sobolev spaces*, Nonlinear Anal. **11** (3) (1987), 379–392.
[16] G. N. JAKOVLEV, *The traces of functions whose derivatives are summable with a certain weight*, (in Russian) Imbedding theorems and their applications (Proc. Sympos., Baku, 1966) (Russian), 225–233, 247. Izdat. "Nauka", Moscow, 1970.
[17] A. KAŁAMAJSKA AND M. KRBEC, *Traces of Orlicz-Sobolev functions under general growth restrictions*, Math. Nachr. **286** (7) (2013), 730–742.
[18] A. KAŁAMAJSKA AND M. KRBEC, *On solutions to heat equation with the initial condition in Orlicz-Slobodetskii space*, to appear in Proc. Roy. Soc. Edinburgh Sec. A.
[19] A. KAŁAMAJSKA AND K. PIETRUSKA-PAŁUBA, *Gagliardo–Nirenberg inequalities in weighted Orlicz spaces*, Studia Math. **173** (1) (2006), 49–71.
[20] H. KITA, *On Hardy-Littlewood maximal functions in Orlicz spaces*, Math. Nachr. **183** (1997), 135–155.
[21] D. KIM, *Trace theorems for Sobolev-Slobodeckij spaces with or without weights*, J. Funct. Spaces Appl. **5** (3) (2007), 243–268.
[22] D. KIM, *Elliptic equations with nonzero boundary conditions in weighted Sobolev spaces*, J. Math. Anal. Appl. **337** (2008) 1465–1479.
[23] K.-H. KIM, N. V. KRYLOV, *On the Sobolev space theory of parabolic and elliptic equations in C^1 domains*, SIAM J. Math. Anal. **36** (2) (2004) 618–642 (electronic).
[24] V. M. KOKILAŠHVILI, *On traces of functions with partial derivatives from Orlicz classes*, Special issue dedicated to Władysław Orlicz on the occasion of his seventy-fifth birthday. Comment. Math. Special Issue 1 (1978), 183–189.
[25] L. D. KUDRYAVCEV, *Direct and inverse imbedding theorems. Applications to the solution of elliptic equations by variational methods*, Trudy Mat. Inst. Steklov. **55** (1959), 182 pp. (in Russian).
[26] A. KUFNER, O. JOHN, S. FUČÍK, *Function spaces. Monographs and Textbooks on Mechanics of Solids and Fluids; Mechanics: Analysis*, Noordhoff International Publishing, Leyden; Academia, Prague, 1977.
[27] A. KUFNER, *Weighted Sobolev spaces*, A Wiley–Interscience Publication, John Wiley & Sons, New York, 1985 (translated from the Czech).
[28] M.-TH. LACROIX, *Espaces de traces des espaces de Sobolev-Orlicz*, J. Math. Pures Appl. **53** (9) (1974), 439–458.
[29] M.-TH. LACROIX, *Echelle d'espaces intermédiaires entre un espace de Sobolev-Orlicz et un espace d'Orlicz. Trace d'espaces de Sobolev-Orlicz avec poids*, (in French), Journées d'Analyse Non Linéaire (Proc. Conf., Besançon, 1977), 121–139, Lecture Notes in Math., **665**, Springer, Berlin, 1978.
[30] J.-L. LIONS, *Un théoreḿe de traces; applications*, (in French) C. R. Acad. Sci. Paris **249** (1959) 2259–2261.
[31] P. I. LIZORKIN, *Boundary properties of functions from "weight" classes*, Dokl. Akad. Nauk SSSR **132** (1960), 514–517 (in Russian); translated as Soviet Math. Dokl. **1** (1960), 589–593.
[32] S. V. LOTOTSKY, *Linear stochastic parabolic equations, degenerating on the boundary of a domain*, Electron. J. Probab. **6** (24) (2001), 1–14.
[33] V. G. MAZ'YA, *Sobolev Spaces*, Springer–Verlag, Berlin, 1985.
[34] V. G. MAZ'YA, J. ROSSMANN, *Schauder estimates for solutions to a mixed boundary value problem for the Stokes system in polyhedral domains*, Math. Methods Appl. Sci. **29** (9) (2006), 965–1017.
[35] J. NEČAS, *Direct methods in the theory of elliptic equations*, Translated from the 1967 French original by Gerard Tronel and Alois Kufner, Editorial coordination and preface by Šárka Nečasová and a contribution by Christian G. Simader, Springer Monographs in Mathematics, Springer, Heidelberg, 2012.
[36] J. NEČAS, *Sur une méthode pour résoudre les équations aux dérivés partielles du type elliptique, voisine de la variationnelle*, (in French), Ann. Scuola Norm. Sup. Pisa **16** (3) (1962), 305–326.
[37] A. NEKVINDA, L. PICK, *On traces on the weighted Sobolev spaces $H^{1,p}_{\varepsilon,M}$*, Funct. Approx. Comment. Math. **20** (1992), 143–151.
[38] A. NEKVINDA, *Characterization of traces of the weighted Sobolev space $W^{1,p}(\Omega, d^{\varepsilon}_{M})$ on M*, Czechoslovak Math. J., Praha **43** (1993), 695–711.
[39] S. M. NIKOLSKII, *Properties of certain classes of functions of several variables on differentiable manifolds*, (in Russian) Mat. Sb. N.S. **33** (75), no. 2 (1953), 261–326.
[40] S. M. NIKOLSKII, P. I. LIZORKIN, N. V. MIROSHIN, *Weighted function spaces and their applications*

to the investigation of boundary value problems for degenerate elliptic equations, (in Russian), Izv. Vyssh. Uchebn. Zaved. Mat. **8** (1988), 4–30; translation in Soviet Math. (Iz. VUZ) **32** (8) (1988), 1–40.

[41] V. R. PORTNOV, *Two embedding theorems for the space* $L^1_{p,b}(\Omega \times \mathbf{R}_+)$ *and their applications*, (in Russian), Dokl. Akad. Nauk SSSR **155** (1964), 761–764.

[42] M. M. RAO AND Z. D. REN, *Theory of Orlicz spaces*, M. Dekker, Inc. New York, 1991.

[43] L. N. SLOBODETSKII, *Sobolev spaces of fractional order and their application to boundary problems for partial differential equations*, (in Russian), Uch. Zapisky Leningrad. Ped. Inst. im. A. I. Gercena **197** (1958), 54–112.

[44] A. A. VAŠARIN, *The boundary properties of functions having a finite Dirichlet integral with a weight*, (in Russian), Dokl. Akad. Nauk SSSR (N.S.) **117** (1957) 742–744.

[45] H. TRIEBEL, *Interpolation Theory, Function Spaces, Differential Operators*, North-Holland Math. Library, vol. 18, North-Holland, Amsterdam, 1978.

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