

## ON OPIAL-TYPE INTEGRAL INEQUALITIES AND APPLICATIONS

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*Abstract.* In the present paper we establish some new Opial-type inequalities involving higher order partial derivatives. Our results in special cases yield some of the recent results on Opial's inequality. As application, we prove the uniqueness of the solution of initial value problem involving higher order partial differential equation.

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