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On p-Wave Pairing Superconductivity under Hexagonal and Tetragonal Symmetries

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A group-theoretical classification of p-wave pairing superconducting states is made for systems with hexagonal and tetragonal crystalline symmetries in the absense of the spin-orbit coupling. For each symmetry ten inert p-pairing states are enumerated. In some of them the energy gap vanishes on lines of the Fermi surface.

Recently much attention has been focused on heavy electron systems[1] such as UBe_{13}, UPt_3 and $CeCu_2Si_2$. One of the most controvertial points is the nature of the superconductivity: Whether it is a convensional singlet pairing or triplet pairing. Volovik and Gor'kov,[2] Ueda and Rice,[3] and Blount[4] have discussed non-trivial pairing superconducting states under certain crystal-field symmetries when the spin-orbit coupling is strong. In particular, Volovik and Gor'kov[2] have listed all possible states for both triplet and singlet pairings in the systems with cubic ($\boldsymbol{O}$), hexagonal ($\boldsymbol{D}_6$) and tetragonal ($\boldsymbol{D}_4$) symmetries. They[2)~4)] all agree that there is no triplet pairing states in which the energy gap vanishes on lines of the Fermi surface.

In an earlier paper[5] (referred to as I) we have listed up 15 inert p-wave pairing states for a crystal with cubic symmetry (e.g., UBe_{13}) in the absence of the spin-orbit coupling. In order to complete our discussion we continue to enumerate all possible p-pairing states in hexagonal (e.g., UPt_3) and tetragonal (e.g., $CeCu_2Si_2$) symmetries in the absence of the spin-orbit coupling. The method and notation used here are the same as those in I.

The order parameter $\Delta(k)$ for p-wave pairing is specified by

$$\Delta(\boldsymbol{k})=\sum_{\lambda=1}^{3}\sum_{j=1}^{2}\sum_{i=1}^{2}A_{\lambda j}^{(i)}\tau_\lambda\hat{k}_j l_i \tag{1}$$

in terms of a 3×3 complex matrix $\boldsymbol{A}=\boldsymbol{A}^{(1)}+i\boldsymbol{A}^{(2)}$($A^{(i)}$ is 3×3 real matrix) where $\tau_\lambda=i\sigma_\lambda\sigma_2$, $\hat{k}_j$ is the j-th component of $\hat{\boldsymbol{k}}=\boldsymbol{k}/|\boldsymbol{k}|$, $l_1=1$ and $l_2=i$. Here τ_λ and l_i transform according to Eq. (2·4) in I for a combined group element $g=pu(e,\theta)\tilde{\phi}$ ($p\in\boldsymbol{D}_6$ or $\boldsymbol{D}_4$, $u(e,\theta)\in S$, $\tilde{\phi}\in M$) where S is the spin rotation group, and $M=\Phi+t\Phi$ in which Φ and t denote the group of the gauge and time reversal transformations. Since (k_x, k_y, k_z) transforms as (x, y, z) for an element p in $\boldsymbol{D}_6$ or $\boldsymbol{D}_4$, the linear space $\{k_x, k_y, k_z\}_R$ spanned by (k_x, k_y, k_z) over the real number field is decomposed into the irreducible components:

$$\{k_x, k_y, k_z\}_R=\{k_z\}_R+\{k_x, k_y\}_R\,, \tag{2}$$

where $\{k_z\}_R$ belongs to the A_2 representation of $\boldsymbol{D}_6$ or $\boldsymbol{D}_4$, and $\{k_x, k_y\}_R$ belongs to the $E_1(E)$ representation of $\boldsymbol{D}_6(\boldsymbol{D}_4)$. In the same way as in I we can enumerate the inert phases by finding the maximal little group. The results are summarized in Tables I and II where all the possible stable phases and the little groups for each representation are listed.

We note first that the physical character of the states enumerated can be found in Table IV of I, namely, the state with the same label (b, c, etc.) has the same character. In particular, the gap vanishes at lines (points) on the Fermi surface in the states labeled by c and g (b, e, f and h). There exists no nodeless state such as the BW phase or α phase described by $\Delta(\boldsymbol{k})=(\tau_x\hat{k}_x+\tau_y\hat{k}_y+\tau_z\hat{k}_z)/\sqrt{3}$ which is stabilized in the cubic case[5] because of the decomposition of the space $\{\hat{k}_x, \hat{k}_y, \hat{k}_z\}_R$ as shown in Eq. (2).

By a method similar to the one in Appendix A in I we can derive the following GL free energies valid up to the fourth order in A under each symmetry:

$$f(\boldsymbol{A})=\alpha\sum_{\mu=1}^{3}A_{\mu3}A_{\mu3}^*+\beta_1\sum_{\mu,\nu=1}^{3}A_{\mu z}A_{\mu z}A_{\nu z}^*A_{\nu z}^* +\beta_2\sum_{\mu,\nu=1}^{3}A_{\mu z}A_{\mu z}^*A_{\nu z}A_{\nu z}^* \tag{3}$$

for A_2 of $\boldsymbol{D}_6$ and $\boldsymbol{D}_4$,

$$f(\boldsymbol{A})=\alpha\sum_{\mu=1}^{3}\sum_{i=1}^{2}A_{\mu i}A_{\mu i}^*+\sum_{i=1}^{5}\beta_i R_i \tag{4}$$

Table I. The inert phase and its little group in the case of $\boldsymbol{D}_6$.

representation	state		order parameter	little group $G(\Delta)$
A_2	polar	c_1	$\tau_z \hat{k}_z$	$(1+C_{2x}\bar{\pi})(1+C_{2x}u_{2x})\{\boldsymbol{C}_6 \times A(e_z) \times T\}$
	β phase	g_1	$(1/\sqrt{2})(\tau_x+i\tau_y)\hat{k}_z$	$(1+C_{2x}u_{2z})(1+tu_{2x})\{\boldsymbol{C}_6 \times \tilde{A}(e_z)\}$
E_1	planar phase	b_1	$(1/\sqrt{2})(\tau_x\hat{k}_x+\tau_y\hat{k}_y)$	$(1+C_{2x}u_{2y}\bar{\pi})\{{}_u\boldsymbol{D}_6 \times T\}$
	polar phase	c_4	$\tau_z\hat{k}_x$	$(1+C_{2z}\bar{\pi})(1+C_{2z}u_{2x})\{\boldsymbol{C}_{2x} \times A(e_z) \times T\}$
		c_5	$\tau_z\hat{k}_y$	$(1+C_{2z}\bar{\pi})(1+C_{2z}u_{2x})\{\boldsymbol{C}_{2y} \times A(e_z) \times T\}$
	bipolar phase	e	$(1/\sqrt{2})(\tau_x\hat{k}_x+i\tau_y\hat{k}_y)$	$(1+tu_{2x})(1+C_{2z}\bar{\pi})_u\boldsymbol{D}_2$
	axial phase	f_1	$(1/\sqrt{2})\tau_z(\hat{k}_x+i\hat{k}_y)$	$(1+u_{2x}\bar{\pi})(1+tC_{2x})\{\tilde{\boldsymbol{C}}_6 \times A(e_z)\}$[a)]
	β phase	g_4	$(1/\sqrt{2})(\tau_x+i\tau_y)\hat{k}_x$	$(1+C_{2z}u_{2z})(1+tu_{2x})\{\boldsymbol{C}_{2x} \times \tilde{A}(e_z)\}$
		g_5	$(1/\sqrt{2})(\tau_x+i\tau_y)\hat{k}_y$	$(1+C_{2z}u_{2z})(1+tu_{2x})\{\boldsymbol{C}_{2y} \times \tilde{A}(e_z)\}$
	γ phase	h_1	$(1/2)(\tau_x+i\tau_y)(\hat{k}_x+i\hat{k}_y)$	$(1+tC_{2x}u_{2x})\{\tilde{\boldsymbol{C}}_6 \times \tilde{A}(e_z)\}$

a) $\tilde{\boldsymbol{C}}_6=\{C_6{}^j\widetilde{(2\pi j/6)}$; $j=0, 1, \cdots 5\}$.

Table II. The inert phase and its little group in the case of $\boldsymbol{D}_4$.

representation	state		order parameter	little group $G(\Delta)$
A_2	polar	c_1	$\tau_z k_z$	$(1+C_{2x}\bar{\pi})(1+C_{2x}u_{2x})\{\boldsymbol{C}_4 \times A(e_z) \times T\}$
		g_1	$(1/\sqrt{2})(\tau_x+i\tau_y)\hat{k}_z$	$(1+u_{2z}C_{2x})(1+tu_{2x})\{\boldsymbol{C}_6 \times \tilde{A}(e_z)\}$
E	planar phase	b_1	$(1/\sqrt{2})(\tau_x\hat{k}_x+\tau_y\hat{k}_y)$	$(1+C_{2x}u_{2y}\bar{\pi})\{{}_u\boldsymbol{D}_4 \times T\}$
	polar phase	c_4	$\tau_z\hat{k}_x$	$(1+C_{2z}\bar{\pi})(1+C_{2z}u_{2x})\{\boldsymbol{C}_{2x} \times A(e_z) \times T\}$
		c_6	$(1/\sqrt{2})\tau_z(\hat{k}_x+\hat{k}_y)$	$(1+C_{2z}\bar{\pi})(1+C_{2b}u_{2x})\{\boldsymbol{C}_{2a} \times A(e_z) \times T\}$
	bipolar phase	e	$(1/\sqrt{2})(\tau_x\hat{k}_x+i\tau_y\hat{k}_y)$	$(1+tu_{2x})_u\boldsymbol{D}_2\cdot{}_s\tilde{\boldsymbol{C}}_4$
	axial phase	f_1	$(1/\sqrt{2})\tau_z(\hat{k}_x+i\hat{k}_y)$	$(1+u_{2x}\bar{\pi})(1+tC_{2x})\{\tilde{\boldsymbol{C}}_4 \times A(e_z)\}$[a)]
	β phase	g_4	$(1/\sqrt{2})(\tau_x+i\tau_y)\hat{k}_x$	$(1+C_{2z}u_{2z})(1+tu_{2x})\{\boldsymbol{C}_{2x} \times \tilde{A}(e_z)\}$
		g_2	$(1/2)(\tau_x+i\tau_y)(\hat{k}_x+\hat{k}_y)$	$(1+C_{2z}u_{2z})(1+tu_{2x})\{\boldsymbol{C}_{2a} \times \tilde{A}(e_z)\}$
	γ phase	h_1	$(1/2)(\tau_x+i\tau_y)(\hat{k}_x+i\hat{k}_y)$	$(1+tC_{2x}u_{2x})\{\tilde{\boldsymbol{C}}_4 \times \tilde{A}(e_z)\}$

a) $\tilde{\boldsymbol{C}}_4=\{C_4{}^j\widetilde{(2\pi j/4)}$, $j=0, 1, \cdots 3\}$.

for E_1 and $\boldsymbol{D}_6$, and

$$f(A)=\alpha\sum_{\mu=1}^{3}\sum_{i=1}^{2}A_{\mu i}A_{\mu i}^{*}+\sum_{i=1}^{7}\beta_i R_i \tag{5}$$

for E of $\boldsymbol{D}_4$. The fourth order invariants R_i ($i=1\sim7$) are the same as before (see Appendix A in I) except that the i,j-sum there is restricted to 1 and 2. Note that within the fourth order $f(\boldsymbol{A})$ in Eq. (4) is invariant to the $\boldsymbol{D}_\infty$ symmetry. Because of this invariance the c_4 and c_5 phases, for example, in $\boldsymbol{D}_6$ give rise to the same free energy. However, due to higher order terms than the fourth order the free energies of the c_4 and c_5 states are different as indicated in Table I. We should remark that the phases listed above are indeed extremum for general GL free energies.

In summary we have enumerated all the possible inert p-wave pairing states under hexagonal and tetragonal crystalline symmetries in the absence of the spin-orbit coupling. In some of our states the energy gap vanishes on lines of the Fermi surface which is contrasted with the strong spin-orbit coupling case[2)~4)] where zeros of the energy gap are isolated points or none.

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