Filomat 31:7 (2017), 1941–1947
DOI 10.2298/FIL1707941D

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

On Para-Sasakian Manifolds Satisfying Certain Curvature Conditions

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Abstract. In this paper, we investigate Ricci pseudo-symmetric and Ricci generalized pseudo-symmetric P-Sasakian manifolds. Next we study P-Sasakian manifolds satisfying the curvature condition $S \cdot R = 0$. Finally, we give an example of a 5-dimensional P-Sasakian manifold to verify some results.

1. Introduction

Let M be an n-dimensional differentiable manifold of class C^∞ in which there are given a $(1,1)$-type tensor field ϕ, a vector field ξ and a 1-form η such that

$$\phi^2 X = X - \eta(X)\xi, \ \phi\xi = 0, \ \eta(\xi) = 1, \ \eta(\phi X) = 0. \tag{1}$$

Then (ϕ, ξ, η) is called an almost paracontact structure and M an almost paracontact manifold. Moreover, if M admits a Riemannian metric g such that

$$g(\xi, X) = \eta(X), \ g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2}$$

then (ϕ, ξ, η, g) is called an almost paracontact metric structure and M an almost paracontact metric manifold [11]. If (ϕ, ξ, η, g) satisfy the following equations:

$$d\eta = 0, \ \nabla_X \xi = \phi X, \ (\nabla_X \phi)Y = -g(X, Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{3}$$

then M is called a para-Sasakian manifold or briefly a P-Sasakian manifold [1]. Further, if a P-Sasakian manifold M admits a 1-form η such that

$$(\nabla_X \eta)(Y) = -g(X, Y) + \eta(X)\eta(Y), \tag{4}$$

then the manifold is called a special para-Sasakian manifold or briefly a SP-Sasakian manifold [12]. We define endomorphisms $R(X, Y)$ and $X \wedge_A Y$ by

$$R(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z \tag{5}$$

2010 *Mathematics Subject Classification*. Primary 53C15; Secondary 53C40
Keywords. Para-Sasakian manifold, Ricci pseudo-symmetric manifold, Ricci generalized pseudo-symmetric manifold, Einstein manifold.
Received: 27 June 2015; Accepted: 14 August 2015
Communicated by Ljubica Velimirović
The second author is supported by NNSF of China (11371194).
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and

$$(X \wedge_A Y)Z = A(Y,Z)X - A(X,Z)Y, \tag{6}$$

respectively, where $X, Y, Z \in \chi(M)$, $\chi(M)$ is the set of all differentiable vector fields on M, A is the symmetric $(0,2)$-tensor, R is the Riemannian curvature tensor of type $(1,3)$ and ∇ is the Levi-Civita connection.

For a $(0,k)$-tensor field T, $k \geq 1$, on (M^n, g) we define the tensors $R \cdot T$ and $Q(g,T)$ by

$$\begin{aligned}(R(X,Y)\cdot T)(X_1, X_2, ..., X_k) &= -T(R(X,Y)X_1, X_2, ..., X_k) - T(X_1, R(X,Y)X_2, ..., X_k) \\ &\quad - ... - T(X_1, X_2, ..., R(X,Y)X_k)\end{aligned} \tag{7}$$

and

$$\begin{aligned}Q(g,T)(X_1, X_2, ..., X_k; X, Y) &= -T((X \wedge Y)X_1, X_2, ..., X_k) - T(X_1, (X \wedge Y)X_2, ..., X_k) \\ &\quad - ... - T(X_1, X_2, ..., (X \wedge Y)X_k),\end{aligned} \tag{8}$$

respectively [14].

If the tensors $R \cdot S$ and $Q(g,S)$ are linearly dependent then M^n is called Ricci pseudo-symmetric [14]. This is equivalent to

$$R \cdot S = fQ(g,S), \tag{9}$$

holding on the set $U_S = \{x \in M : S \neq 0 \text{ at } x\}$, where f is some function on U_S. Analogously, if the tensors $R \cdot R$ and $Q(S,R)$ are linearly dependent then M^n is called Ricci generalized pseudo-symmetric [14]. This is equivalent to

$$R \cdot R = fQ(S,R), \tag{10}$$

holding on the set $U_R = \{x \in M : R \neq 0 \text{ at } x\}$, where f is some function on U_R. A very important subclass of this class of manifolds realizing the condition is

$$R \cdot R = Q(S,R).$$

Every three dimensional manifold satisfies the above equation identically. Other examples are the semi-Riemannian manifolds (M,g) admitting a non-zero 1-form ω such that the equality $\omega(X)R(Y,Z)+\omega(Y)R(Z,X)+\omega(Z)R(X,Y) = 0$, holds on M. The condition $R \cdot R = Q(S,R)$ also appears in the theory of plane gravitational waves.

Furthermore we define the tensors $R \cdot R$ and $R \cdot S$ on (M^n, g) by

$$\begin{aligned}(R(X,Y)\cdot R)(U,V)W &= R(X,Y)R(U,V)W - R(R(X,Y)U,V)W \\ &\quad - R(U,R(X,Y)V)W - R(U,V)R(X,Y)W\end{aligned} \tag{11}$$

and

$$(R(X,Y)\cdot S)(U,V) = -S(R(X,Y)U,V) - S(U,R(X,Y)V), \tag{12}$$

respectively.

Recently, Kowalczyk [7] studied semi-Riemannian manifolds satisfying $Q(S,R) = 0$ and $Q(S,g) = 0$, where S, R are the Ricci tensor and curvature tensor respectively.

An almost paracontact Riemannian manifold M is said to be an η-Einstein manifold if the Ricci tensor S satisfies the condition

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y),$$

where a and b are smooth functions on the manifold. In particular, if $b = 0$, then M is an Einstein manifold.

De and Tarafdar [5] studied P-Sasakian manifolds satisfying the condition $R(X,Y)\cdot R = 0$. In [4], De and Pathak studied P-Sasakian manifolds satisfying the conditions $R(X,Y)\cdot P = 0$ and $R(X,Y)\cdot S = 0$. Özgür [9] studied Weyl-pseudosymmetric P-Sasakian manifolds and also P-Sasakian manifolds satisfying the condition $C\cdot S = 0$. Also P-Sasakian manifolds have been studied by several authors such as Adati and Miyazawa [2], Deshmukh and Ahmed [6], De et al [3], Sharfuddin, Deshmukh, Husain [13], Matsumoto, Ianus and Mihai [8], Özgür and Tripathi [10] and many others.

Motivated by the above studies, we characterize P-Sasakian manifolds satisfying certain curvature conditions on the Ricci tensor. The paper is organized as follows: After preliminaries in section 3, we study Ricci pseudo-symmetric P-Sasakian manifolds and it is proved that a P-Sasakian manifold is Ricci pseudo-symmetric if and only if it is an Einstein manifold provided $f \neq -1$. Section 4 is devoted to study Ricci generalized pseudo-symmetric P-Sasakian manifolds and it is proved that a Ricci generalized pseudo-symmetric P-Sasakian manifold is Einstein provided $nf \neq 1$. Some corollaries have been obtained. In section 5, we characterize a P-Sasakian manifold satisfying the curvature condition $S\cdot R = 0$. Finally, we construct an example of a 5-dimensional P-Sasakian manifold to verify some results.

2. Preliminaries

In a P-Sasakian manifold the following relations hold ([1], [9]):

$$S(X,\xi) = -(n-1)\eta(X),\ Q\xi = -(n-1)\xi, \tag{13}$$

$$\eta(R(X,Y)Z) = g(X,Z)\eta(Y) - g(Y,Z)\eta(X), \tag{14}$$

$$R(X,\xi)Y = g(X,Y)\xi - \eta(Y)X, \tag{15}$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{16}$$

$$\eta(R(X,Y)\xi) = 0, \tag{17}$$

for any vector fields $X, Y, Z \in \chi(M)$.

3. Ricci Pseudo-Symmetric *P*-Sasakian Manifolds

In this section we study Ricci pseudo-symmetric manifold, that is, the manifold satisfying the condition $R\cdot S = fQ(g,S)$. Assume that M is a Ricci pseudo-symmetric P-Sasakian manifold and $X, Y, U, V, \in \chi(M)$. We have from (9)

$$(R(X,Y)\cdot S)(U,V) = fQ(g,S)(X,Y;U,V). \tag{18}$$

It is equivalent to

$$(R(X,Y)\cdot S)(U,V) = f((X\wedge_g Y)\cdot S)(U,V). \tag{19}$$

From (12) and (8) we have

$$-S(R(X,Y)U,V) - S(U,R(X,Y)V) = f[-S((X\wedge_g Y)U,V) - S(U,(X\wedge_g Y)V)]. \tag{20}$$

Using (6) we obtain from (20)

$$\begin{aligned} -S(R(X,Y)U,V) - S(U,R(X,Y)V) \;&=\; f[-g(Y,U)S(X,V) + g(X,U)S(Y,V) \\ &\quad -g(Y,V)S(U,X) + g(X,V)S(U,Y)]. \end{aligned} \tag{21}$$

Substituting $X = U = \xi$ in (21) and using (13), (15) yields

$$(1+f)\{S(Y,V) + (n-1)g(Y,V)\} = 0. \tag{22}$$

Then either $f = -1$ or, the manifold is an Einstein manifold of the form

$$S(Y,V) = -(n-1)g(Y,V). \tag{23}$$

By the above discussions we have the following:

Proposition 3.1. *Every n-dimensional Ricci pseudo-symmetric P-Sasakian manifold is of the form* $R \cdot S = -Q(g, S)$, *provided the manifold is non-Einstein.*

Conversely, if the manifold is an Einstein manifold of the form (23), then it is clear that $R \cdot S = fQ(g, S)$. This leads the following:

Theorem 3.2. *An n-dimensional P-Sasakian manifold is Ricci pseudo-symmetric if and only if the manifold is an Einstein manifold provided* $f \neq -1$.

In particular, if we consider $Q(g, S) = 0$, then by the similar argument of Theorem 3.2 we can state the following:

Corollary 3.3. *An n-dimensional P-Sasakian manifold satisfies the condition* $Q(g, S) = 0$ *if and only if the manifold is an Einstein one.*

4. Ricci Generalized Pseudo-Symmetric *P*-Sasakian Manifolds

In this section we deal with Ricci generalized pseudo-symmetric P-Sasakian manifolds. Let us suppose that M be an n-dimensional Ricci generalized pseudo-symmetric P-Sasakian manifold. Then from (10) we have

$$R \cdot R = fQ(S, R), \tag{24}$$

that is,

$$(R(X, Y) \cdot R)(U, V)W = f((X \wedge_S Y) \cdot R)(U, V)W. \tag{25}$$

Using (11) and (8) we get from (25)

$$\begin{aligned} &R(X, Y)R(U, V)W - R(R(X, Y)U, V)W \\ &-R(U, R(X, Y)V)W - R(U, V)R(X, Y)W \\ &= f[(X \wedge_S Y)R(U, V)W - R((X \wedge_S Y)U, V)W \\ &-R(U, (X \wedge_S Y)V)W - R(U, V)(X \wedge_S Y)W]. \end{aligned} \tag{26}$$

In view of (6) and (26) we obtain

$$\begin{aligned} &R(X, Y)R(U, V)W - R(R(X, Y)U, V)W - R(U, R(X, Y)V)W \\ &-R(U, V)R(X, Y)W = f[S(Y, R(U, V)W)X - S(X, R(U, V)W)Y \\ &-S(Y, U)R(X, V)W + S(X, U)R(Y, V)W - S(Y, V)R(U, X)W \\ &+S(X, V)R(U, Y)W - S(Y, W)R(U, V)X + S(X, W)R(U, V)Y]. \end{aligned} \tag{27}$$

Substituting $X = U = \xi$ in (27) and using (13), (15) and (16) yields

$$\begin{aligned} &-g(V, W)Y + g(V, W)\eta(Y)\xi - R(Y, V)W \\ &+\eta(Y)\eta(W)V - g(V, W)\eta(Y)\xi - \eta(W)\eta(Y)V + g(Y, W)V \\ &= f[\eta(W)S(Y, V)\xi - (n-1)g(V, W)Y - (n-1)R(Y, V)W \\ &+(n-1)g(Y, W)\eta(V)\xi - S(Y, W)V + S(Y, W)\eta(V)\xi + (n-1)g(V, Y)\eta(W)\xi]. \end{aligned} \tag{28}$$

Taking the inner product of (28) with Z we obtain

$$\begin{aligned} &-g(V, W)g(Y, Z) + g(V, W)\eta(Y)\eta(Z) - g(R(Y, V)W, Z) \\ &-g(V, W)\eta(Y)\eta(Z) + g(Y, W)g(V, Z) \\ &= f[S(Y, V)\eta(W)\eta(Z) - (n-1)g(V, W)g(Y, Z) - (n-1)g(R(Y, V)W, Z) \\ &+(n-1)g(Y, W)\eta(V)\eta(Z) - S(Y, W)g(V, Z) \\ &+S(Y, W)\eta(V)\eta(Z) + (n-1)g(V, Y)\eta(W)\eta(Z)]. \end{aligned} \tag{29}$$

Let $\{e_i\}(1 \leq i \leq n)$ be an orthonormal basis of the tangent space at any point. Now taking summation over $i = 1, 2, ..., n$ of the relation (29) for $V = W = e_i$ gives

$$S(Y,Z) + (n-1)g(Y,Z) = nf[S(Y,Z) + (n-1)g(Y,Z)].$$

This implies

$$(1 - nf)[S(Y,Z) + (n-1)g(Y,Z)] = 0.$$

Then either $f = 1/n$ or, the manifold is an Einstein manifold of the form

$$S(Y,Z) = -(n-1)g(Y,Z).$$

This leads to the following:

Theorem 4.1. *An n-dimensional Ricci generalized pseudo-symmetric P-Sasakian manifold is an Einstein manifold provided* $nf \neq 1$.

By the above discussions we have the following:

Proposition 4.2. *Every n-dimensional Ricci generalized pseudo-symmetric P-Sasakian manifold is of the form* $R \cdot R = \frac{1}{n}Q(S,R)$, *provided the manifold is non-Einstein.*

In particular, if we consider $Q(S,R) = 0$, then by the similar argument of Theorem 4.1 we can state the following:

Corollary 4.3. *If an n-dimensional P-Sasakian manifold satisfies the condition* $Q(S,R) = 0$ *then the manifold is an Einstein one.*

Corollary 4.4. *If a P-Sasakian manifold satisfies the condition* $R \cdot R = Q(S,R)$, *then the manifold is an Einstein manifold.*

5. *P*-Sasakian Manifolds Satisfying the Curvature Condition $S \cdot R = 0$

In this section we consider a P-Sasakian manifold satisfying the curvature condition $S \cdot R = 0$. Thus we have

$$(S(X,Y) \cdot R)(U,V)W = 0, \tag{30}$$

which implies

$$(X \wedge_S Y)R(U,V)W + R((X \wedge_S Y)U,V)W + R(U,(X \wedge_S Y)V)W + R(U,V)(X \wedge_S Y)W = 0. \tag{31}$$

Using (6) we have from (31)

$$\begin{aligned} &S(Y,R(U,V)W)X - S(X,R(U,V)W)Y + S(Y,U)R(X,V)W - S(X,U)R(Y,V)W \\ &+S(Y,V)R(U,X)W - S(X,V)R(U,Y)W + S(Y,W)R(U,V)X - S(X,W)R(U,V)Y = 0. \end{aligned} \tag{32}$$

Substituting $U = W = \xi$ in (32) and using (15), (16) yields

$$\begin{aligned} &2S(Y,V)X - 2S(X,V)Y + 2(n-1)\eta(V)\eta(Y)X - 2(n-1)\eta(X)\eta(V)Y - S(Y,V)\eta(X)\xi \\ &+S(X,V)\eta(Y)\xi + (n-1)g(V,X)\eta(Y)\xi - (n-1)g(V,Y)\eta(X)\xi = 0. \end{aligned} \tag{33}$$

Taking the inner product of (33) with ξ and replacing X by ξ and also using (13), we have

$$S(Y,V) = (n-1)g(Y,V) - 2(n-1)\eta(Y)\eta(V).$$

This leads to the following:

Theorem 5.1. *If an n-dimensional P-Sasakian manifold satisfying the curvature condition* $S \cdot R = 0$, *then the manifold is an* η*-Einstein manifold.*

6. Example of a 5-Dimensional *P*-Sasakian Manifold

We consider the 5-dimensional manifold $M = \{(x, y, z, u, v) \in \mathbb{R}^5\}$, where (x, y, z, u, v) are the standard coordinates in $\mathbb{R}^5$.
We choose the vector fields

$$e_1 = \frac{\partial}{\partial x},\ e_2 = \frac{\partial}{\partial y},\ e_3 = \frac{\partial}{\partial z},\ e_4 = \frac{\partial}{\partial u},\ e_5 = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z} + u\frac{\partial}{\partial u} + \frac{\partial}{\partial v},$$

which are linearly independent at each point of M.
Let g be the Riemannian metric defined by

$$g(e_i, e_j) = \begin{cases} 1 & \text{if} \quad i = j \\ 0 & \text{if} \quad i \neq j;\ i, j = 1, 2, 3, 4, 5. \end{cases}$$

Let η be the 1-form defined by

$$\eta(Z) = g(Z, e_5),$$

for any $Z \in \chi(M)$.
Let ϕ be the $(1, 1)$-tensor field defined by

$$\phi(e_1) = e_1,\ \phi(e_2) = e_2,\ \phi(e_3) = e_3,\ \phi(e_4) = e_4,\ \phi(e_5) = 0.$$

Using the linearity of ϕ and g, we have

$$\eta(e_5) = 1,\ \phi^2 Z = Z - \eta(Z)e_5 \text{ and } g(\phi Z, \phi U) = g(Z, U) - \eta(Z)\eta(U),$$

for any vector fields $Z, U \in \chi(M)$. Thus for $e_5 = \xi$, the structure (ϕ, ξ, η, g) defines an almost paracontact metric structure on M.
Then we have

$$\begin{aligned} &[e_1, e_2] = 0, [e_1, e_3] = 0, [e_1, e_4] = 0, [e_1, e_5] = e_1, \\ &[e_2, e_3] = [e_2, e_4] = 0, [e_2, e_5] = e_2, \\ &[e_3, e_4] = 0, [e_3, e_5] = e_3, [e_4, e_5] = e_4. \end{aligned}$$

The Levi-Civita connection ∇ of the metric tensor g is given by Koszul's formula which is given by

$$2g(\nabla_X Y, Z) = Xg(Y, Z) + Yg(Z, X) - Zg(X, Y) - g(X, [Y, Z]) - g(Y, [X, Z]) + g(Z, [X, Y]). \tag{34}$$

Taking $e_5 = \xi$ and using (34), we get the following:

$$\begin{aligned} &\nabla_{e_1} e_1 = -e_5,\ \nabla_{e_1} e_2 = 0,\ \nabla_{e_1} e_3 = 0,\ \nabla_{e_1} e_4 = 0,\ \nabla_{e_1} e_5 = e_1, \\ &\nabla_{e_2} e_1 = 0,\ \nabla_{e_2} e_2 = -e_5,\ \nabla_{e_2} e_3 = 0,\ \nabla_{e_2} e_4 = 0,\ \nabla_{e_2} e_5 = e_2, \\ &\nabla_{e_3} e_1 = 0,\ \nabla_{e_3} e_2 = 0,\ \nabla_{e_3} e_3 = -e_5,\ \nabla_{e_3} e_4 = 0,\ \nabla_{e_3} e_5 = e_3, \\ &\nabla_{e_4} e_1 = 0,\ \nabla_{e_4} e_2 = 0,\ \nabla_{e_4} e_3 = 0,\ \nabla_{e_4} e_4 = -e_5,\ \nabla_{e_4} e_5 = e_4, \\ &\nabla_{e_5} e_1 = 0,\ \nabla_{e_5} e_2 = 0,\ \nabla_{e_5} e_3 = 0,\ \nabla_{e_5} e_4 = 0,\ \nabla_{e_5} e_5 = 0. \end{aligned}$$

By the above results, we can easily obtain the non-vanishing components of the curvature tensor as follows:

$$\begin{aligned} &R(e_1, e_2)e_1 = e_2,\ R(e_1, e_2)e_2 = -e_1,\ R(e_1, e_3)e_1 = e_3,\ R(e_1, e_3)e_3 = -e_1, \\ &R(e_1, e_4)e_1 = e_4,\ R(e_1, e_4)e_4 = -e_1,\ R(e_1, e_5)e_1 = e_5,\ R(e_1, e_5)e_5 = -e_1, \\ &R(e_2, e_3)e_2 = e_3,\ R(e_2, e_3)e_3 = -e_2,\ R(e_2, e_4)e_2 = e_4,\ R(e_2, e_4)e_4 = -e_2, \\ &R(e_2, e_5)e_2 = e_5,\ R(e_2, e_5)e_5 = -e_2,\ R(e_3, e_4)e_3 = e_4,\ R(e_3, e_4)e_4 = -e_3, \\ &R(e_3, e_5)e_3 = e_5,\ R(e_3, e_5)e_5 = -e_3,\ R(e_4, e_5)e_4 = e_5,\ R(e_4, e_5)e_5 = -e_4. \end{aligned}$$

Clearly, $R(X, Y)Z = k\{g(Y, Z)X - g(X, Z)Y\}$, for any vector fields $X, Y, Z \in \chi(M)$, where $k = -1$. Thus the manifold is of constant curvature. Also

$$S(e_1, e_1) = S(e_2, e_2) = S(e_3, e_3) = S(e_4, e_4) = S(e_5, e_5) = -4.$$

It can be easily verified that the manifold is an Einstein manifold. Thus Theorem 3.2 and Corollary 3.3 are verified.

References

[1] T. Adati, K. Matsumoto, On conformally recurrent and conformally symmetric *P*-Sasakian manifolds, TRU Math. 13(1977), 25–32.
[2] T. Adati, T. Miyazawa, On *P*-Sasakian manifolds satisfying certain conditins, Tensor (N.S.) 33(1979), 173–178.
[3] U.C. De, C. Özgür, K. Arslan, C. Murathan, A. Yildiz, On a type of *P*-Sasakian manifolds, Mathematica Balkanica, 22(2008), 25–36.
[4] U.C. De, G. Pathak, On *P*-Sasakian manifolds satisfying certain conditions, J. Indian Acad. Math., 16(1994), 72–77.
[5] U.C. De, D. Tarafdar, On a type of *P*-Sasakian manifold, Math. Balkanica (N. S.) 7(1993), 211–215.
[6] S. Deshmukh, S. Ahmed, Para-Sasakian manifolds isometrically immersed in spaces of constant curvature, Kyungpook J. Math. 20(1980), 112–121.
[7] D. Kowalczyk, On some subclass of semisymmetric manifolds, Soochow J. Math. 27(2001), 445–461.
[8] K. Matsumoto, S. Ianus, I. Mihai, On *P*-Sasakian manifolds which admit certain tensor fields, Publ. Math. Debrecen 33(1986), 61–65.
[9] C. Özgür, On a class of Para-Sasakian manifolds, Turkish J. Math. 29(2005), 249–257.
[10] C. Özgür, M.M. Tripathi, On *P*-Sasakian manifolds satisfying certain conditions on the concircular curvature tensor, Turkish J. Math. 31(2007), 171–179.
[11] I. Satō, On a structure similar to the almost contact structure, Tensor (N.S.) 30(1976), 219–224.
[12] I. Satō, K. Matsumoto, On *P*-Sasakian manifolds satisfying certain conditions, Tensor, N. S., 33(1979), 173–178.
[13] A. Sharfuddin, S. Deshmukh, S.I. Husain, On Para-Sasakian manifolds, Indian J. pure appl. Math., 11(1980), 845–853.
[14] L. Verstraelen, Comments on pseudo-symmetry in sense of R. Deszcz, in: Geometry and Topology of submanifolds, World Sci. Publishing, 6(1994), 199-209.