

ON POSSIBLE OBSERVABLE EFFECTS OF ELECTRON PAIR PRODUCTION IN QSOs

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SUMMARY

The energy distribution of the electrons produced in the annihilation of γ -rays by softer ambient photons is calculated, assuming different possible spectra for the soft photons. The relevance of this process to QSOs is discussed, and the effects of the electrons produced in this way are quantitatively studied in the frame of simplified models. Some results about the spectral index, and possible bends of the photon spectrum, are obtained.

I. INTRODUCTION

There can be little doubt that the bulk of the non-thermal continuum radiation from quasars and active galactic nuclei emerges from a compact region < 1 pc in size. Although present observations refer mostly to the optical, infra-red and radio parts of the spectrum, X-ray emission has already been reported from the quasar 3C 273 and from the radio galaxies M 87 and Centaurus A (Friedmann & Byram 1967) (Bowyer *et al.* 1970). There is every likelihood that a large flux of γ -rays may be generated within these sources. Indeed, if the electron synchrotron process is responsible for the low-frequency radiation, the requirement that the same electrons should *not* transform an embarrassingly large fraction of this radiation into γ -rays in the inverse Compton mechanism sets a severe constraint on acceptable models for these objects (Hoyle, Burbidge & Sargent 1966). If the sources are exceedingly compact, however, the internal radiation density may be so high that a γ -ray interacts with a second photon, giving rise to an electron-positron pair, before escaping (Jelley 1966). The pairs will in general have relativistic kinetic energies, and will themselves participate in the synchrotron and inverse Compton emission. Because these pairs will generally have a different energy distribution from the original relativistic electrons, they will modify the spectrum of the source in other wavebands. In this paper we explore some of the consequences of this process quantitatively.

As an illustration, consider a spherical source with a spectrum emitting a power P in each decade of frequencies. The optical depth for γ -rays of energy ϵ would be > 1 if the radius of the source satisfies

$$r \lesssim 10^{16} \left(\frac{\epsilon}{100 \text{ MeV}} \right) \left(\frac{P}{10^{46} \text{ erg s}^{-1}} \right) \text{ cm.} \quad (1.1)$$

The relevant values of P range from 10^{43} erg s^{-1} for nearby objects such as M 87, up to perhaps more than 10^{46} erg s^{-1} for QSO. There are several indications that the continuum sources may indeed be so small that (1.1) is satisfied for γ -rays above, say, 10–100 MeV. Cromwell & Weymann (1970) have argued that the *thermal*

optical emission from the Seyfert galaxy NGC 4151 comes from a region of $\sim 3 \times 10^{15}$ cm in radius: the continuum source which energizes the thermal gas would presumably be substantially smaller. If the energy were gravitational in origin, and resulted from accretion by a central mass, the radiative power would be concentrated within only a few Schwarzschild radii. (For a mass of $\sim 10^8 M_\odot$, the Schwarzschild radius is only 3×10^{13} cm.) In addition to these admittedly indirect theoretical arguments, the rapid optical variability, with time scales of only a few hours in some cases, indicates dimensions of $\sim 10^{15}$ cm.

In such a small source pair creation may play a key role. In addition to suppressing γ -ray emission from the source, this process modifies the spectrum at all lower frequencies, because the energy which would otherwise be emitted in the γ -spectrum, but which (because of pair production) has to be channelled into other wavebands, may constitute a major part of the electro-magnetic emission of the source.

In this paper we present a quantitative discussion of some highly simplified models, our aim being to show how they are modified by the inclusion of pair production. Our first step (Section 2) is to calculate the energy distribution of the electron-positron pairs which result when γ -rays interact with power-law spectra of softer photons. For comparison a δ -law is also studied. The results of the calculation are summarized in Section 3. It turns out that for power-law absorbers it is an adequate approximation to suppose that the energy of the γ -ray is shared between the electron and positron in every possible way with equal probability. In Section 4 and Section 5, making this approximation, we derive the equations relating the electron and radiation spectra, and calculate self-consistent solutions for homogeneous model sources emitting Compton-synchrotron radiation. The relevance of these models to present and future observations is briefly considered in a final section.

In the following $\hbar = c = 1$ units will be used. We shall also put the electron mass equal to 1.

2. ENERGY DISTRIBUTION OF CREATED PAIRS: THEORY

The importance, for non-stellar astrophysics, of electron pair production in γ - γ annihilation due to the reaction

$$\gamma + \gamma \rightarrow e^+ + e^-$$

(hereafter we shall refer to it as PP) was first outlined by Nikishov (1961) and later by Gould & Schröder (1967a). The occurrence of PP inside QSOs was proposed by Jelley (1966), as a possible explanation of the observational the low upper limits on the γ -emission from these objects.

We shall first briefly review some results from the paper by Gould & Schröder (1967b) and then present general formulae for the energy distribution of created electrons for different soft-photons spectra. Readers who are not interested in the quantum-electrodynamical details should omit the rest of this section.

PP has a threshold. To show this, let us define the invariant

$$s = -\frac{1}{4}(p_\mu + p'_\mu)^2 = \frac{1}{4}(k_\mu + k'_\mu)^2 \quad (2.1)$$

where p_μ and p'_μ are the 4-momenta of the outgoing electrons, and $k_\mu(k'_\mu)$ is the 4-momentum of the hard (soft) photon. By looking at the explicit expression of

$(p_\mu + p'_\mu)^2$ one easily sees that the requirement $s \geq 1$ must be fulfilled; this implies the threshold $\omega\omega' \geq 1$ (where $\omega^{(\prime)} = -ik_4^{(\prime)}$ are the photons energies).

The invariant s is also connected with the C.M. frame velocity β and energy ϵ of the electrons by the relations

$$s = \epsilon^2 = \frac{1}{1 - \beta^2}; \quad \beta = (1 - s^{-1})^{1/2}. \quad (2.2)$$

Let us now consider an isotropic spatial distribution of soft photons calling $n(\omega') d\omega'$ the number of photons of energy between ω' and $\omega' + d\omega'$ in the unit volume. The rate of absorption (i.e. the inverse of the mean free path), for a photon of energy ω travelling in this soft photon gas, will be

$$\alpha(\omega) = \iint d\omega' d(\cos \vartheta) n(\omega') w(\omega, \omega', \cos \vartheta) \quad (2.3)$$

where ϑ is the angle between the directions of \mathbf{k} and \mathbf{k}' , and

$$w = (1 - \cos \vartheta) \sigma \quad (2.4)$$

while σ is the total cross-section for PP.

By using (2.1) one can easily derive the relation between $\cos \vartheta$ and the invariant s . Using the relations

$$1 - \cos \vartheta = \frac{2s}{\omega\omega'}, \quad d(\cos \vartheta) = -\frac{2}{\omega\omega'} ds$$

one can change the integration variables in (2.3); by also taking into account (2.4), and the limitation on the integration intervals arising from the existence of a threshold, one can write

$$\alpha(\omega) = \frac{4}{\omega^2} \int_1^\infty ds s \sigma(s) \int_{s\omega^{-1}}^\infty d\omega' \omega'^{-2} n(\omega'). \quad (2.5)$$

This formula for the absorption rate of a photon of energy ω is fully equivalent to the one in Gould & Schreder's (1967a) paper.

The rate of absorption $\alpha(\omega)$ calculated above enables us to determine the probability that a photon of energy ω will 'decay' into two electrons of energies E , E' (where $\omega \simeq E + E'$).

We now seek the probability that a decay will lead to a particular energy E for one of the electrons (the other will have an energy $\omega - E$). Let $\sigma(s, E)$ be a differential cross-section giving the distribution on energy of the electrons produced in PP, and let us insert it in the r.h.s. of (2.5), instead of $\sigma(s)$. The distribution

$$g(\omega, E) = 4 \frac{1}{\omega^2} \int_1^\infty ds s \sigma(s, E) \int_{s\omega^{-1}}^\infty d\omega' \omega'^{-2} n(\omega') \quad (2.6)$$

so obtained will express the rate of absorption of the photons of energy ω , leading to the creation of a pair of electrons of energies E , $\omega - E$.

We shall use, like Gould and Schröder, the quantum-electrodynamical formulas given in the book by Jauch & Rohrlich (1955) for $\sigma(\beta, \cos \theta)$ and for the dependence on β of the total cross-section. By using (2.2) one can easily work out the total cross-section as a function of s . The same book also gives the differential cross-section as a distribution on $\cos \theta$, where θ is the angle between the direction of

ingoing and outgoing particles in the C.M. frame. This may be put into the form

$$\sigma(\beta, \beta \cos \theta) = \frac{\pi r_0^2}{2} (1 - \beta^2) \frac{1 - (\beta \cos \theta)^4 + 2(1 - \beta^2)[\beta^2 - (\beta \cos \theta)^2]}{[1 - (\beta \cos \theta)^2]^2} \quad (2.7)$$

where r_0 is the classical radius of the electron.

What we in fact require is a relation directly connecting $\beta \cos \theta$ and E . Now a very good *approximate* one-to-one correspondence holds when $\omega' \ll \omega$: in this case we may take the relative velocity of the C.M. frame, and of the frame where $n(\omega')$ is defined, to be in the direction of \mathbf{k} .

This allows us to write

$$\begin{aligned} \omega_{CM'} &= \gamma(1 - v \cos \vartheta) \omega' \\ \omega_{CM} &= \gamma(1 - v) \omega \end{aligned} \quad (2.8)$$

where $\omega_{CM}^{(')}$ is the energy in the C.M. frame of the photon of energy $\omega^{(')}$; $\gamma = (1 - v^2)^{-1/2}$, and $v = |\mathbf{v}|$. As $\omega_{CM'} = \omega_{CM}$ we get from (2.8)

$$v = \frac{\omega - \omega'}{\omega - \omega' \cos \vartheta} = \frac{\omega - \omega'}{\omega - \omega' + \frac{2s}{\omega}} \simeq 1 - \frac{2s}{\omega^2} \quad (2.9)$$

and also

$$\gamma = (1 - v^2)^{-1/2} = \frac{\omega}{2\sqrt{s}}; \quad s = \frac{\omega^2}{4\gamma^2}. \quad (2.10)$$

Concerning the electron energies, we can write

$$E = \gamma(\epsilon + v\sqrt{\epsilon^2 - 1} \cos \theta)$$

and then, by using (2.10) and (2.2),

$$E = \frac{1}{2} \{ \omega + [(1 - x^{-1})(\omega^2 - 4x)]^2 \cos \theta \} \quad (2.11)$$

where $x = \gamma^2$. As x is a function of s and ω alone, (2.11) states a one-to-one correspondence between E and $\cos \theta$. We have introduced the new x variable because, for reasons outlined later, we shall use it instead of s as an integration variable. As we may write

$$\beta = \left(1 - \frac{4x}{\omega^2} \right)^{1/2} \quad (2.12a)$$

and

$$\beta \cos \theta = \frac{2E - \omega}{\omega(1 - x^{-1})^{1/2}} \quad (2.12b)$$

we can express the differential cross-section as

$$\begin{aligned} &\sigma(\beta, \beta \cos \theta) \eta(\beta - \beta \cos \theta) \eta(\beta \cos \theta + \beta) d(\beta \cos \theta) \\ &= \sigma \left(\left[1 - \frac{4x}{\omega^2} \right]^{1/2}, \frac{2E - \omega}{\omega(1 - x^{-1})^{1/2}} \right) \frac{2}{\omega(1 - x^{-1})^{1/2}} \eta \left[\left(1 - \frac{4x}{\omega^2} \right)^{1/2} - \frac{2E - \omega}{\omega(1 - x^{-1})^{1/2}} \right] \\ &\quad \times \eta \left[\frac{2E - \omega}{\omega(1 - x^{-1})^{1/2}} + \left(1 - \frac{4x}{\omega^2} \right)^{1/2} \right] dE \quad (2.13) \end{aligned}$$

where the η are step distributions, which in general are not explicitly written when the dependence on $\cos \theta$ is stressed, since they mean only $-1 \leq \cos \theta \leq 1$. But, as the

$\cos \theta$ variable now disappears, the former relation gives much more complex relations among x , E , ω which one must take into account when the limits of integration intervals are set.

Let us now change the integration variable from s to x . The equation (2.6) then has the form

$$g(\omega, E) = \frac{\omega^2}{4} \int_1^{\omega^{3/4}} dx x^{-3} \sigma \left[\left(1 - \frac{4x}{\omega^2} \right)^{1/2}, \frac{2E - \omega}{\omega(1 - x^{-1})^{1/2}} \right] \frac{2}{\omega(1 - x^{-1})^{1/2}} \\ \times \eta[2E - \omega - (1 - x^{-1})^{1/2}(\omega^2 - 4x)^{1/2}] \\ \times \eta[\omega - (1 - x^{-1})^{1/2}(\omega^2 - 4x)^{1/2} - 2E] \int_{\omega/4x}^{\infty} d\omega' \omega'^{-2} n(\omega'). \quad (2.14)$$

It is at this point that the choice of x as an integration variable becomes useful. In effect the two η distributions inside the first integral may be interpreted as limiting the interval of integration over x , and one can prove that the choice of x as an integration variable allows the new integration interval to be internal to $[1, \omega^2/4]$. These simple proofs will be given in the Appendix. Also $\alpha(\omega)$ can be written using the x variable:

$$\alpha(\omega) = \frac{\omega^2}{4} \int_1^{\omega^{3/4}} dx x^{-3} \sigma \left(\frac{\omega^2}{4x} \right) \int_{\omega/4x}^{\infty} d\omega' \omega'^{-2} n(\omega') \quad (2.15)$$

and then it is easy to see that

$$\frac{\int_0^{\infty} g(\omega, E) dE}{\alpha(\omega)} = \frac{\alpha(\omega)}{\alpha(\omega)} = 1 \quad (2.16)$$

by interpreting the two step distributions as distributions on E . We finally obtain the following expression for $g(\omega, E)$.

$$g(\omega, E) = \frac{\omega^2}{2} \int_{x_-}^{x_+} dx x^{-3} \sigma \left[\left(1 - \frac{4x}{\omega^2} \right), \frac{2E - \omega}{\omega(1 - x^{-1})^{1/2}} \right] \frac{1}{\omega(1 - x^{-1})^{1/2}} \\ \times \int_{\omega/4x}^{\infty} d\omega' \omega'^{-2} n(\omega') \quad (2.17)$$

where

$$x_{\pm} = \frac{E\omega - p^2 \pm p\sqrt{p^2 + \omega^2 - 2E\omega}}{2}. \quad (2.18)$$

The integral on ω' extends only formally up to ∞ . In effect our approximations are valid if there are no photons of energy $\omega' \sim \omega$ in the radiation field. Therefore $n(\omega')$ must be cut off at an energy much lower than ω .

Before concluding this section let us stress the fact that the energy dependence of g will be contained in the extremes of integration x_{\pm} , as well as in the $\cos \theta$ dependence of the differential cross-section. Besides, in a number of cases, when the most important contributions from σ will be those near the threshold of the reaction, the only relevant energy dependence will be that contained in x_{\pm} . This is because the differential cross-section, near threshold, is almost independent of $\cos \theta$, and therefore on E also (which depends linearly on it). The energy dependence of x_{\pm} comes from the relation $-1 \leq \cos \theta \leq 1$. This was the reason why we have stressed the importance of the relations among x , ω , E , which can be got from this simple relation.

3. ENERGY DISTRIBUTION FOR CREATED PAIRS: RESULTS

As we have seen in Section 2, the rate of transition of photons, of energy ω , into two electrons of energies E and $\omega - E$, is given by the distribution $g(\omega, E)$ (equation (2.17)). Reference is made to the differential cross-section $\sigma(\beta, \beta \cos \theta)$ given by (2.7).

In this section we present some curves which show the energy distribution for different values of ω and different spectra $n(\omega')$ for the soft ambient photons.

To facilitate comparison of the results at different energies ω , we give, instead of the energy dependence of $g(\omega, E)$, that of

$$K(\omega, E) = \frac{g(\omega, E)}{\alpha(\omega)} \quad (3.1)$$

which, as shown by (16), is normalized to 1. This means that a uniform distribution over the range of E allowed by a certain value of ω would correspond to

$$K(\omega, E) = \omega^{-1}.$$

The explicit shape of K , when $n(\omega)$ has a power law or a δ -function spectrum, are given for different values of the spectral index, and of ω , in Figs 1, 2, 3.

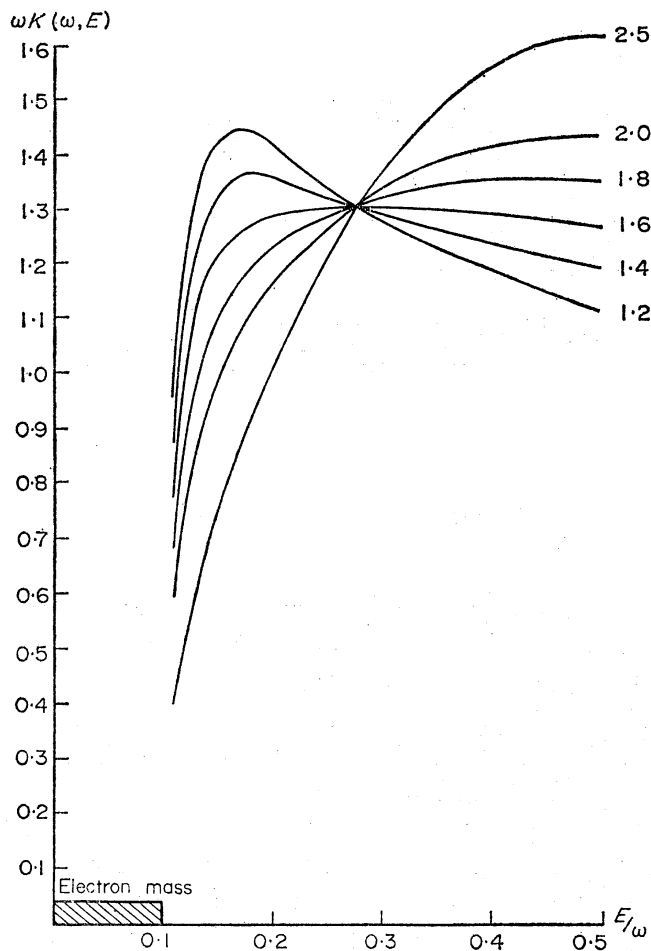


FIG. 1. Energy distributions of the electrons produced in the scattering of a photon of energy $\omega = 10 m_e$ with soft ambient photons of number distribution $\propto \omega^{-\alpha}$. The numbers at the end of each line correspond to the value of α it is calculated for.

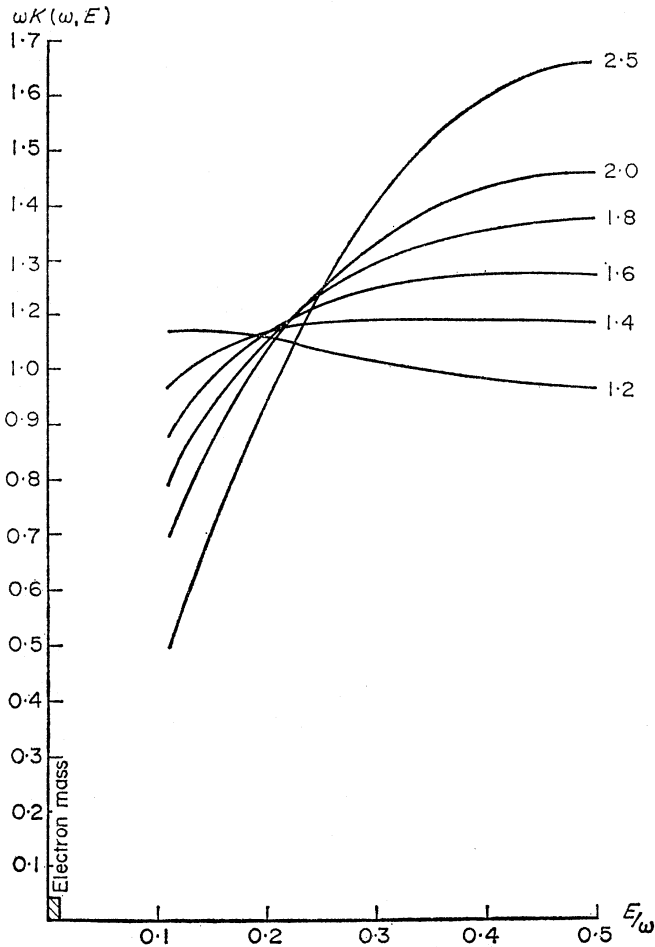


FIG. 2. Energy distributions of the electrons produced when photons of energy $\omega = 100 m_e$ are scattered by soft ambient photons of number distribution $\propto \omega^{-\alpha}$. The numbers at the end of each line correspond to the value of α it is calculated for. The same curves have also been obtained for $\omega = 10^3 m_e$; therefore one expects them to be valid for any energy larger than $100 m_e$.

For power law absorbers it is important to note that the energy distributions are rather flat and not so much dependent on the spectral index (for δ absorbers we can always look at the relation between $\omega\omega'$ which is fixed, and the threshold. From the curves of Fig. 3 we observe the interesting result that for $\omega\omega' \gg 1$ one of the two electrons has $E \sim \omega$, the other carrying away only a small portion of the energy of the incoming photon.)

Before concluding this section we make some qualitative remarks. The energy dependence of $K(\omega, E)$ is connected with the $\cos \theta$ dependence of the differential cross-section at different C.M. energies. At threshold, this cross-section does not depend on $\cos \theta$; for relativistic C.M. energies, on the other hand, it is strongly peaked forward and backward. In the latter case, the highest and lowest energy decay electrons become strongly favoured. However, the total cross-section is maximized very near to the threshold, so when all energies of target photons are present (as in the case for power law soft photon spectra), the most likely scattering is that near threshold. The probability of a scattering taking place far from threshold is then depressed both by the total cross-section decrease and by the decrease with energy of the number of the target photons. Thus we can understand why the

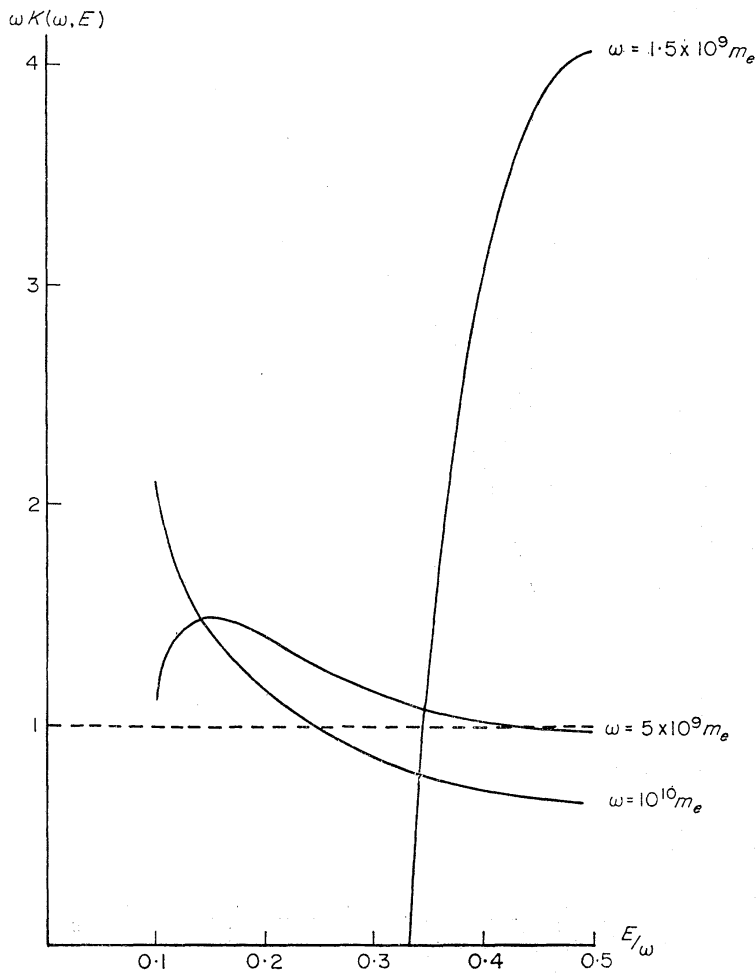


FIG. 3. Energy distribution of the electrons produced in the scattering of a hard photon with soft ambient monochromatic photons of an energy $\omega' = 3/2 kT$ with $T = 3^\circ K$. The difference curves correspond to different energies of the hard photon (they are given by the numbers at the end of each line).

electron energy distribution will not be strongly dependent on the soft photons distribution, provided that their number is decreasing with energy.

Throughout the following, the approximation

$$K(\omega, E) = \omega^{-1} \quad (3.2)$$

will be made.

4. EQUATIONS FOR A SOURCE WHERE PP TAKES PLACE

In our discussion of the equilibrium radiation and electron spectra in compact sources, we shall make the following further approximations in addition to (3.2)

(a) The photons resulting from synchrotron radiation by an electron of energy E all have energy

$$\omega = 2 \times 10^{-14} HE^2 \quad (4.1)$$

(where ω and E are measured in electron masses, and H is in Gauss).

(b) When an electron scatters a photon of energy ω , and $E\omega < 1$ then a photon of energy

$$\omega' = \frac{4}{3}\omega E^2 \quad (4.2)$$

will arise. If $E\omega > 1$ the scattered photon has energy $\sim E$, but the cross-section is less than the Thomson cross-section σ_T .

(c) The energy loss rate of an electron due to both synchrotron emission and inverse Compton scattering is given by

$$-\dot{E} = \sigma_T E^2 \left\{ \frac{H^2}{8\pi} + \int^{\lambda/E} \omega n(\omega) d\omega \right\} \quad (4.3)$$

(i.e. we ignore the energy loss associated with Compton scatterings for which $E\omega > 1$).

Consider a homogeneous source into which relativistic electrons with energy between E and $E + dE$ are injected at a rate $\mathcal{N}(E) dE$ per unit volume. If the age of the source is $\gg -(E/\dot{E})$ and it is in a steady state, the equilibrium density $N(E)$ of electrons is given by

$$\frac{d}{dE} [N(E)\dot{E}] = \mathcal{N}(E). \quad (4.4)$$

If the electrons produce γ -rays (as a result of synchrotron—Compton processes), then pair production induced by these photons will provide an *extra* source of relativistic particles. For typical power-law soft photon spectra, the absorption probability of a γ -ray increases monotonically with energy. But for simplicity we make the further approximation that pair production can be neglected at energies below some value E_1 , and that *no* photons with $\omega > E_1$, escape. (In fact, of course, E_2 will be a function of position within the source.) There would then be, in addition to $\mathcal{N}(E)$, an additional term

$$\mathcal{N}_{e^+}(E) = 2 \int_E^{E_0} \mathcal{N}_\gamma(\omega) K(\omega, E) \eta(\omega - E_1) d\omega \quad (4.5)$$

where $\mathcal{N}_\gamma(\omega) d\omega$ is the rate of production of photons with energy between ω and $\omega + d\omega$ and E_0 is the maximum energy of the injected electrons (and therefore of the photons produced). If the γ -rays themselves arise from inverse Compton scattering of softer photons, then

$$\mathcal{N}_\gamma(\omega) = \sigma_T \int_1^{E_0} \frac{dE}{E^2} N(E) n \left(\frac{3}{4} \frac{\omega}{E^2} \right) \eta \left(1 - \frac{3\omega}{4E} \right) \quad (4.6)$$

where $n(\omega)$ denotes the equilibrium density of photons of energy ω . Therefore, by combining expression (4.3)–(4.6) we derive

$$\frac{d}{dE} [N(E)\dot{E}] = \mathcal{N}(E) + 2\sigma_T \int_{\frac{3}{2}(E, E_1)}^{E_0} d\epsilon \frac{N(\epsilon)}{\epsilon^2} \int_{\frac{3}{2}E^2(E, E_1)}^{E_0} d\omega \frac{n(\omega)}{\omega} \quad (4.7)$$

where one must take the *larger* of E_1 and E , in the lower limits of integration.

Equation (4.7) provides one relation between the equilibrium electron and photon spectra in the source. But the fact that the photons are *emitted* by the electrons provides another obvious connection between $N(E)$ and $n(\omega)$. These two relations suffice, in principle, to determine $N(E)$ and $n(\omega)$ self-consistently in terms

of $\mathcal{N}(E)$, for a source of given dimensions and magnetic field. But the solution is generally very difficult, because of the intrinsic non-linearity of the problem. We have, however, been able to derive simple approximate solutions for cases when

$$\mathcal{N}(E) = \mathcal{N}_0 \delta(E - E_0). \quad (4.8)$$

We discuss this solution in the next section. Since (4.7) is non-linear, the solution cannot be straightforwardly extended to more general electron spectra. (However it should be applicable to power-law spectra $\mathcal{N}(E) \propto E^{-\gamma}$ if $\gamma < 2$, since in this case most of the energy is concentrated towards the high-energy cut-off.)

5. SPECTRA OF A SOURCE WITHOUT AND WITH PP

Although E_1 is itself a function of $\mathcal{N}(E)$, it will simplify the exposition if we treat it as an independent parameter and discuss separately the alternatives $E_1 > E_0$ and $E_1 < E_0$.

When $E_1 > E_0$ the situation is comparatively simple because pair production can be ignored. If the electrons lost most of their energy by the emission of photons with $\omega < E_0^{-1}$ (which would generally be the case if inverse Compton losses were small compared with synchrotron losses), then we should have the standard case $N(E) \propto E^{-2}$. However, if the main energy loss is due to inverse Compton scattering of synchrotron radiation—i.e. to ‘SC’ emission—then the ‘SC²’ emission may be even more important, and it is inevitable that, for most of the radiation, $\omega > E_0^{-1}$. Then, as we can clearly see from (4.3), \dot{E} is not proportional to E^2 , so the equilibrium spectrum will no longer be E^{-2} . Some particular self-consistent solutions for $N(E)$ and $n(\omega)$ were presented by Rees (1967) but the typical form of electron spectrum in this situation can be inferred from the following argument.

The synchrotron spectrum would cut-off above an energy of $2 \times 10^{-14} H E_0^2$, and (from 4.2) the ‘SC’ spectrum would extend up to a photon energy

$$\omega_n = 2 \times 10^{-14} \left(\frac{4}{3}\right)^n H E_0^{2n+2} \quad (5.1)$$

provided that this does not exceed E_0 . If Compton losses dominate, most of the energy will generally emerge as ‘SC ^{n_m} ’ radiation, where n_m is the largest value of n for which $\omega_n < E_0$. If the electrons have a power-law spectrum

$$N(E) \propto E^{-A} \quad (5.2)$$

then the SC ^{n_m} spectrum will also have a power-law form,

$$n(\omega) \propto \omega^{-B} \quad (5.3)$$

where $B = \frac{1}{2}(A + 1)$. Integrating (4.7) from E to E_0 , and taking (4.3) into account, we obtain

$$N(E) \propto E^{-2} \int^{1/E} d\omega \omega n(\omega). \quad (5.4)$$

If $\omega_{n_m} > 1$, we thus derive $A = B = 1$ for the whole electron spectrum; if $\omega_{n_m} < 1$ then this holds only for energies between $\omega_{n_m}^{-1}$ and E_0 , and at lower energies we have $A = 2$ and $B = 3/2$.

We turn now to the case when $E_0 > E_1$. If we substitute (5.2) and (5.3) into (4.7), we get

$$\frac{A-B}{2-B} E^{-A+B-1} \simeq \frac{1}{B} \left(\frac{3}{4}\right)^{-B} \int_{\frac{3}{4}E}^{E_0} d\epsilon \epsilon^{2B-A-2} E^{-B}. \quad (5.5)$$

If the electron spectrum (5.2) is generating the photon spectrum (5.3), we again have

$$2B - A - 2 = -1 \quad (5.6)$$

so (5.5) becomes

$$\frac{A-B}{2-B} E^{-A+B-1} \simeq \frac{E^{-B}}{B} \left(\frac{3}{4}\right)^{-B} [\ln E_0 - \ln \frac{3}{4}E]. \quad (5.7)$$

Thus, apart from a slowly varying logarithmic term, the hypothesis of power laws still works, as it does when $E_1 > E_0$, provided there is a suitable relation between A and B .

A further constraint is needed in order to fix A and B , and this can be obtained by equating the coefficients on both sides of (5.7). For $E \ll E_0$ the coefficient of E^{-B} on the r.h.s. will be of the order of 10 or more, and so we require $B - 2 \lesssim 0.1$. (Actually B will reach the value 1.9 when E is about two orders of magnitude less than E_0 . Moreover, if we take into account the logarithmic terms which appear in a more accurate computation of the SC radiation spectrum, we improve the situation by about one more order of magnitude.)

These admittedly approximate arguments therefore suggest that, in a source where most of the synchrotron radiation is boosted to γ -ray energies by repeated Compton scattering, but where the γ -rays generate electron-positron pairs before escaping, a δ -function injection spectrum leads to an equilibrium electron spectrum $N(E) \propto E^{-3}$. The fact that this is at least one power steeper than occurs in the absence of pair production can be qualitatively understood if we remember that a single electron injected with energy E_0 gives rise, via a repeated cascade, to a large number of lower energy electrons and positrons.

To consider the range of validity of this solution, let us recall the various connections between the photon and electron spectra:

(i) The photons are emitted by the electrons, and in our approximation there is a relation of the form $\omega = \alpha E^2$ relating each point on the photon spectrum to a particular energy. The value of α depends on the magnetic field, and on the order n_m of the SC^n spectrum containing most of the energy. Because of the quadratic dependence of ω on E , the photon spectrum for each order n of the SC^n spectrum spans twice the range of the electron spectrum on a logarithmic scale.

(ii) If the photon spectrum has an index $B < 2$, an electron of energy E loses most of its energy via inverse Compton scattering of photons with $\omega \simeq 1/E$. Thus the electron spectrum between E_1 and E_0 depends on the photon spectrum between $1/E_0$ and $1/E_1$. (Apart from a logarithmic term, this is still true when $B = 2$.)

(iii) Photons with energy $> E_1$ arising from inverse Compton scattering interact with softer photons within the source and yield electron-positron pairs whose energy distribution affects the whole injected spectrum. These photons come from the part of the spectrum $\frac{3}{4}(E_1/E_0^2) < \omega < 1/E_1$. We showed in Section 2 that the energy distribution of the pairs is insensitive to the spectrum of the soft photons, and we can in all cases adopt the simplification (3.2).

Connections (i) and (ii) apply even when there is no pair production, and we have seen that (for δ -function injection) they can lead to an E^{-1} electron spectrum and a ω^{-1} photon spectrum—in contrast to the ‘standard’ case E^{-2} and $\omega^{-3/2}$

which holds when synchrotron losses alone are important.

When pair production is important, it is clear from (i)–(iii) that our solution would hold for electron energies $> E_1$, if the part of the SC^{n_m} spectrum produced by these electrons covered the photon energy range $[\frac{3}{4}(E_1/E_0^2), 1/E_1]$. But the interval has the same logarithmic extent as the entire SC^{n_m} spectrum produced by electrons with energies in the interval $[E_1, E_0]$ so it would be an unlikely coincidence for this condition to be fulfilled. However we find from (4.7) that, for $n(\omega) \propto \omega^{-2}$ and $N(E) \propto E^{-3}$, the production rate of photons with a particular energy $E > E_1$ (which will produce pairs with energy $\sim E$) is given by an integral of the form

$$\int_{\frac{3}{4}(EE_0^{-2})}^{E^{-1}} \frac{d\omega}{\omega}.$$

Thus, if the photon spectrum were flatter than ω^{-2} below (say) $1/E_0$, the integral would change by less than a factor 2. Thus, at least to within a logarithmic term, our solution holds provided that the SC^{n_m} spectrum from electrons in the interval $[E_1, E_0]$ covers the range $[1/E_0, 1/E_1]$. If one assigns equal probabilities to equal logarithmic intervals, one may loosely say that this condition would be expected in ~ 50 per cent of cases. Moreover, it can easily be verified that a modification of our solution holds under even less restrictive conditions: if the SC^{n_m} spectrum cuts off at a photon energy kE_0^2 lying *within* the interval $[1/E_0, 1/E_1]$, then we expect $N(E) \propto E^{-3}$ for $\frac{3}{4}(1/kE_0) < E < E_0$, and in consequence $n(\omega) \propto \omega^{-2}$ for

$$\frac{3}{4} \frac{1}{kE_0^4} < \omega < kE_0^2.$$

Since $1/E_0$ lies almost exactly on the mid-point of this interval, it would in principle be possible to determine E_0 observationally for sources when this solution was believed to be relevant.

CONCLUSIONS

It is necessary to make a number of approximations even in order to derive the simple equilibrium spectrum discussed in the preceding section. The assumption that the source is homogeneous and the radiation field isotropic is perhaps the most serious oversimplification. If E_0 and H varied through the source, the spectral features would tend to be smeared out (moreover, the breaks and cut-offs in the SC^n spectra are not, of course, completely sharp even in the homogeneous case). Although our calculations are based on δ -function injection, they would still apply for a power law injection spectrum flatter than $\mathcal{N}(E) \propto E^{-2}$. We also, of course, have supposed that pair production provides the only important opacity for hard photons. This is likely to be a realistic assumption unless there is a large amount of cool thermal gas in the emitting region. Processes involving interactions with the relativistic particles themselves could provide a significant opacity only in the unrealistically extreme situation when *every* low energy photon undergoes inverse Compton scattering (a much stronger statement than that most of the *energy* results from inverse Compton scattering).

Equation (1.1) shows that pair production would only be important if the hard continuum radiation came from a region $\leq 10^{16}$ cm in size. We have mentioned in the introduction some indirect arguments that this may be so, but more direct evidence would be provided if the X-ray intensity were found to vary on the same

short time-scale as the known optical variability of some QSOs. The high frequency radio emission also may originate in this very compact region. Although extragalactic X-ray and γ -ray astronomy is still in its infancy, we believe that the considerations of this paper will eventually provide evidence on the nature of compact continuum sources. In particular, a ω^{-2} photon spectral index would be some indication that the electron spectrum may be affected by pair production, whereas a flatter spectrum (in X-rays or even in the visible) would argue against the source being so compact that pair production occurred. It should also be possible to obtain some estimate of the injection energy E_0 . Direct γ -ray measurements would of course provide further tests.

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REFERENCES

- Bowyer, C. S., Lampton, M., Mack J. & Mendonca, F. De, 1970. *Astrophys. J. Lett.*, **161**, 1.
 Cromwell, R. & Weymann, R. J., 1970. *Astrophys. J.*, **159**, L147.
 Fazio, G. G. & Stecker, F. W., 1970. *Nature, Lond.*, **226**, 135.
 Friedmann, H. & Byram, E. T., 1967. *Science*, **158**, 287.
 Gould, R. J. & Schröder, G. P., 1967a. *Phys. Rev.*, **155**, 4104.
 Gould, R. J. & Schröder, G. P., 1967b. *Phys. Rev.*, **155**, 1408.
 Hoyle, F., Burbidge, G. R. & Sargent, W. L. W., 1966. *Nature, Lond.*, **209**, 751.
 Jauch, J. M. & Rohrlich, F., 1955. *Theory of Photons and Electrons*, Addison Wesley Publishing Company, Inc., Reading, Massachusetts).
 Jelley, J. V., 1966. *Nature, Lond.*, **211**, 472.
 Nikishov, A. I., 1961. *Zh. Eksperim. i Teor. Fiz.*, **41**, 549. English translation *Sov. Phys.—JEPT*, **14**, 393 (1962).
 Rees, M. J., 1967. *Mon. Not. R. astr. Soc.*, **137**, 429.

APPENDIX

The two step distributions in (2.14) mean

$$E_- \leq E \leq E_+ \quad (a)$$

with

$$E_{\pm} = \frac{\omega}{2} \left\{ 1 \pm (1 - x^{-1})^{1/2} \left(1 - \frac{4}{\omega^2} x \right)^{1/2} \right\}.$$

From $E \leq E_+$,

$$\frac{2E}{\omega} - 1 < (1 - x^{-1})^{1/2} \left(1 - \frac{4}{\omega^2} x \right)^{1/2}. \quad (b)$$

Now, if

$$\frac{2E}{\omega} < 1$$

no restriction is set by (b).

Otherwise, if

$$\left(\frac{2E}{\omega} - 1\right)^2 \leq (1 - x^{-1}) \left(1 - \frac{4}{\omega^2} x\right)^{1/2}, \quad (c)$$

$E \geq E_-$ implies

$$\frac{2E}{\omega} - 1 \geq -(1 - x^{-1}) \left(1 - \frac{4}{\omega^2} x\right)^{1/2}, \quad (d)$$

If

$$\frac{2E}{\omega} > 1$$

(b) again sets no restriction, otherwise (c) must be satisfied.

Therefore, instead of (a), one can require (c) to be fulfilled. It is easy to prove that (c) is equivalent to

$$4x^2 + 4(p^2 - E\omega)x + \omega^2 < 0. \quad (e)$$

Then, x must lie between the two roots of the equation one obtains equating to zero the l.h.s. of (e).

These roots are

$$x_{\pm} = \frac{E\omega - p^2 \pm p\sqrt{p^2 + \omega^2 - 2E\omega}}{2}.$$

We notice that, if $\omega > E + 1$, then $p^2 + \omega^2 - 2E\omega > 0$ always. Now we prove that

$$x_- \geq 1, \quad x_+ \leq \frac{\omega^2}{4}. \quad (f)$$

The first of the (f) is equivalent to

$$E\omega - p^2 - p\sqrt{p^2 + \omega^2 - 2E\omega} \geq 2,$$

i.e. by putting the square root at the r.h.s. and squaring

$$E(\omega - E) - 1 \geq p^2(p^2 + \omega^2 - 2E\omega).$$

Now, using only the relation $E^2 = p^2 + 1$, we hence obtain

$$(\omega - 2E)^2 \geq 0$$

which is always true.

The second of (f), instead, is equivalent to

$$E\omega - p^2 + p\sqrt{p^2 + \omega^2 - 2E\omega} \leq \omega^2/2,$$

i.e. by putting the square root at the r.h.s.

$$E\omega - p^2 - \omega^2 \leq -p\sqrt{p^2 + \omega^2 - 2E\omega}.$$

Owing to the fact that, as $E < \omega$,

$$E\omega - p^2 - \omega^2 = \omega(E - \omega) - p^2 < 0,$$

the relation to be proved is

$$(E\omega - p^2 - \omega^2)^2 \geq p^2(p^2 + \omega^2 - 2E\omega).$$

By using the relation $E^2 = p^2 + 1$ we see that it is equivalent to

$$\omega^2 \left(E - \frac{\omega}{2} \right)^2 \geq 0$$

which again always holds.