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ON PROJECTIVE φ-RECURRENT KENMOTSU MANIFOLDS

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ABSTRACT

In this paper we study a projective φ-recurrent Kenmotsu manifold and show that projective φ-recurrent Kenmotsu manifold having a non-zero constant sectional curvature is locally projective φ-symmetric.

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1. Introduction

In 1977, T. Takahashi [5] introduced the notion of locally φ-symmetric Sasakian manifold and obtain few of its interesting properties. The authors [2] and [7], have extended this notion to 3-dimensional Kenmotsu and trans-Sasakian manifolds respectively. Also the authors [3] and [6] studied φ-recurrent Sasakian and Kenmotsu manifold respectively. In this paper we study a projective φ-recurrent Kenmotsu manifold and obtain some interesting results. Here we show that a projective φ-recurrent Kenmotsu manifold is an Einstein manifold and a projective φ-recurrent Kenmotsu manifold having a non-zero constant sectional curvature is locally projective φ-symmetric.

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2. Preliminaries

Let $M^{2n+1}(\varphi, \xi, \eta, g)$ be a Kenmotsu manifold with the structure (φ, ξ, η, g). Then the following relations hold [4]:

(2.1) $\varphi^2 = -I + \eta \otimes \xi$

(2.2) (a) $\eta(\xi) = 1$, (b) $g(X, \xi) = \eta(X)$, (c) $\varphi\xi = 0$ (d) $\eta \circ \varphi = 0$

(2.3) $g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y)$,

(2.4) $(\nabla_x \varphi)Y = -g(X, \varphi Y)\xi - \eta(Y)\varphi X$,

(2.5) (a) $\nabla_X \xi = X - \eta(X)\xi$, (b) $(\nabla_X \eta)Y = g(X, Y) - \eta(X)\eta(Y)$

(2.6) $R(X, Y)\xi = \eta(X)Y - \eta(Y)X$

(2.7) $R(X, \xi)Y = g(X, Y)\xi - \eta(Y)X$

(2.8) $\eta(R(X, Y)Z) = g(X, Z)\eta(Y) - g(Y, Z)\eta(X)$

(2.9) $S(X, \xi) = -2n\,\eta(X)$

(2.10) $S(\varphi X, \varphi Y) = S(X, Y) + 2n\,\eta(X)\eta(Y)$

for all vector fields X, Y, Z, where ∇ denotes the operator of covariant differentiation with respect to g, φ is a (1, 1) tensor field, S is the Ricci tensor of type (0, 2) and R is the Riemannian curvature tensor of the manifold.

Definition 2.1. [2] A Kenmotsu manifold is said to be locally φ-symmetric if

$$\varphi^2((\nabla_W R)(X, Y)Z) = 0, \qquad (2.11)$$

for all vector fields X, Y, Z, W orthogonal to ξ.

Definition 2.2. A Kenmotsu manifold is said to be locally projective φ-symmetric if

$$\varphi^2((\nabla_W P)(X, Y)Z) = 0, \qquad (2.12)$$

for all vector fields X, Y, Z, W orthogonal to ξ.

Definition 2.3. A Kenmotsu manifold is said to be projective φ-recurrent manifold if there exists a non-zero 1-form A such that

$$\varphi^2((\nabla_W P)(X,Y)Z) = A(W)P(X,Y)Z \tag{2.13}$$

for arbitrary vector fields X, Y, Z, W, where P is a projective curvature tensor given by [1],

$$P(X,Y)Z = R(X,Y)Z - \frac{1}{2n}[S(Y,Z)X - S(X,Z)Y] \tag{2.14}$$

If the 1-form A vanishes, then the manifold reduces to a locally projective φ-symmetric manifold.

3. Projective φ-recurrent Kenmotsu Manifold

Let us consider a Projective φ-recurrent Kenmotsu manifold. Then by virtue of (2.1) and (2.13) we have

$$-(\nabla_W P)(X,Y)Z + \eta((\nabla_W P)(X,Y)Z)\xi = A(W)P(X,Y)Z \tag{3.1}$$

from which it follows that

$$-g((\nabla_W P)(X,Y)Z,U) + \eta((\nabla_W P)(X,Y)Z)\eta(U) = A(W)g(P(X,Y)Z,U) \tag{3.2}$$

Again from (3.1) we have

$$(\nabla_W P)(X,Y)Z = \eta((\nabla_W P)(X,Y)Z)\xi - A(W)P(X,Y)Z \tag{3.3}$$

This implies,

$$\begin{aligned}(\nabla_W R)(X,Y)Z = {} & \eta((\nabla_W R)(X,Y)Z)\xi - A(W)R(X,Y)Z \\ & + \frac{1}{2n}[(\nabla_W S)(Y,Z)X - (\nabla_W S)(X,Z)Y] \\ & - \frac{1}{2n}[(\nabla_W S)(Y,Z)\eta(X) - (\nabla_W S)(X,Z)\eta(Y)]\xi \\ & + \frac{1}{2n}A(W)[S(Y,Z)X - S(X,Z)Y\end{aligned} \tag{3.4}$$

From (3.4) and the Bianchi identity we get

$$A(W)\eta(R(X,Y)Z) + A(X)\eta(R(Y,W)Z) + A(Y)\eta(R(W,X)Z)$$

$$= \frac{1}{2n} A(W)[S(Y,Z)\eta(X) - S(X, Z)\eta(Y)]$$

$$+ \frac{1}{2n} A(X)[S(W,Z)\eta(Y) - S(Y,Z)\eta(W)]$$

$$+ \frac{1}{2n} A(Y)[S(X,Z)\eta(W) - S(W,Z)\eta(X)] \tag{3.5}$$

By virtue of (2.8) we obtain from (3.5) that

$$A(W)[\eta(Y)g(X, Z) - \eta(X)g(Y, Z) + A(X)[\eta(W)g(Y, Z) - \eta(Y)g(W, Z)]$$

$$+ A(Y)[\eta(X)g(W,Z) - \eta(W)g(X,Z)]$$

$$= \frac{1}{2n} A(W)[S(Y,Z)\eta(X) - S(X,Z)\eta(Y)]$$

$$+ \frac{1}{2n} A(X)[S(W,Z)\eta(Y) - S(Y,Z)\eta(W)]$$

$$+ \frac{1}{2n} A(Y)[S(X,Z)\eta(W) - S(W,Z)\eta(X)] \tag{3.6}$$

Putting Y = Z = e_i in (3.6) and taking summation over i, $1 \le i \le 2n+1$, we get

$$A(W)\,\eta(X) = A(X)\,\eta(W), \tag{3.7}$$

for all vector fields X, W.

Replacing X by ξ in (3.7), we get

$$A(W) = \eta(W)\,\eta(\rho), \tag{3.8}$$

for any vector field W, where $A(\xi) = g(\xi,\rho) = \eta(\rho)$, ρ being the vector field associated to the 1-form A i.e., $g(X,\rho) = A(X)$.

From (3.7) and (3.8), we state the following:

Theorem 3.1. *In a projective φ-recurrent Kenmotsu manifold (M^{2n+1}, g), (n > 1), the characteristic vector field ξ and the vector field ρ associated to the 1-form A are co-directional and the 1-form A is given by (3.8).*

From (2.14) it follows that

$$(\nabla_W P)(X,Y)\xi = (\nabla_W R)(X,Y)\xi - \frac{1}{2n}[(\nabla_W S)(Y,\xi)X - (\nabla_W S)(X,\xi)Y]$$

Using (2.5), (2.6) and (2.9) in the above equation, we have

$$(\nabla_W P)(X,Y)\xi = [g(W,X) - \eta(W)\eta(X)]Y - [g(W,Y)\,\eta(W)\eta(Y)]X$$

$$-R(X,Y)(W - \eta(W)\xi) \tag{3.9}$$

By virtue of (2.8), it follows from (3.9) that

$$\eta((\nabla_W P)(X,Y)\xi) = 0 \tag{3.10}$$

Using (3.9) and (3.10) in (3.1), we have

$$-[g(W,X) - \eta(W)\eta(X)] + [g(W,Y) - \eta(W)\eta(Y)] + R(X,Y)(W - \eta(W)\xi) \tag{3.11}$$

$$= A(W)[R(X,Y)\xi - (\eta(X)Y - \eta(Y)X)]$$

Again using (3.8) and (2.6) in (3.11) we obtain

$$R(X, Y)W = [g(X, W)Y - g(Y, W)X] \tag{3.12}$$

Hence we can state the following:

Theorem 3.2. *A projective φ-recurrent Kenmotsu manifold (M^{2n+1}, g), (n > 1), is a space of constant curvature.*

We now suppose that a Kenmotsu manifold (M^{2n+1}, g), $(n > 1)$, is projective φ-recurrent. Then from (3.3) and (3.9), it follows that

$$(\nabla_W P)(X,Y)Z = [g(Y,W)g(X,Z) - g(X,W)g(Y,Z) + g(R(X,Y)W,Z)]\xi \quad (3.13)$$

$$- A(W)P(X,Y)Z$$

Next, we suppose that in a projective φ-recurrent Kenmotsu manifold, the sectional curvature of a plane $\pi \subset T_p M$ defined by

$$K_p(\pi) = g(R(X,Y)Y,X),$$

is a non-zero constant k, where $\{X, Y\}$ is any orthonormal basis of π. Then we have

$$g((\nabla_Z R)(X,Y)Y,X) = 0 \quad (3.14)$$

From (2.14), we have

$$(\nabla_Z P)(X,Y)Y = (\nabla_Z R)(X,Y)Y - \frac{1}{2n}[(\nabla_Z S)(Y,Y)X - (\nabla_Z S)(X,Y)Y]$$

Using (3.14) and the above equation we have

$$g((\nabla_Z P)(X,Y)Y,X) = 0 \quad (3.15)$$

By virtue of (3.15) and (3.1) we obtain

$$g((\nabla_Z P)(X,Y)Y,\xi)\eta(X) = A(Z)g(P(X,Y)Y,X) \quad (3.16)$$

Since in a projective φ-recurrent Kenmotsu manifold, the relation (3.13) holds good. Using (3.13) in (3.16) and on simplification we get

$$\eta(\rho) = 0$$

Hence by (3.8) we obtain from (2.13) that

$$\varphi^2((\nabla_W P)(X,Y)Z) = 0$$

This leads to the following theorem:

Theorem 3.3. *If a projective φ-recurrent Kenmotsu manifold (M^{2n+1}, g), $(n > 1)$, has a non-zero constant sectional curvature, then it reduces to a locally projective φ-symmetric manifold.*

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