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ON PROPER (STRONG) RAINBOW CONNECTION OF GRAPHS

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Abstract

A path in an edge-colored graph G is called a *rainbow path* if no two edges on the path have the same color. The graph G is called *rainbow connected* if between every pair of distinct vertices of G, there is a rainbow path. Recently, Johnson *et al.* considered this concept with the additional requirement that the coloring of G is proper. The *proper rainbow connection number* of G, denoted by $prc(G)$, is the minimum number of colors needed to properly color the edges of G so that G is rainbow connected. Similarly, the *proper strong rainbow connection number* of G, denoted by $psrc(G)$, is the minimum number of colors needed to properly color the edges of G such that for any two distinct vertices of G, there is a rainbow geodesic (shortest path) connecting them. In this paper, we characterize those graphs with proper rainbow connection numbers equal to the size or within 1 of the

size. Moreover, we completely solve a question proposed by Johnson *et al.* by proving that if $G = K_{p_1}\Box\cdots\Box K_{p_n}$, where $n \geq 1$, and $p_1, \ldots, p_n > 1$ are integers, then $prc(G) = psrc(G) = \chi'(G)$, where $\chi'(G)$ denotes the chromatic index of G. Finally, we investigate some sufficient conditions for a graph G to satisfy $prc(G) = rc(G)$, and make some slightly positive progress by using a relation between $rc(G)$ and the girth of the graph.

Keywords: proper (strong) rainbow connection number, Cartesian product, chromatic index.

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