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ON PSEUDO-SYMMETRY CURVATURE CONDITIONS OF GENERALIZED (k,μ)-PARACONTACT METRIC MANIFOLDS

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Abstract. In this paper we investigate Ricci pseudo-symmetric and Ricci generalized pseudo-symmetric generalized (k,μ)-paracontact metric manifolds. Besides this we characterize generalized (k,μ)-paracontact metric manifolds satisfying the curvature conditions $Q(S,R)=0$ and $Q(S,g)=0$, where S, R are the Ricci tensor and curvature tensor respectively. Several corollaries are also obtained.

1. Introduction

The notion of paracontact geometry was introduced by Kaneyuki and Williams [16] in 1985. A systematic investigation on paracontact metric manifolds done by Zamkovoy [19]. Recently, Cappelletti-Montano et al [6] introduced a new type of paracontact geomerty so-called paracontact metric (k,μ) space, where k and μ are constant. It is known [1] that in contact case $k \leq 1$, but in paracontact case there is no restriction for k.

The conformal curvature tensor C is invariant under conformal transformation and vanishes identically for 3-dimensional manifolds. Using this result several authors studied different types of 3-dimensional manifolds ([10], [11], [12]).

A semi-Riemannian manifold (M,g) is called locally symmetric if its curvature tensor R is parallel (that is, $\nabla R = 0$) and semi-symmetric if its curvature tensor R satisfies the condition

$$R(X,Y)\cdot R = 0, \tag{1.1}$$

where R is the Riemannian curvature tensor and $R(X,Y)$ is considered as a derivation of the tensor algebra at each point of the manifold for tangent vector fields

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X, Y. A complete intrinsic classification of these manifolds was given by Szabo in [18].

A (k,μ)-paracontact metric manifold is called an Einstein manifold if the Ricci tensor satisfies the condition $S = \lambda g$, where λ is some constant.
We define endomorphisms $R(X,Y)$ and $X \wedge_A Y$ by

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z \tag{1.2}$$

and

$$(X \wedge_A Y)Z = A(Y,Z)X - A(X,Z)Y, \tag{1.3}$$

respectively, where $X, Y, Z \in \chi(M)$, $\chi(M)$ is the set of all differentiable vector fields on M, A is the symmetric (0,2)-tensor, R is the Riemannian curvature tensor of type (1,3) and ∇ is the Levi-Civita connection. For a $(0,k)$-tensor field T, $k \geq 1$, on (M,g) we define the tensor $R \cdot T$ and $Q(g,T)$ by

$$\begin{aligned}(R(X.Y) \cdot T)(X_1, X_2, ..., X_k)) &= -T(R(X,Y)X_1, X_2, ..., X_k) \\ &\quad -T(X_1, R(X,Y)X_2, ..., X_k) \\ &\quad ..... - T(X_1, X_2, ..., R(X,Y)X_k)\end{aligned} \tag{1.4}$$

and

$$\begin{aligned}Q(g,T)(X_1, X_2, ..., X_k, Y) &= -T((X \wedge Y)X_1, X_2, ..., X_k) \\ &\quad -T(X_1, (X \wedge Y)X_2, ..., X_k) \\ &\quad ..... - T(X_1, X_2, ..., (X \wedge Y)X_k)\end{aligned} \tag{1.5}$$

respectively [17]. If the tensors $R \cdot S$ and $Q(g,S)$ are linearly dependent, then M is called Ricci pseudo-symmetric [17]. This is equivalent to

$$R \cdot S = fQ(g,S), \tag{1.6}$$

holding on the set $U_S = \{x \in M : S \neq 0 \text{ at } x\}$, where f is some function on U_S. Also if the tensors $R \cdot R$ and $Q(S,R)$ are linearly dependent, then M is said to be Ricci generalized pseudo-symmetric [17]. This is equivalent to

$$R \cdot R = fQ(S,R). \tag{1.7}$$

Recently, 3-dimensional generalized (k,μ)-paracontact metric manifolds have been studied by Kupeli Erken et al ([15], [14]). Kowalczyk [13] studied semi-Riemannian manifolds satisfying $Q(S,R) = 0$ and $Q(g,S) = 0$, where S, R are the Ricci tensor and curvature tensor respectively. De et al. [9] studied Ricci pseudo-symmetric and Ricci generalized pseudo-symmtric P-sasakian manifolds.

The paper is organized in the following way:

In Section 2, we discuss about some basic results of paracontact metric manifolds. Next, we investigate Ricci pseudo-symmetric generalized (k,μ)-paracontact metric manifolds. Section 4 deals with Ricci generalized pseudo-symmetric generalized (k,μ)-paracontact metric manifolds. In Section 5 and 6 we study generalized (k,μ)-paracontact metric manifolds satisfying $Q(S,R) = 0$ and $Q(S,g) = 0$, where S, R are the Ricci tensor and curvature tensor respectively.

2. Preliminaries

A $(2n+1)$-dimensional smooth manifold M is said to be has an alomost paracontact structure if it carries a (1,1)-tensor ϕ, a vector field ξ and a 1-form η satisfying [16]:
(i) $\phi^2 X = X - \eta(X)\xi$, for all $X \in \chi(M)$, $\eta(\xi) = 1$,
(ii) the tensor field ϕ induces an almost paracomplex structure on each fibre of $D = ker(\eta)$, that is, the eigendistributions D^+_ϕ and D^-_ϕ of ϕ corresponding the eigenvalues 1 and -1, respectively, have equal dimension n.
From the above conditions it follows that $\phi(\xi) = 0$, $\eta \circ \phi = 0$.

An almost paracontact structure is said to be normal [16] if and only if the (1,2) type torsion tensor $N_\phi = [\phi,\phi] - 2d\eta \otimes \xi$ vanishes identically, where $[\phi,\phi](X,Y) = \phi^2[X,Y] + [\phi X, \phi Y] - \phi[\phi X, Y] - \phi[X, \phi Y]$. If an almost paracontact manifold admits a pseudo-Riemannian metric g such that

$$g(\phi X, \phi Y) = -g(X,Y) + \eta(X)\eta(Y), \tag{2.1}$$

for $X, Y \in \chi(M)$, then we say that (M, ϕ, ξ, η, g) is an almost paracontact metric manifold. Any such pseudo-Riemannian metric manifold is of signature $(n+1, n)$. An almost paracontact structure is said to be a paracontact structure if $g(X, \phi Y) = d\eta(X,Y)$ [19]. In a paracontact metric manifold we define (1,1)-type tensor fields h by $h = \frac{1}{2}\pounds_\xi \phi$, where $\pounds_\xi \phi$ is the Lie derivative of ϕ along the vector field ξ. Then we observe that h is symmetric and anti-commutes with ϕ. Also h satisfies the following conditions [19]:

$$h\xi = 0,\ tr(h) = tr(\phi h) = 0, \tag{2.2}$$

$$\nabla_X \xi = -\phi X + \phi h X. \tag{2.3}$$

for all $X \in \chi(M)$, where ∇ denotes the Levi-Civita connection of the pseudo-Riemannian manifold.
Moreover h vanishes identically if and only if ξ is a Killing vector field and then (M, ϕ, ξ, η, g) is said to be a K-paracontact manifold. (k,μ)-paracontact manifolds have been studied by Calvasuso et al. ([3],[4], [5]) and Cappellaeti-Montano et al. ([7], [8]) and many others.

Generalized (k,μ)-paracontact metric manifolds were studied by Murathan and Kupeli Erken in [15]. A generalized (k,μ)-paracontact metric manifolds mean a 3-dimensional paracontact metric manifold which satisfy the nullity condition

$$R(X,Y)\xi = k(\eta(Y)X - \eta(X)Y) + \mu(\eta(Y)hX - \eta(X)hY). \tag{2.4}$$

In a generalized $(k \neq -1, \mu)$-paracontact manifold the following results hold ([2], [14]):

$$h^2 = (1+k)\phi^2, \tag{2.5}$$

$$\xi(k) = 0, \tag{2.6}$$

$$Q\xi = 2k\xi, \tag{2.7}$$

$$QX = (\frac{r}{2} - k)X + (-\frac{r}{2} + 3k)\eta(X)\xi + \mu hX, k \neq -1, \tag{2.8}$$

where X is any vector fields on M, Q is the Ricci operator of M, r denotes the scalar curvature of M.

$$h\, grad\mu = grad\, k. \tag{2.9}$$

We recall the following:

Lemma 2.1. [14] *Let $M(\phi, \xi, \eta, g)$ be a generalized (k, μ)-paracontact metric manifold with $k > -1$ and $\xi\mu = 0$. Then*

1. *At any point of M, precisely one of the following relations is valid: $\mu = 2(1 + \sqrt{1 + k})$, or $\mu = 2(1 - \sqrt{1 + k})$*
2. *At any point $P \in M$ there exists a chart $(U, (x, y, z))$ with $P \in U \subseteq M$, such that the functions k, μ depend only on the variable z.*

3. Ricci pseudo-symmetric generalized (k, μ)-paracontact metric manifolds

In this section we study Ricci pseudo-symmetric generalized (k, μ)-paracontact metric manifolds, that is, the manifold satisfying the curvature condition $R \cdot S = fQ(g, S)$. Then we have from (1.6)

$$(R(X, Y) \cdot S)(U, V) = fQ(g, S)(X, Y; U, V). \tag{3.1}$$

It is equivalent to

$$(R(X, Y) \cdot S)(U, V) = f((X \wedge_g Y \cdot S)(U, V)). \tag{3.2}$$

Using (1.7) in (3.2), we get

$$\begin{aligned} -S(R(X, Y)U, V) - S(U, R(X, Y)V) &= f[-g(Y, U)S(X, V) \\ &+g(X, U)S(Y, V) - g(Y, V)S(U, X) + g(X, V)S(U, Y)]. \end{aligned} \tag{3.3}$$

Substituting $X = U = \xi$, we obtain

$$\begin{aligned} &-S(R(\xi, Y)\xi, V) - S(\xi, R(\xi, Y)V) \\ &= f[-g(Y, \xi)S(\xi, V) + g(\xi, \xi)S(Y, V) - g(Y, V)S(\xi, \xi) + g(\xi, V)S(\xi, Y)]. \end{aligned} \tag{3.4}$$

Applying (2.4) and (2.7) in (3.4), we get

$$(k - f)[S(Y, V) - 2kg(Y, V)] + \mu[S(hY, V) - 2kg(hY, V)] = 0. \tag{3.5}$$

Putting hY for Y in (3.5) yields

$$(k - f)[S(hY, V) - 2kg(hY, V)] + \mu(k + 1)[S(Y, V) - 2kg(Y, V)] = 0. \tag{3.6}$$

Multiplying (3.5) by $(k - f)$ and (3.6) by μ and subtracting the results we have

$$[(k - f)^2 - \mu^2(k + 1)][S(Y, V) - 2kg(Y, V)] = 0. \tag{3.7}$$

Then either $S(Y, V) = 2kg(Y, V)$ or, $(k - f)^2 = \mu^2(k + 1)$.
Case 1: Let $S(Y, V) = 2kg(Y, V)$. Then the manifold is an Einstein manifold.
Case 2: Let $(k-f)^2 = \mu^2(k+1)$. Therefore $f = k \pm \mu\sqrt{1 + k}$. Hence the manifold is of the form $R \cdot S = (k \pm \mu\sqrt{1 + k})Q(g, S)$.
By the above discussions we have the following:

Theorem 3.1. *A Ricci pseudo-symmetric generalized (k, μ)-paracontact metric manifold is either an Einstein manifold or of the form $R \cdot S = (k \pm \mu\sqrt{1 + k})Q(g, S)$.*

Also we can state the following:

Proposition 3.1. *Every Ricci pseudo-symmetric generalized (k,μ)-paracontact metric manifold is of the form $R\cdot S = (k \pm \mu\sqrt{1+k})Q(g,S)$, provided the manifold is non-Einstein.*

If the manifold is an Einstein manifold, then obviously the manifold is Ricci pseudo-symmetric. This leads to the following:

Corollary 3.1. *A generalized (k,μ)-paracontact metric manifold is Ricci pseudo-symmetric if and only if the manifold is an Einstein manifold, provided $f \neq k \pm \mu\sqrt{1+k}$.*

4. Ricci generalized pseudo-symmetric generalized (k,μ)-paracontact metric manifolds

This section is devoted to study Ricci generalized pseudo-symmetric generalized (k,μ)-paracontact metric manifolds. Then we have $R\cdot R = fQ(S,R)$, that is,

$$(R(X,Y)\cdot R)(U,V)W = f((X\wedge_S Y)\cdot R)(U,V)W). \tag{4.1}$$

Then using (1.6) in (4.1), we get

$$\begin{aligned}&R(X,Y)R(U,V)W - R(R(X,Y)U,V)W - R(U,R(X,Y)V)W\\ &-R(U,V)R(X,Y)W = f[S(Y,R(U,V)W)X - S(X,R(U,V)W)Y\\ &-S(Y,U)R(X,V)W + S(X,U)R(Y,V)W - S(Y,V)R(U,X)W\\ &+S(X,V)R(U,Y)W - S(Y,W)R(U,V)X + S(X,W)R(U,V)Y].\end{aligned} \tag{4.2}$$

Putting $X = U = \xi$ in (4.2), we have

$$\begin{aligned}&R(\xi,Y)R(\xi,V)W - R(R(\xi,Y)\xi,V)W - R(\xi,R(\xi,Y)V)W\\ &-R(\xi,V)R(\xi,Y)W = f[S(Y,R(\xi,V)W)\xi - S(\xi,R(\xi,V)W)Y\\ &-S(Y,\xi)R(\xi,V)W + S(\xi,\xi)R(Y,V)W - S(Y,V)R(\xi,\xi)W\\ &+S(\xi,V)R(\xi,Y)W - S(Y,W)R(\xi,V)\xi + S(\xi,W)R(\xi,V)Y].\end{aligned} \tag{4.3}$$

Applying (2.4) and (2.7) in (4.3), we get

$$\begin{aligned}&-k^2g(V,W)Y - \mu kg(V,W)hY - \mu k\eta(W)g(hV,Y)\xi\\ &-\mu kg(hW,V)Y - \mu^2 g(hW,V)hY + \mu k\eta(W)g(Y,hV)\xi\\ &+kR(Y,V)W + \mu R(hY,V)W + \mu kg(hY,W)\eta(V)\xi -\\ &\mu k\eta(V)\eta(W)hY + \mu^2(k+1)\eta(V)g(Y,W)\xi - \mu^2(k+1)\eta(V)\eta(W)Y\\ &+k^2g(Y,W)V + \mu kg(Y,W)hV + +\mu kg(hW,Y)V\\ &+\mu^2 g(hW,Y)hV = f[-k\eta(W)S(Y,V)\xi - \mu\eta(W)S(Y,hV)\xi\\ &-2k^2g(V,W)Y - 2k\mu g(hW,V)Y + 2kR(Y,V)W\\ &+2k^2\eta(V)g(Y,W)\xi + 2k\mu g(hW,Y)\eta(V)\xi - 2k\mu\eta(V)\eta(W)hY\\ &-k\eta(V)S(Y,W)\xi + kS(Y,W)V + \mu S(Y,W)hV + 2k^2\eta(W)g(V,Y)\xi\\ &+2k\mu\eta(W)g(hY,V)\xi].\end{aligned} \tag{4.4}$$

Taking inner product with T, we obtain

$$\begin{aligned}
&-k^2 g(V,W)g(Y,T) - \mu k g(V,W)g(hY,T) - \mu k\eta(W)g(hV,Y)\eta(T)\\
&-\mu k g(hW,V)g(Y,T) - \mu^2 g(hW,V)g(hY,T) + \mu k\eta(W)g(Y,hV)\eta(T)\\
&+kg(R(Y,V)W,T) + \mu g(R(hY,V)W,T) + \mu k g(hY,W)\eta(V)\eta(T)\\
&-\mu k\eta(V)\eta(W)g(hY,T) + \mu^2(k+1)\eta(V)g(Y,W)\eta(T)\\
&-\mu^2(k+1)\eta(V)\eta(W)g(Y,T) + k^2 g(Y,W)g(V,T)\\
&+\mu k g(Y,W)g(hV,T) + +\mu k g(hW,Y)g(V,T) + \mu^2 g(hW,Y)g(hV,T)\\
&= f[-k\eta(W)S(Y,V)\eta(T) - \mu\eta(W)S(Y,hV)\eta(T) - 2k^2 g(V,W)Y\\
&-2k\mu g(hW,V)g(Y,T) + 2kg(R(Y,V)W,T) + 2k^2\eta(V)g(Y,W)\eta(T)\\
&+2k\mu g(hW,Y)\eta(V)\eta(T) - 2k\mu\eta(V)\eta(W)g(hY,T) - k\eta(V)S(Y,W)\eta(T)\\
&+kS(Y,W)g(V,T) + \mu S(Y,W)g(hV,T) + 2k^2\eta(W)g(V,Y)\eta(T)\\
&+2k\mu\eta(W)g(hY,V)\eta(T)]. \qquad (4.5)
\end{aligned}$$

Let $\{e_i\}$, $i = 1,2,3$ be a local orthonormal basis in the tangent space $T_P M$ at each point p $\in M$. Substituting $Y = T = e_i$ in (4.5) and summing over $i = 1$ to 3, we infer that

$$(1-3f)k\{S(Y,T) - 2kg(Y,T)\} + \mu(1-f)\{S(hY,T) - 2kg(hY,T)\} = 0. \qquad (4.6)$$

Setting hY for Y in (4.6), we get

$$(1-3f)k\{S(hY,T)-2kg(hY,T)\}+\mu(1-f)(k+1)\{S(Y,T)-2kg(Y,T)\} = 0. \qquad (4.7)$$

Multiplying (4.6) by $(1-3fk)$ and (4.7) by $\mu(1-f)$ and then subtracting the result, we have

$$\{(1-3f)^2k^2 - \mu^2(1-f)^2(k+1)\}\{S(Y,T) - 2kg(Y,T)\} = 0. \qquad (4.8)$$

Then either $S(Y,T) = 2kg(Y,T)$
or, $(1-3f)^2k^2 - \mu^2(1-f)^2(k+1) = 0$.
Thus we can state the following:

Theorem 4.1. *A Ricci generalized pseudo-symmetric generalized (k,μ)-paracontact metric manifold is an Einstein manifold, provided $(1-3f)^2k^2-\mu^2(1-f)^2(k+1) \neq 0$.*

Now if we consider $\mu = 0$, then from $(1-3f)^2k^2 - \mu^2(1-f)^2(k+1) = 0$, we infer $f = \frac{1}{3}$.
Thus we can state that

Corollary 4.1. *A Ricci generalized pseudo-symmetric generalized $N(k)$-paracontact metric manifold is of the form $R \cdot R = \frac{1}{3}Q(S,R)$, provided the manifold is non-Einstein.*

Again if we consider $f = 0$, then from $(1-3f)^2k^2 - \mu^2(1-f)^2(k+1) = 0$, we obtain

$$k^2 - \mu^2(k+1) = 0, \qquad (4.9)$$

which implies $(2k-\mu^2)(\xi k) - 2\mu(k+1)(\xi\mu) = 0$. Now by using (2.6) we have $\mu(k+1)(\xi\mu) = 0$. Taking account of $\mu \neq 0$ and $k < -1$, we have $\xi\mu = 0$. Hence using Lemma 2.1 we have the following:

Corollary 4.2. *If a generalized (k,μ)-paracontact metric manifold with $k > -1$ satisfy the curvature condition $R \cdot R = 0$ then at any point $P \in M$ there exists a chart $(U,(x,y,z))$ with $P \in U \subseteq M$, such that the functions k, μ depend only on the variable z and either $\mu = 2(1+\sqrt{1+k})$, or $\mu = 2(1-\sqrt{1+k})$ is valid.*

5. GENERALIZED (k,μ)-PARACONTACT METRIC MANIFOLDS SATISFYING Q(S,R)=0

In this section we study generalized (k,μ)-paracontact metric manifolds satisfying the curvature condition Q(S,R)=0. Therefore

$$(X \wedge_S Y) \cdot R)(U,V)W = 0. \tag{5.1}$$

Then using (1.7) in (5.1), we get

$$\begin{aligned}&S(Y,R(U,V)W)X - S(X,R(U,V)W)Y - S(Y,U)R(X,V)W\\&+S(X,U)R(Y,V)W - S(Y,V)R(U,X)W + S(X,V)R(U,Y)W\\&-S(Y,W)R(U,V)X + S(X,W)R(U,V)Y = 0.\end{aligned} \tag{5.2}$$

Substituting $X = U = \xi$ in (5.2) yields

$$\begin{aligned}&S(Y,R(\xi,V)W)\xi - S(\xi,R(\xi,V)W)Y - S(Y,\xi)R(\xi,V)W\\&+S(\xi,\xi)R(Y,V)W - S(Y,V)R(\xi,\xi)W + S(\xi,V)R(\xi,Y)W\\&-S(Y,W)R(\xi,V)\xi + S(\xi,W)R(\xi,V)Y = 0.\end{aligned} \tag{5.3}$$

Applying (2.4) and (2.7) in (5.3), we get

$$\begin{aligned}&-k\eta(W)S(Y,V)\xi - \mu\eta(W)S(Y,hV)\xi - 2k^2g(V,W)Y - 2k\mu g(hW,V)Y\\&+2kR(Y,V)W + 2k^2\eta(V)g(Y,W)\xi + 2k\mu g(hW,Y)\eta(V)\xi - 2k\mu\eta(V)\eta(W)hY\\&-k\eta(V)S(Y,W)\xi + kS(Y,W)V + \mu S(Y,W)hV + 2k^2\eta(W)g(V,Y)\xi\\&+2k\mu\eta(W)g(hY,V)\xi = 0.\end{aligned} \tag{5.4}$$

Taking inner product with T, we obtain

$$\begin{aligned}&-k\eta(W)S(Y,V)\eta(T) - \mu\eta(W)S(Y,hV)\eta(T) - 2k^2g(V,W)Y\\&-2k\mu g(hW,V)g(Y,T) + 2kg(R(Y,V)W,T) + 2k^2\eta(V)g(Y,W)\eta(T)\\&+2k\mu g(hW,Y)\eta(V)\eta(T) - 2k\mu\eta(V)\eta(W)g(hY,T) - k\eta(V)S(Y,W)\eta(T)\\&+kS(Y,W)g(V,T) + \mu S(Y,W)g(hV,T) + 2k^2\eta(W)g(V,Y)\eta(T)\\&+2k\mu\eta(W)g(hY,V)\eta(T) = 0.\end{aligned} \tag{5.5}$$

Let $\{e_i\}$, $i = 1,2,3$ be a local orthonormal basis in the tangent space T_PM at each point p $\in M$. Substituting $Y = T = e_i$ in (5.5) and summing over $i = 1$ to 3, we have

$$-6k^2g(Y,T) + 3kS(Y,T) - 2k\mu g(hY,T) + \mu S(hY,T) = 0 \tag{5.6}$$

Putting $Y = hY$ in (5.6), we get

$$-6k^2g(hY,T) + 3kS(hY,T) - 2(k+1)k\mu g(Y,T) + \mu(k+1)S(Y,T) = 0. \tag{5.7}$$

Multiplying (5.6) by $3k$ and (5.7) by μ and then subtracting the result we have

$$(9k^2 - \mu^2(k+1))\{S(Y,T) - 2kg(Y,T)\} = 0. \tag{5.8}$$

Then either $9k^2 - \mu^2(k+1) = 0$ or, $S(Y,T) = 2kg(Y,T)$.
Thus we can state the following:

Theorem 5.1. *If a generalized (k, μ)-paracontact metric manifold satisfy the condition $Q(S, R) = 0$, then the manifold is an Einstein manifold, provided $9k^2 - \mu^2(k+1) \neq 0$*

6. GENERALIZED (k, μ)-PARACONTACT METRIC MANIFOLDS SATISFYING $Q(g, S)=0$

In this section we investigate generalized (k, μ)-paracontact metric manifolds satisfying $Q(g, S) = 0$. Therefore

$$(X \wedge_g Y \cdot S)(U, V) = 0 \tag{6.1}$$

Using (1.6)in (6.1), we get

$$-g(Y, U)S(X, V) + g(X, U)S(Y, V) - g(Y, V)S(U, X) + g(X, V)S(U, Y) = 0. \tag{6.2}$$

Substituting $X = U = \xi$, we obtain

$$-g(Y, \xi)S(\xi, V) + g(\xi, \xi)S(Y, V) - g(Y, V)S(\xi, \xi) + g(\xi, V)S(\xi, Y) = 0. \tag{6.3}$$

Applying (2.4) and (2.7) in (6.3), we get

$$S(Y, V) - 2kg(Y, V) = 0. \tag{6.4}$$

This leads to the following:

Theorem 6.1. *If a generalized (k, μ)-paracontact metric manifold satisfy the condition $Q(g, S) = 0$, then the manifold is an Einstein manifold.*

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