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Abstract—**Large scale integration of fluctuating and non-dispatchable generation and variable transmission patterns induce high uncertainty in power system operation. In turn, transmission system operators (TSOs) need explicit information about available flexibility to maintain a desired reliability level at a reasonable cost. In this paper,** *locational flexibility* **is defined and a unified framework to compare it against forecast uncertainty is introduced. Both metrics are expressed in terms of ramping rate, power and energy and consider the network constraints. This framework is integrated into the operational practice of the TSO using a robust reserve procurement strategy which guarantees optimal system response in the worst-case realization of the uncertainty. An illustrative three-node system is used to investigate the procurement method. Finally, the locational flexibility for a larger test system is presented.**

Keywords—*Operational flexibility, reserve procurement, robust optimization, uncertainty.*

Nomenclature

t	Index of time periods, from 1 to N_T.
k	Index of generating unit, from 1 to N_K.
ω	Index of wind power scenarios, from 1 to N_Ω.
$\otimes^{\min}$	Lower bound of value $\otimes$.
$\otimes^{\max}$	Upper bound of value $\otimes$.
$\hat{\otimes}$	Scheduled operation point of value $\otimes$.
C	Storage capacity.
η_l, η_g	Charge and discharge efficiencies.
$x^{k,t}$	State of charge of unit k in period t.
$P_{\text{gen}}^{k,t}$	Power production of unit k in period t.
$P_{\text{load}}^{k,t}$	Power consumption of unit k in period t.
ω, υ, ξ	Energy curtailment, storage losses and consumption of primary energy carrier.
$P_l^{\max}$	Vector of maximum transmission capacities.
H	Power transfer distribution factor (PTDF).
B	Mapping of the set of generating units into the set of buses.
C_{proc}	Vector of flexibility capacity offer prices.
C_{op}	Vector of operational costs.

I. Introduction

In recent years, power systems have undergone significant changes primarily due to developments in the generation mix and the electricity market. Owing to the high shares of fluctuating renewable energy sources (RES), e.g., wind power, system operation is subject to increased *variability*, i.e., random fluctuations of the production driven by the ambient conditions, as well as *uncertainty*, i.e., partial predictability of the actual power output [1]. Meanwhile, the establishment of common electricity markets covering large geographical areas leads to high cross-border power flows with changing patterns driven by price differences over the network. The aforementioned challenges require system operators to incorporate flexibility metrics in their decision-making process and account for the stochastic behavior of the system parameters. Flexibility in power systems has been discussed in various publications. In [2] a method to estimate the probability of insufficient ramping capabilities is presented and [3] tries to optimize the flexibility of a generation mix.

For the scope of this paper, we define the following terms:

Definition 1: Operational flexibility is the ability of a power system to contain a disturbance sufficiently fast in order to keep the system secure.

Most common disturbances are component outages, such as line or generator trippings, or a deviation of power injection, e.g., due to forecast errors. In the context of this work, operational flexibility is determined in terms of ramping rate (R), power capacity (P) and energy limitations (E) as proposed in [4]. However, operational flexibility may differ depending on the grid location and the current network utilization. Therefore, we introduce also the term locational flexibility as:

Definition 2: Locational flexibility is the operational flexibility that can be accessed at a given bus in the grid. It describes the disturbance at a given node of the system that could be contained by suitable and available remedial actions.

Suitable actions comprise re-dispatching measures such as deployment of reserves, demand side participation as well as changes in network topology and power flow set-points.

In this paper, we focus on locational flexibility and demonstrate its relevance in power systems with increased uncertainties. The contributions of this paper are twofold: *i)* to introduce a unified framework for the characterization of uncertainty and available locational flexibility; *ii)* to formulate a robust reserve procurement method to guarantee sufficient flexibility under the worst case realization of the uncertainty.

Operational flexibility has been characterized using various different metrics, e.g. [5], but usually transmission constraints are neglected. The authors of [6] propose a flexibility metric, which accounts for transmission limits. Here we formulate a flexibility set that describes the system flexibility limits based on individual contributions from all units and respects transmission limits. In accordance with the properties of this flexibility metric, an uncertainty set that encapsulates the plausible realizations of the pertaining stochastic processes is constructed. The main advantage of this uncertainty characterization, compared with analogous frameworks that focus solely on the power output stochasticity, e.g., statistical scenarios, is that it provides explicit information for the flexibility needs expressed in terms of $[R, P, E]$ and thus enables its direct comparison with the flexibility capabilities in each grid location.

The proposed framework can be readily applied by a transmission system operator (TSO) in order to assess whether the available locational flexibility is sufficient to cover the predicted uncertainty (visualization and monitoring). To incorporate the interaction between flexibility and uncertainty into the dispatch decision-making of the TSO, we propose a two-stage adaptive robust optimization model for the procurement of control reserves. Similar applications of adaptive robust optimization in power systems are presented in [7], which however do not explicitly account for ramping and energy limitations.
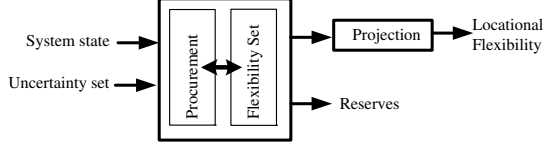

Fig. 1. Overview of the method presented in this paper.

In Fig. 1 an overview of the method presented in this paper is shown. Based on a given system state and a description of uncertainty described in Section III sufficient reserves are procured (Section IV) such that the flexibility set defined in Section II is sufficiently large. In Section V, we use a projection method to determine the locally available flexibility.

II. MODELING OF OPERATIONAL FLEXIBILITY

Following the definitions of section I, we present a methodology that allows to express locational flexibility in terms of $[R, P, E]$ and consists of the following steps.

Step 1: Generic Modeling of the Flexibility of a Single Unit

Based on a generic modeling framework, which allows to model different types of units, we determine the available up-/down- ramping rate, the maximum allowed deviations from the scheduled power outputs as well as the additional energy that can be stored/discharged over the considered time horizon. The size of the deviations depends on the scheduled system state, i.e., the scheduled production of the generation units and the demand which is the result of an economic dispatch or a market operation.

The dynamic behaviour of a wide range of different generators, storages as well as loads can be approximately represented using the first-order discrete-time differential equation as

$$C\left(x^{k,t+1} - x^{k,t}\right) = \eta_l P_{\text{load}}^{k,t} - \frac{1}{\eta_g} P_{\text{gen}}^{k,t} + \omega^{k,t} + v^{k,t} + \xi^{k,t}. \quad (1)$$

Details and examples to be found in [8]. The left side of Eq. (1) corresponds to the change in the normalized state of charge $x^{k,t}$ of the unit, i.e., $0 \leq x^{k,t} \leq 1$. The unit's storage capacity is C. The right-hand side represents the net power exchange of the unit with the grid and the primary fuel source ξ as well as terms for curtailment and storage losses.

The net infeed to the grid, denoted as $P^{k,t}$, can be split into two components as

$$P^{k,t} = P_{\text{gen}}^{k,t} - P_{\text{load}}^{k,t} = \hat{P}^{k,t} + \Delta P^{k,t}, \quad (2)$$

where $\hat{P}^{k,t} = \hat{P}_{gen}^{k,t} - \hat{P}_{load}^{k,t}$ are the scheduled contributions and $\Delta P^{k,t}$ the deviations from initial dispatch. The flexibility a unit can provide is given by the allowed deviations, in terms of ramping rate ΔR, power ΔP and energy ΔE. The bounds on possible deviations can be expressed through constraints

$$\hat{E}^k + \Delta E^k = \sum_{t=1}^{t_N-1} t_s \left(\hat{P}^{k,t} + \Delta P^{k,t}\right) \in \mathcal{U}$$

$$\hat{P}^{k,t} + \Delta P^{k,t} = P_{\text{gen}}^{k,t} - P_{\text{load}}^{k,t} \in \mathcal{U}, \ \forall t = 1 \ldots N_T \quad (3)$$

$$\hat{R}^{k,t} + \Delta R^{k,t} = \frac{1}{t_s}\left(P^{k,t+1} - P^{k,t}\right) \in \mathcal{U}, \ \forall t = 1 \ldots N_T - 1,$$

where N_T is the number of time steps of the considered horizon in the flexibility calculation and t_s the corresponding time step length. The energy E is given by the sum over all time steps. The operational constraints $\mathcal{U}$, depending on each unit type, are defined by the following set of equations:

$$0 \leq P_{\text{load}}^{k,t} \leq P_{\text{load}}^{k,max}, \qquad \forall t = 1 \ldots t_N$$
$$0 \leq P_{\text{gen}}^{k,t} \leq P_{\text{gen}}^{k,max}, \qquad \forall t = 1 \ldots t_N$$
$$x^{\text{min}} \leq x^{k,t} \leq x^{\text{max}}, \qquad \forall t = 1 \ldots t_N \quad (4)$$
$$R_{\text{load}}^{\text{min},k} \leq P_{\text{load}}^{k,t+1} - P_{\text{load}}^{k,t} \leq R_{\text{load}}^{\text{max},k}, \quad \forall t = 1 \ldots t_N - 1$$
$$R_{\text{gen}}^{\text{min},k} \leq P_{\text{gen}}^{k,t+1} - P_{\text{gen}}^{k,t} \leq R_{\text{gen}}^{\text{max},k}, \quad \forall t = 1 \ldots t_N - 1.$$

Units with storage capabilities, e.g., batteries or pumped hydro plants, may face energy limitations. In that case, the evolution of the state of charge x influences the available flexibility. The dynamics of the storage are integrated according to Eq. (1).

Step 2: Generic Disturbance Modeling

In order to model a disturbances in the system, we now define a *generic disturbance*, which can be considered as a unit with the following basic properties:

$$\Delta R^{k,t} = \frac{1}{t_s}\left(\Delta P^{k,t+1} - \Delta P^{k,t}\right), \quad \forall t = 1 \ldots t_N$$

$$\Delta E = \frac{t_s}{2} \sum_{t=1}^{t_N-1} \left(\Delta P^{k,t+1} + \Delta P^{k,t}\right). \quad (5)$$

The disturbance is modeled without explicit limits on the ramping, power and energy capacity, but given that the system as a whole has to stay stable, which is ensured by Step 3, the limitations on the disturbance are given implicitly by constraints of other units and the transmission grid.

Step 3: Grid Modeling

The transmission grid is modeled by the standard power flow equations: i) active power balance ensuring that the consumption and production are equal and ii) transmission limits are not violated. We use a linearized power flow formulation.

The sum of all active power injections and consumptions, including the disturbance, need to be balanced at all times. As the scheduled system state without any deviation is assumed to be selected such that the system is stable, the sum of all deviations need to be zero:

$$\sum_{k=1}^{N_K} \Delta P^{k,t} = 0, \quad \forall t = 1 \ldots N_T. \quad (6)$$

The set of transmission constraints, i.e., maximum transmission line loading, are written as

$$-P_l^{\text{max}} \leq HB\left(\hat{P}^t + \Delta P^t\right) \leq P_l^{\text{max}}, \quad \forall t = 1 \ldots N_T, \quad (7)$$

where the PTDF matrix H represents the sensitivity of bus injections on power flows [9]. Other constraints, such as the N-1 security criterion, could easily be integrated. The inclusion of the grid can be adapted depending on the study, e.g., if transmission capacity is sufficiently available in all operating conditions, one might consider to model the grid as a copperplate and Eq. (7) can be omitted.

Step 4: Flexibility Set

Equality constraints can easily be rewritten as inequality constraints. Thus, the constraints from steps 1-3 can be compiled as a linear matrix inequality of the form

$$F = \{(\delta, f_s) \in \mathbb{R}^{n_d + n_s} | C_s f_s + C_d \delta \leq b\}, \qquad (8)$$

where the vector f_s corresponds to all state variables associated with units providing flexibility and the vector δ contains the state variables associated with the generic disturbances.

$$f_s = \begin{bmatrix} \mathbf{x} \\ \mathbf{\Delta P} \\ \mathbf{\Delta P_{gen}} \\ \mathbf{\Delta P_{load}} \end{bmatrix}, \quad \delta = \begin{bmatrix} \mathbf{\Delta R} \\ \mathbf{\Delta P} \\ \mathbf{\Delta E} \end{bmatrix}. \qquad (9)$$

The vectors are stacked versions of the corresponding state variables of all units k and all time steps t considered. The states $x^{k,t}, \Delta P_{gen}^{k,t}, \Delta P_{load}^{k,t}$ are only needed for units with storage. For conventional units, these state variables can be discarded. The generic disturbance needs the states $\Delta R^{k,t}$ and ΔE^k. Multiple generic disturbances can be attached to different buses. In that way, also the relations between different disturbances can be investigated, e.g. the relations of the maximum power of a disturbance at two different locations.

This inequality can be formulated considering multiple time steps and one may call F the *flexibility set* of the system, where n_d and n_s are the dimensions of the vector and C_s, C_d, b are appropriately stacked versions of the constraints. The polytope F contains all possible setpoints of the system that are stable.

The flexibility set will now be applied in two applications taking different perspectives: in section III, possible disturbances are pre-defined and one wants to determine the needed flexibility to contain these disturbances. In section V, we reverse the problem and determine the set of disturbances that the system can cope with for a given available flexibility.

III. RESERVE PROCUREMENT WITH EXPLICIT FLEXIBILITY NEEDS

To demonstrate this methodology we formulate a robust reserve procurement problem. Our goal here is to procure an appropriate set of reserves Δb_i, in addition to day-ahead dispatch b_0, in order to guarantee that the flexibility set F is sufficiently large, i.e., the system will be able to respond optimally to any realization of the stochastic parameters within the uncertainty set $\mathcal{W}$.

The reserve procurement problem is formulated as

$$\min_{\Delta b_i, f_s} C_{proc}^T \Delta b_i + \mathcal{L}(\Delta b_i) \qquad (10)$$

$$\text{s.t. } \Delta b_i^{min} \leq \Delta b_i \leq \Delta b_i^{max}, \qquad (11)$$

where

$$\mathcal{L}(\Delta b_i) = \max_{\delta} \quad \min_{f_s} C_{op}^T f_s \qquad (12)$$

$$\text{s.t. } C_s f_s + C_d \delta \leq b_0 + \Delta b_i : \mu \quad (13)$$

$$\text{s.t. } \quad \delta \in \mathcal{W} \qquad (14)$$

This is an adaptive robust optimization problem where the objective function (10) to be minimized is the sum of the reserve procurement cost with the balancing operation cost under the worst-case realization of the uncertainty $\mathcal{L}(\Delta b_i)$. The first-stage decisions Δb_i, represent the reserve procurement and constraints (11) enforce the bounds of the available reserves. The second-stage variables (recourse actions) account for the balancing dispatch f_s, as a response to the uncertainty realization δ. The $max-min$ programming problem (12)-(14) finds the minimum balancing cost for the worst-case realization of the uncertainty. Constraint (13) ensures the feasibility of the balancing dispatch for every $\delta \in \mathcal{W}$ given in (14). Constraint (13) incorporates the flexibility set as in Eq. (8).

Considering that the above optimization problem cannot be solved directly given its $min - max - min$ structure, we reformulate the $max - min$ problem according to [7] using the dual of the right-hand side problem which is written as

$$\max_{\mu} (C_d \delta - \Delta b_i - b_0)\mu$$

$$\text{s.t.} \begin{cases} -C_s^T \mu \leq C_{op}^T \\ \mu \geq 0. \end{cases} \qquad (15)$$

Hence, we obtain the following $min-max$ optimization problem, where we have merged the two maximization problems into a single problem with decision variables δ and μ.

$$\min_{\Delta b_i} C_{proc}^T \Delta b_i + \max_{\delta, \mu} C_d \delta \mu + (\Delta b_i - b_0)\mu$$

$$\text{s.t.} \begin{cases} -C_s^T \mu \leq C_{op}^T \\ \mu \geq 0 \\ \delta \in \mathcal{W} \end{cases} \qquad (16)$$

$$\text{s.t. } \Delta b_i^{min} \leq \Delta b_i \leq \Delta b_i^{max}.$$

It can be noted that the objective function of (16) includes a cross-product of the variables δ and μ and thus the resulting optimization problem is bilinear. If the uncertainty set $\mathcal{W}$ is a polyhedral set, then the optimal solution will be one of the vertices of this set. In addition, since the first-stage variables Δb_i do not appear in the constraints of the bilinear problem, the feasible polyhedron has a finite number of vertices $v = A, ..., H$, i.e., the feasible polyhedron in independent of the first-stage decisions. Hence, the model can be further reformulated introducing the auxiliary variable C_{op}^{wc} representing the worst-case resource cost $\mathcal{L}(\Delta b_i)$, equal to the optimal objective function of the $max - min$ problem (12)-(14). This model could also be viewed as an *extension* to prevalent procurement procedures, i.e., every realization of the uncertainty is checked whether it can be covered by the flexibility set given by the procured amount.

$$\min_{\Xi} \quad C_{proc}^T \Delta b_i + C_{op}^{wc}$$

$$\text{s.t.} \quad C_{op}^{wc} \geq C_{op}^T f_{s,v}, \; \forall v = A, ..., H$$

$$C_s f_{s,v} + C_d \delta_v \leq b_0 + \Delta b_i, \; \forall v = A, ..., H \qquad (17)$$

$$\Delta b_i^{min} \leq \Delta b_i \leq \Delta b_i^{max},$$

where $\Xi = \{\Delta b_i, C^{wc}_{op}, f_{s,v}\}$ is the set of optimization variables. The solution of model (17) requires the enumeration of all the vertices v of the uncertainty set $\mathcal{W}$, which may result in intractable problems if the number of vertices becomes very large. However, the structure of this problem allows to employ more efficient solution schemes, e.g., based on Benders Decomposition [10]. In this case, the optimal solution of the procurement problem can be obtained iteratively, adding a Benders cut for each vertex of the uncertainty set until the convergence criterion is met.

IV. Modeling of Explicit Flexibility Needs

The operation of the power system is inherently related with stochasticity both in production and consumption due to partly predictable energy sources and load. In turn this uncertainty translates into flexibility needs for the power system which should be taken into account during the stage of reserve procurement. Here we focus on the uncertainty arising from the fluctuating in-feed of RES, but other sources of uncertainty, e.g., load deviations or equipment failures (N-1 security criterion), can be also represented. Aiming to express the flexibility needs in a common framework as the locational flexibility, using the $[R, P, E]$ metric, we construct a polytope of the form $S\delta \leq h$ bounding the disturbances δ based on uncertainty description in form of scenarios. If these scenarios respect the spatio-temporal dependence structure of the prediction errors, e.g., for a number of wind farms in several locations and multiple forecast lead times, the resulting polytope would preserve this information. A complete methodological framework for the generation of spatio-temporal scenarios is provided in [11].

Following an approach similar to [12], the uncertainty set is constructed using a set of $\omega \in \Omega$ scenarios. The flexibility metrics for each time interval $t - t + 1$ are calculated as

$$
\begin{aligned}
R_{t,\omega} &= (P_{t+1,\omega} - P_{t,\omega})/t_s \\
P_{t,\omega} &= P_{t+1,\omega} - \hat{P}_t \\
E_{t,\omega} &= \left[(P_{t+1,\omega} - \hat{P}_t) + (P_{t,\omega} - \hat{P}_t) \right] t_s/2
\end{aligned}
\tag{18}
$$

and a cloud of N_Ω points is obtained in a space with coordinates R, P and E. The uncertainty set $\mathcal{W}$ is defined as the convex hull (Fig. 2) of these points constructed using the Quickhull Algorithm [13].

Since the dispatch of the wind turbine influences the possible deviations, it should be noted that the uncertainty set is constructed based on the expected system state, i.e., $\hat{P}$ is equal to the conditional mean forecast. In general, higher forecast lead-time is expected to increase the volume of the corresponding uncertainty sets due to higher forecast uncertainty, as reflected by the wider range of scenarios in Fig. 2. In addition, the number vertices of the convex hull may increase significantly if a larger amount of wind power locations and time steps is considered. This problem can be tackled by applying scenario reduction techniques [14] on the initial scenario set Ω.

V. Locational Flexibility

For a given flexibility set, we determine the flexibility that is available at a selected bus of the system. This *locational flexibility* will explicitly be determined and characterizes the
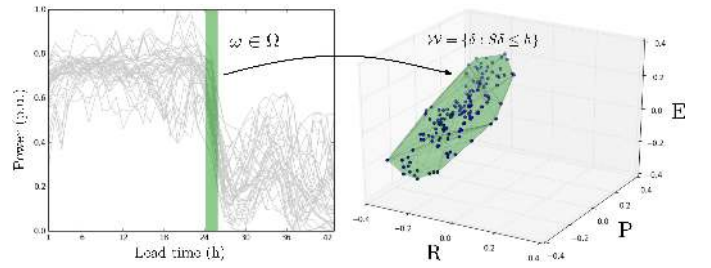

Fig. 2. From spatio-temporal scenarios to uncertainty set (convex hull) for a period with predicted steep ramping-down event. For illustration, only one time interval (hours 24-25) is displayed.

disturbances that could be balanced at the selected node. It is described by the set F_d as

$$
\begin{aligned}
F_d &= \{\delta \in \mathbb{R}^{n_d} | \exists f_s, (\delta, f_s) \in F\} \\
&= \{\delta \in \mathbb{R}^{n_d} | G\delta \leq g\},
\end{aligned}
\tag{19}
$$

which contains all the possible values of δ such that there exists a vector (δ, f_s) that points in the flexibility set. The set F_d is determined by the projection of the polytope F from a high-dimensional space into the space spanned by the elements of δ, using the equality set projection method described in detail in [15], [16]. The method is illustrated in Fig. 3. For example, the projection of F on the dimensions associated to $\Delta P^{k,t=\{1,2,3\}}$ of the generic disturbance results in the possible combinations of power deviations over the time steps 1 to 3.
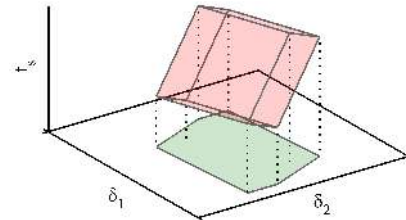

Fig. 3. Illustration of the projection method: Projecting the flexibility set F (red polytope) on the axes of the generic disturbance (δ_1, δ_2) results in feasible combinations of disturbances (green polytope). Axis f_s represents all the remaining dimensions of the flexibility set.

VI. Case Studies

The case study section has four parts, where for the first three parts we use a 3-bus system in combination with the procurement algorithm. This simple system allows a straight-forward investigation of the influence of different parameters. For the last part, we use the IEEE RTS96 bus system and calculate the locational flexibility in two different locations.

A. Setup

The model in Fig. 4 consists of three buses, where each bus is reduced to a predominant characteristic: bus 1 has units with storage capabilities, e.g. pumped-hydro storage power plants with high ramping rates. The conversion efficiencies are selected to be 90% per conversion, ergo 81% for a storage cycle. Bus 2 represents fluctuating energy sources and bus 3 corresponds to a load center where also conventional generation units are found with low ramping rates. The load is assumed to be inflexible. The uncertainty arises from uncertain production at bus 2. Curtailment is available up to a predefined percentage of the current production. As the grid represents a simplified system with three zones, the grid is modeled

Unit	$P^{\min}$, $\hat{P}$, $P^{\max}$	$R^{\min}$, $R^{\max}$	Procurement Costs $\Delta E, \Delta P, \Delta R$
Storage	0.05,0.50,1	-1,1	0,10,1
Wind farm	0,1,1	-1,1	0,2,1
Conv. Gen.	0,1,2	-0.05, 0.05	0,3,50
Load	2,2,2	-	-

using a transport model, i.e., up to the transmission limits, the energy can be moved on the interconnections. The data of the units and the grid are summarized in Tab. I. The costs for the flexibility procurement in C_{proc}, i.e. the costs for reserving ΔE, ΔP or ΔR at some units are selected such that costs of conventional units has low power reservation costs but high ramping costs. Storage units have low ramping costs but high power reservation costs. All values are given in per-unit, where 1 p.u. corresponds to 1000MVA. The wind power scenarios used to construct the uncertainty set $\mathcal{W}$ according to method presented in Section IV are based on a publicly available dataset provided by the Australian Energy Market Operator (AEMO) and further descriptions can be found in [11].
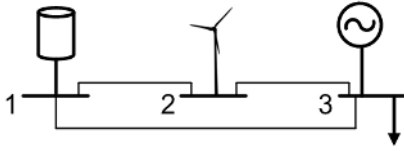

Fig. 4.   Three bus system with flexible units with storage capabilities at bus 1, intermittent energy sources at bus 2 and load and inflexible conventional generation. Transmission capacity is variable and curtailment at bus 2 is available up to a predefined percentage of the current production.

B. Procurement Costs

Curtailment of intermittent energy sources and units with storage capacities (e.g. pumped hydro storage) are two possibilities to add flexibility to the system operation. In this case study, we vary the amount of wind energy that can be curtailed (in percent of current production) as well as the storage capacity available at bus 1. In Fig. 5 (Left) we investigate the capacity costs for the procured reserves as a function of the share that can be curtailed and the size of the storage at bus 1. The storage is charged to 50% of the capacity. The reserves are procured according to (17) using a given uncertainty set. The costs are given by $c_{\text{proc}}^T \Delta b_i$. It can be observed that the costs decrease in the given system for increasing storage sizes as well as the possibility to curtail intermittent infeeds. The least costs are achieved by a suitable combination of both remedies.
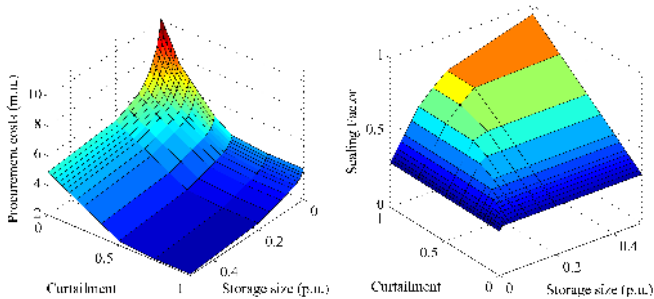

Fig. 5.   Left: Capacity reservation costs as a function of curtailment and storage size. Right: Scaling factors of normalized feasible disturbances for different curtailment possibilities and storage sizes. For illustration purposes, the axes are rotated.

C. Resilience against disturbances

In different applications the knowledge on disturbances that can be balanced at a certain bus is essential. For example a TSO wants to know whether the available reserves are sufficient for the forecasted system states or how investment in grid infrastructure influences the flexibility. Exemplarily, we determine the largest scaling factor with which the uncertainty set can be linearly scaled without compromising the security. Fig. 5 (Right) shows the scaling factors as a function of the share that is allowed to be curtailed and the size of the storage. It can be seen that the disturbance can be increased substantially if curtailment and the storage size are increased, but similarly to the previous case study, a proper trade-off between has to be made between the two options.

D. Influence of Transmission Capacity on Flexibility

The available transmission capacity influences the flexibility. The locational flexibility is determined using the projection method from section V. As long the transmission capacity is large, the locational flexibilities will not differ much from bus to bus. However in the case of congested lines the flexibility might reduce. In Fig. 8 the locational flexibility at bus 2 is shown for three different transmission capacities. Illustrated are the feasible deviations from the scheduled power injections over three time steps. This could be considered as the bounds on the evolution of a forecast error of the power injection, such that the system remains stable. For simplicity we assume that after the system is dispatched, all the remaining capacity can be used for balancing. We consider three cases: *i*) unlimited transmission capacity between the buses, i.e., "copperplate", *ii*) transmission capacity limited to 0.4 p.u. between 1-2 and 1-3 and *iii*) same situation but capacity limited to 0.3 p.u.
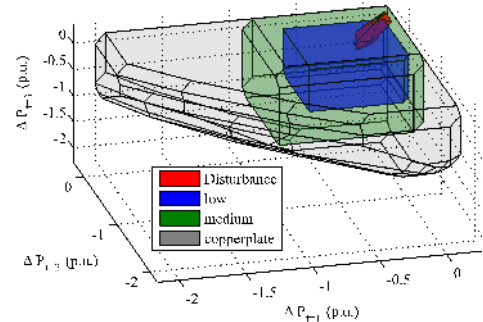

Fig. 6.   Locational flexibility at bus 2 for different transmission capacities compared to possible disturbances.

In red, the uncertainty is shown. In order for the system to cope with all possible disturbances, the flexibility set has to cover the polytope bounding all the possible disturbances. It can be observed, that for the case with little transmission capacity, not all deviations in the red polytope could be contained. In the case of unlimited transmission capacity the locational flexibility is substantially larger, i.e. the locational flexibility at this bus could be increased by increasing the transmission capacities or by taking measures to increase the controllability directly, e.g. allowing more wind energy to be curtailed. Further, the origin is contained in all polytopes, as no disturbances corresponds to the scheduled operation which has to be feasible.

E. Locational Flexibility

Similar to the previous case study, we determine the available flexibility for the IEEE RTS96 system with 2 zones [17] (Fig. 7). In Fig. 8 the locally available flexibility at the central bus 217 and a peripheral bus 207 is shown. Additionally, for the flexibility at bus 217, the transmission capacity of the tie-lines is reduced to a third of the original capacity (congested case). The ramping capabilities per time step are assumed to be either 100% of the installed capacity or 10%. One observes a strong dependence between the available flexibility and the transmission capacity as well as the ramping availability at bus 217. Although for the flexibility at bus 207 no reduction in ramping or transmission capacity is assumed, the flexibility is still comparably low due to its peripheral location.
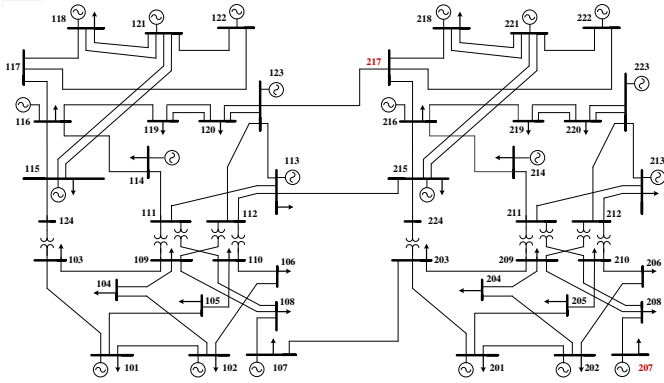

Fig. 7. IEEE RTS96 system with two zones that are connected with three tie-lines. The locational flexibility is determined for the central bus 217 and the peripheral bus 207.
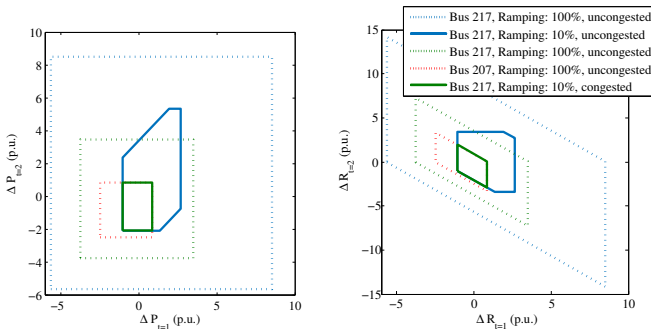

Fig. 8. Locational flexibility at buses 217 and 207 for time steps $t = \{1, 2\}$.

VII. CONCLUSION

This paper introduces the term locational flexibility to define the ability of the system to contain a certain disturbance at a given node in terms of ramping rate, power and energy. In addition, a unified framework to quantify and compare the available flexibility with the forecast uncertainty is formulated. The proposed framework is integrated in the operational strategy of the TSO through a robust procurement algorithm that guarantees sufficient locational flexibility for the worst-case realization of the uncertainty. The locational flexibility is then determined by a projection method. The case studies show that the flexibility vary for different locations in the grid and that storage and curtailment could be efficient ways to locally increase the flexibility.

Future work will investigate the scalability of the proposed procurement algorithm as well as further applications of the flexibility set in operational and electricity market applications.

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