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ON REAL HYPERSURFACES OF A COMPLEX SPACE FORM WITH η-PARALLEL RICCI TENSOR

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Introduction.

Let $M_n(c)$ denote an n-dimensional complex space form with constant holomorphic sectional curvature c. It is well known that a complete and simply connected complex space form consists of a complex projective space CP^n, a complex Euclidean space C^n or a complex hyperbolic space CH^n, according as $c>0$, $c=0$ or $c<0$. In this paper we consider a real hypersurface M of CP^n or CH^n.

The study of real hypersurfaces of CP^n was initiated by Takagi [10], who proved that all homogeneous hypersurfaces of CP^n could be divided into six types which are said to be of type A_1, A_2, B, C, D and E. Moreover, he showed that if a real hypersurface M of CP^n has two or three distinct constant principal curvatures, then M is locally congruent to one of the homogeneous ones of type A_1, A_2 and B ([11]). Recently, to give another charac terization of homogeneous hypersurfaces of type A_1, A_2 and B in CP^n Kimura and Maeda [6] introduced the notion of an η-parallel second fundamental form, which was defined by $g((\nabla_X A)Y, Z)=0$ for any vector fields X, Y and Z orthogonal to the structure vector field ξ, where A means the second fundamental form of M in CP^n, and g and ∇ denote the induced Riemannian metric and the induced Riemannian connection, respectively.

On the other hand, real hypersurfaces of CH^n have also been investigated by many authors (Berndt [1], Montiel [8], Montiel and Romero [9]).

Using some results about focal sets, Berndt [1] proved the following.

THEOREM A. *Let M be a connected real hypersurface of $CH^n(n\geqq 2)$. Then M has constant principal curvatures and ξ is principal if and only if M is locally congruent to one of the following.*

(A_0) *a horosphere in CH^n.*

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(A_1) *a geodesic hypersphere or a tube over a complex hyperbolic hyperplane* CH^{n-1}.

(A_2) *a tube over a totally geodesic submanifold* CH^k *for* $k=1, \cdots, n-2$.

(B) *a tube over a totally real hyperbolic space* RH^n.

It is necessary to remark that real hypersurfaces of type A_0 or A_1 appearing in Theorem A, are *totally* η*-umblical* hypersurfaces with two distinct constant principal curvatures. In the paper of Montiel [7] the real hypersurface of type A_0 in Theorem A is said to be self-tube.

In §3 we also consider the η*-parallel* second fundamental form in CH^n and give a further characterization of type A_0, A_1, A_2, and B in CH^n. Now we introduce the notion of an η*-parallel* Ricci-tensor of M in $M_n(c)$, $c\neq 0$, which is defined by $g((\nabla_X S)Y, Z)=0$ for any X, Y, and Z orthogonal to ξ, where S is the Ricci-tensor of M in $M_n(c)$, $c\neq 0$. It is easily seen that if the second fundamental form is η*-parallel*, then so is the Ricci-tensor, under the condition such that ξ is principal. Thus the purpose of this paper is to investigate this converse problem. By using the classification theorem due to Takagi [10] and Kimura and Maeda [6], we get the following.

THEOREM B. *Let* M *be a real hypersurface of* CP^n. *Then the Ricci-tensor of* M *is* η*-parallel and* ξ *is principal if and only if* M *is locally congruent to one of homogeneous real hypersurfaces of type* A_1, A_2 *and* B.

By applying the Theorem A we can also prove the following.

THEOREM C. *Let* M *be a real hypersurface of* $CH^n (n\geqq 2)$. *Then the Ricci-tensor of* M *is* η*-parallel and* ξ *is principal if and only if* M *is locally congruent to one of type* A_0, A_1, A_2 *and* B.

§1. Preliminaries.

Let M be a real hypersurface of a complex n-dimensional complex space form $M_n(c)$, and let C be its unit normal vector field. Since $M_n(c)$ admits an almost complex structure, let us denote by F its almost complex structure. For any tangent vector field X and normal vector field C on M, the transformations of X and C under F can be given by

$$FX=\phi X+\eta(X)C, \qquad FC=-\xi,$$

where ϕ defines a skew-symmetric transformation on the tangent bundle TM of