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On Restricting to One Loop Order the Radiative Effects in Quantum Gravity

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The dimensionful nature of the coupling in the Einstein-Hilbert action in four dimensions implies that the theory is non-renormalizable; explicit calculation shows that beginning at two loop order, divergences arise that cannot be removed by renormalization without introducing new terms in the classical action. It has been shown that, by use of a Lagrange multiplier field to ensure that the classical equation of motion is satisfied in the path integral, radiative effects can be restricted to one loop order. We show that by use of such Lagrange multiplier fields, the Einstein-Hilbert action can be quantized without the occurrence of non-renormalizable divergences. We then apply this mechanism to a model in which there is in addition to the Einstein-Hilbert action, a fully covariant action for a self-interacting scalar field coupled to the metric. It proves possible to restrict loop diagrams involving internal lines involving the metric to one-loop order; diagrams in which the scalar field propagates occur at arbitrary high order in the loop expansion. This model also can be shown to be renormalizable. Incorporating spinor and vector fields in the same way as scalar fields is feasible, and so a fully covariant Standard Model with a dynamical metric field can also be shown to be renormalizable

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I. INTRODUCTION

Removing divergences arising from loop momentum integrals is a particularly acute problem in quantum gravity due to the dimensionful nature of the coupling. The divergences occurring at one-loop order when using the Einstein-Hilbert action can be removed by a field redefinition on account of the divergences vanishing if the equations of motion are satisfied if the Gauss-Bonnet identity is used [1, 2], but once the metric interacts with a scalar [1], vector [3] or spinor [4] field this is no longer possible even at one-loop order [1]. Not even the Einstein-Hilbert action by itself is renormalizable, in the power-counting sense, beyond one-loop order [6, 7]. It is well known [8, 9], that quantum gravity based on the Einstein-Hilbert action is renormalizable if there is a counter-term available to cancel every ultraviolet divergence. However, this procedure requires an infinite number of counter-terms, which lessens the predictive power of the theory.

A way has been found to eliminate all radiative effects beyond one-loop order in the loop expansion. This has been illustrated in Yang-Mills theory [10] and the Proca model [11]. By using a Lagrange multiplier field to impose the condition that when evaluating the quantum path integral, only field configurations that satisfy the classical equations of motion contribute and one no longer encounters radiative effects beyond one loop. The tree-level diagrams are reproduced and the one-loop contribution is twice that of the usual one-loop diagrams that occur without this Lagrange multiplier field; all contributions beyond one-loop order are absent. The problem of showing renormalizability is thus greatly simplified as only one-loop effects need to be considered. This procedure is also consistent with unitarity.

We first show how this approach using a Lagrange multiplier field can be used in conjunction with the Einstein-Hilbert action alone. In this case, upon using the Gauss-Bonnet theorem, the divergences arising from one-loop effects can be removed by a shift of the Lagrange multiplier field. Next, we add to this action, the fully covariant action of a self interacting scalar field. This results in diagrams of arbitrary high order in the loop expansion, but we still find that propagators involving the metric field only contribute to one-loop diagrams. The model remains renormalizable,

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[1] By using analytic continuation, divergences can be avoided completely [5].

even when considering these higher loop diagrams involving internal scalar field lines. It is possible to couple the metric not only to a scalar field, but also to all fields contributing to the Standard Model, again in a way that leaves the theory renormalizable.

II. USE OF A LAGRANGE MULTIPLIER

In general, an action

$$S[\phi_i] = \int dx\,(\mathcal{L}[\phi_i(x)] + j_i\phi_i) \tag{1}$$

can be considered in conjunction with the path integral [10, 11]

$$Z_j^2 = \lim_{\eta\to\infty}\int \mathcal{D}\phi_i\mathcal{D}\lambda_i \exp i\left[\left(\frac{1+\eta}{2}\right)S[\phi_{+i}] + \left(\frac{1-\eta}{2}\right)S[\phi_{-i}]\right] \tag{2}$$

$$\left(\phi_{\pm_i} \equiv \phi_i \pm \frac{1}{\eta}\lambda_i\right)$$

$$= \int \mathcal{D}\phi_i\mathcal{D}\lambda_i \exp i\int dx\left[\mathcal{L}[\phi_k] + \lambda_i\frac{\delta\mathcal{L}[\phi_k]}{\delta\phi_i} + j_i\left(\phi_i + \lambda_i\right)\right]. \tag{3}$$

Integration over the Lagrange multiplier field λ_i leads to

$$Z_j^2 = \int \mathcal{D}\phi_i\;\delta\left[\frac{\delta\mathcal{L}}{\delta\phi_i} + j_i\right]\exp i\int dx\left[\mathcal{L}[\phi_i] + j_i\phi_i\right]. \tag{4}$$

The functional analogue to

$$\int dx\;\delta\left(f(x)\right)g(x) = \sum_i \frac{g(x_i)}{|f'(x_i)|} \qquad (f(x_i) = 0) \tag{5}$$

reduces Eq. (4) to

$$Z_j^2 = \sum_i \exp i\int dx\;\left[\mathcal{L}[\overline{\phi}_i] + j_i\overline{\phi}_i\right]/\det\left(\frac{\delta^2\mathcal{L}[\overline{\phi}_k]}{\delta\phi_i\delta\phi_j}\right), \tag{6}$$

where $\frac{\delta\mathcal{L}[\overline{\phi}_k]}{\delta\phi_i} + j_i = 0$ defines $\overline{\phi}_i$. In Eq. (6), the exponential is the sum of all tree-level diagrams while the functional determinant is the square of the usual one-loop contribution when there is no Lagrange multiplier λ_i present.

A diagrammatic approach to the path integral of Eq. (3) uses the expansion

$$\mathcal{L}[\phi_i] = \frac{1}{2!}a_{ij}\phi_i\phi_j + \frac{1}{3!}a_{ijk}\phi_i\phi_j\phi_k + \dots. \tag{7}$$

The bilinear $\frac{1}{2}a_{ij}(\phi_i\phi_j + 2\phi_i\lambda_j)$ leads to the propagators $<\phi_i\phi_j>=0$, $<\phi_i\lambda_j>=a_{ij}^{-1}=-<\lambda_i\lambda_j>$ since $\begin{pmatrix} a & a \\ a & 0\end{pmatrix}^{-1} = \begin{pmatrix} 0 & a^{-1} \\ a^{-1} & -a^{-1}\end{pmatrix}$. As $<\phi_i\phi_j>=0$ and since all vertices are at most linear in λ_i, the only Feynman diagrams that can contribute have mixed propagators $<\phi_i\lambda_j>$ with only the fields ϕ_i on external legs. A combinatorial analysis shows that these diagrams are twice the corresponding one-loop diagrams that come from Eq. (1) [10].

If there is an infinitesimal gauge symmetry

$$\phi_i \to \phi_i' = \phi_i + H_{ij}(\phi_k)\xi_j \tag{8}$$

in Eq. (1), then a_{ij} in Eq. (7) cannot be inverted. In this case Eq. (1) requires the addition of a gauge fixing Lagrangian

$$\mathcal{L}_{gf} = -\frac{1}{2\alpha}\left(F_{ij}\phi_j\right)^2 \tag{9}$$

and a ghost Lagrangian

$$\mathcal{L}_{ghost} = \bar{c}_i F_{ij} H_{jk} c_k \tag{10}$$

when using the path integral [12, 13].

The invariance of Eq. (8) means that

$$\begin{aligned}\int dx \mathcal{L}[\phi_i'] &= \int dx \mathcal{L}[\phi_i] \\ &= \int dx \left(\mathcal{L}[\phi_k] + H_{ij}[\phi_k]\xi_j \frac{\delta\mathcal{L}[\phi_k]}{\delta\phi_i}\right)\end{aligned} \tag{11}$$

and consequently $\int dx \lambda_i \frac{\delta\mathcal{L}[\phi_k]}{\delta\phi_i}$ is invariant under the transformation

$$\lambda_i \to \lambda_i + H_{ij}[\phi_k]\zeta_k. \tag{12}$$

Furthermore, since by Eq. (11)

$$\begin{aligned}&\int dx \left(\mathcal{L}[\phi_k] + \lambda_i \frac{\delta\mathcal{L}[\phi_k]}{\delta\phi_i}\right) \\ &= \int dx \left(\mathcal{L}[\phi_k'] + \lambda_i \frac{\delta\phi_j'}{\delta\phi_i}\frac{\delta\mathcal{L}[\phi_k']}{\delta\phi_j'}\right)\end{aligned} \tag{13}$$

and so if ϕ_i undergoes the transformation of Eq. (8) while

$$\lambda_i \to \lambda_i + \lambda_k \frac{\delta H_{ij}}{\delta\phi_k}\xi_j \tag{14}$$

then

$$S_T = \int dx \mathcal{L}_T[\phi_k, \lambda_k] = \int dx \left(\mathcal{L}[\phi_k] + \lambda_i \frac{\delta\mathcal{L}[\phi_k]}{\delta\phi_i}\right) \tag{15}$$

is left invariant.

Following the Faddeev-Popov procedure [12, 13], the path integral associated with S_T is supplemented with the factor

$$\int \mathcal{D}\xi_i \mathcal{D}\zeta_i \delta\left(F_{ij}\left(\left(\begin{pmatrix}\phi_j \\ \lambda_j\end{pmatrix} + \begin{pmatrix}0 & H_{jk} \\ H_{jk} & \lambda_l \frac{\delta H_{jk}}{\delta\phi_l}\end{pmatrix}\begin{pmatrix}\zeta_k \\ \xi_k\end{pmatrix}\right) - \begin{pmatrix}p_j \\ q_j\end{pmatrix}\right)\right)\left|\det F_{ij}\begin{pmatrix}0 & H_{jk} \\ H_{jk} & \lambda_l \frac{\delta H_{jk}}{\delta\phi_l}\end{pmatrix}\right| \tag{16}$$

as well as

$$\int \mathcal{D}p_i \mathcal{D}q_i \exp i \int dx \left[-\frac{1}{2\alpha}\left(p_i p_i + 2p_i q_i\right)\right] \tag{17}$$

if we choose the gauge fixing conditions

$$F_{ij}\phi_j = 0 = F_{ij}\lambda_j. \tag{18}$$

Upon exponentiating the determinant in Eq. (16) by using Fermionic ghost fields, we are left with the generating functional

$$\begin{aligned}Z^2 &= \int \mathcal{D}\phi_i \mathcal{D}\lambda_i \mathcal{D}\bar{c}_i \mathcal{D}c_i \mathcal{D}\bar{\gamma}_i \mathcal{D}\gamma_i \exp i \int dx \Big[\mathcal{L}_T[\phi_k, \lambda_k] - \frac{1}{2\alpha}(F_{ij}\phi_j)^2 - \frac{1}{\alpha}(F_{ij}\phi_j)(F_{ik}\lambda_k) \\ &+ \bar{c}_i F_{ij}\left(H_{jk} + \lambda_l \frac{\delta H_{jk}}{\delta\phi_l}\right)c_k + \bar{\gamma}_i F_{ij} H_{jk} c_k + \bar{c}_i F_{ij} H_{jk}\gamma_k + j_i(\phi_i + \lambda_i)\Big]\end{aligned} \tag{19}$$

once we make use of the identity

$$\det\begin{pmatrix} 0 & A \\ A & B \end{pmatrix} = \det\begin{pmatrix} 0 & A \\ A & A+B \end{pmatrix}. \tag{20}$$

In the path integral in Eq. (19), λ_i, γ_i, $\bar{\gamma}_i$ are Lagrange multipliers associated with the equations of motion of the fields ϕ_i, c_i and $\bar{c}_i$ respectively. It is interesting to note that we also have

$$\left|\det\begin{pmatrix} 0 & A \\ A & B \end{pmatrix}\right| = \det{}^2 A \tag{21}$$

and so the effect of the functional integrals over c_i, $\bar{c}_i$, γ_i $\bar{\gamma}_i$ is to give the square of the one loop contributions coming from the usual Faddeev-Popov factor in Eq. (10) (as expected).

This general formalism has been used when considering the Yang-Mills [10] and Proca [11] model. We now will apply it to the Einstein-Hilbert action.

III. THE EINSTEIN-HILBERT ACTION WITH A LAGRANGE MULTIPLIER

We now consider the second order Einstein-Hilbert action

$$S_{2EH} = \frac{1}{\kappa^2}\int d^4x\sqrt{g}R[g_{\mu\nu}] \quad (\kappa^2 \equiv 16\pi G_N) \tag{22}$$

The gauge invariance of this action is diffeomorphism invariance. If $g_{\mu\nu}$ is split into a background metric $\bar{g}_{\mu\nu}$ and a quantum field $\phi_{\mu\nu}$ [13]

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa\phi_{\mu\nu} \tag{23}$$

with indices raised and lowered and covariant differentiation defined using $\bar{g}_{\mu\nu}$, then a convenient gauge fixing action is

$$S_{gf} = -\frac{1}{2\alpha}\int dx\sqrt{\bar{g}}\left(\phi^{\mu\nu}_{;\bar{\nu}} - \phi^{\nu}_{\nu;}{}^{\bar{\mu}}\right)^2 \equiv -\frac{1}{2\alpha}\int dx\sqrt{\bar{g}}\left[F^{\mu,\alpha\beta}(\bar{g})\phi_{\alpha\beta}\right]^2. \tag{24}$$

where "$;\bar{\mu}$" denotes a covariant derivative using the background metric $\bar{g}_{\mu\nu}$.

The gauge transformation associated with the action of Eq. (22) is an infinitesimal coordinate transformation

$$\begin{aligned}\delta g_{\mu\nu} &= \kappa\left[g_{\mu\lambda}\partial_\nu\xi^\lambda + g_{\nu\lambda}\partial_\mu\xi^\lambda + \xi^\lambda\partial_\lambda g_{\mu\nu}\right] \\ &= \kappa\left[g_{\mu\lambda}\xi^\lambda_{;\nu} + g_{\nu\lambda}\xi^\lambda_{;\mu}\right]\end{aligned} \tag{25}$$

and so under Eq. (23)

$$\delta\left(\bar{g}_{\mu\nu} + \kappa\phi_{\mu\nu}\right) = \kappa\left[\bar{g}_{\mu\lambda}\xi^\lambda_{;\bar{\nu}} + \bar{g}_{\nu\lambda}\xi^\lambda_{;\bar{\mu}} + \kappa\left(\phi_{\mu\lambda}\partial_\nu\xi^\lambda + \phi_{\nu\lambda}\partial_\mu\xi^\lambda + \xi^\lambda\partial_\lambda\phi_{\mu\nu}\right)\right]. \tag{26}$$

There are two types of gauge transformations associated with that of Eq. (26). In the first type,

$$\delta\bar{g}_{\mu\nu} = \kappa\left(\bar{g}_{\mu\lambda}\xi^\lambda_{;\bar{\nu}} + \bar{g}_{\nu\lambda}\xi^\lambda_{;\bar{\mu}}\right) \tag{27a}$$

$$\delta\phi_{\mu\nu} = \kappa\left(\phi_{\mu\lambda}\partial_\nu\xi^\lambda + \phi_{\nu\lambda}\partial_\mu\xi^\lambda + \xi^\lambda\partial_\lambda\phi_{\mu\nu}\right) \tag{27b}$$

while in the second type

$$\delta\bar{g}_{\mu\nu} = 0 \tag{28a}$$

$$\begin{aligned}\delta\phi_{\mu\nu} &= \bar{g}_{\mu\lambda}\xi^\lambda_{;\bar{\nu}} + \bar{g}_{\nu\lambda}\xi^\lambda_{;\bar{\mu}} + \kappa\left(\phi_{\mu\lambda}\partial_\nu\xi^\lambda + \phi_{\nu\lambda}\partial_\mu\xi^\lambda + \xi^\lambda\partial_\lambda\phi_{\mu\nu}\right) \\ &\equiv H_{\mu\nu,\lambda}(\phi)\xi^\lambda.\end{aligned} \tag{28b}$$

The gauge fixing of Eq. (24) does not break the gauge invariance of Eqs. (27), but breaks that of Eq. (28). By use of Eqs. (9) and (10) we can find the gauge fixing and Faddeev-Popov ghost Lagrangians that follow from Eqs. (24) and (28) for S_{2EH} of Eq. (22) alone.

If the background metric is flat (ie $\bar{g}_{\mu\nu} = \delta_{\mu\nu}$) then

$$S_{gf} = -\frac{1}{2\alpha}\int d^4x\left(\phi_{\mu\nu,\nu} - \phi_{\nu\nu,\mu}\right)^2 \tag{29}$$

and [12, 13]

$$S_{FP} = \int d^4x\bar{c}_\mu\left(\partial^2\delta_{\mu\nu} + \kappa M_{\mu\nu}(\phi)\right)c_\nu \tag{30}$$

where

$$M_{\mu\nu}(\phi) = \overleftarrow{\partial}_\sigma\left(\phi_{\mu\sigma,\nu} + \phi_{\nu\sigma,\mu}\right) - \frac{1}{2}\overleftarrow{\partial}_\mu\left(\phi_{\lambda\lambda,\nu} + \phi_{\lambda\nu,\lambda}\right). \tag{31}$$

Variation of $g_{\mu\nu}$ in Eq. (22) leads to [14]

$$\delta S_{2EH} = -\frac{1}{\kappa^2}\int dx\delta g_{\mu\nu}\sqrt{g}\,G^{\mu\nu}(g) \quad \left(G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right). \tag{32}$$

We will now adapt the arguments of the preceding section to deal with background field quantization of the Einstein-Hilbert action when using a Lagrange multiplier field to suppress higher loop contributions to the effective action. The Lagrange multiplier field $\lambda_{\mu\nu}$ associated with the metric $g_{\mu\nu}$ has a background part $\bar{\lambda}_{\mu\nu}$ and a quantum part $\psi_{\mu\nu}$

$$\lambda_{\mu\nu} = \bar{\lambda}_{\mu\nu} + \kappa\psi_{\mu\nu}. \tag{33}$$

We consider the action (much like that in Eq. (3))

$$S_T = \int d^4x\mathcal{L}_T = \frac{1}{\kappa^2}\int d^4x\sqrt{\bar{g}+\kappa\phi}\left[R(\bar{g}+\kappa\phi) - \left(\bar{\lambda}^{\mu\nu} + \kappa\psi^{\mu\nu}\right)G_{\mu\nu}(\bar{g}+\kappa\phi)\right]. \tag{34}$$

Eq. (34) follows from Eq. (22) just as Eq. (3) follows from Eq. (1).

If $\phi_{\mu\nu}$ undergoes the transformation of Eq. (28b), then using the arguments leading to Eqs. (12, 14) we see that

$$\delta\psi_{\mu\nu} = H_{\mu\nu,\lambda}\zeta^\lambda \tag{35a}$$

and

$$\delta\lambda_{\mu\nu} = \frac{1}{\kappa}\left(\bar{\lambda}_{\alpha\beta} + \kappa\psi_{\alpha\beta}\right)\frac{\delta H_{\mu\nu,\lambda}}{\delta\phi_{\alpha\beta}}\xi^\lambda \tag{35b}$$

are gauge transformations associated with $\psi_{\mu\nu}$ (with $\delta\bar{\lambda}_{\mu\nu} = 0$).

Next we insert into the path integral associated with quantizing S_T a factor of unity much like that of Eq. (16)

$$\int\mathcal{D}\xi_\mu\mathcal{D}\zeta_\mu\delta\left(F^{\mu,\alpha\beta}\left(\begin{pmatrix}\phi_{\alpha\beta}\\ \psi_{\alpha\beta}\end{pmatrix} + \begin{pmatrix}0 & H_{\alpha\beta,\rho}\\ H_{\alpha\beta,\rho} & \frac{1}{\kappa}\left(\bar{\lambda}_{\pi\tau} + \kappa\psi_{\pi\tau}\right)\frac{\delta H_{\alpha\beta,\rho}}{\delta\phi_{\pi\tau}}\end{pmatrix}\begin{pmatrix}\zeta_\rho\\ \xi_\rho\end{pmatrix}\right) - \begin{pmatrix}p^\mu\\ q^\mu\end{pmatrix}\right)$$
$$\left|\det F^{\mu,\alpha\beta}\begin{pmatrix}0 & H_{\alpha\beta,\rho}\\ H_{\alpha\beta,\rho} & \frac{1}{\kappa}\left(\bar{\lambda}_{\pi\tau} + \kappa\psi_{\pi\tau}\right)\frac{\delta H_{\alpha\beta,\rho}}{\delta\phi_{\pi\tau}}\end{pmatrix}\right|. \tag{36}$$

In addition, we insert a constant

$$\int\mathcal{D}p^\mu\mathcal{D}q^\mu\exp - \frac{1}{2\alpha}\int d^4x\sqrt{\bar{g}}\left(p^\mu p_\mu + 2p^\mu q_\mu\right) \tag{37}$$

so that much like Eq. (19) we have the generating functional

$$\begin{aligned} Z^2 &= \int\mathcal{D}\phi_{\mu\nu}\mathcal{D}\psi_{\mu\nu}\mathcal{D}\bar{c}_\mu\mathcal{D}c_\mu\mathcal{D}\bar{\gamma}_\mu\mathcal{D}\gamma_\mu\exp - \int d^4x\Bigg\{\mathcal{L}_T + \sqrt{\bar{g}}\Bigg[-\frac{1}{2\alpha}\left((F^{\mu,\alpha\beta}\phi_{\alpha\beta})^2 + 2(F^{\mu,\alpha\beta}\phi_{\alpha\beta})(F_{\mu,\gamma\delta}\psi^{\gamma\delta})\right) \\ &+ \bar{c}_\mu F^{\mu,\alpha\beta}\left(H_{\alpha\beta,\nu} + \frac{1}{\kappa}\left(\bar{\lambda}_{\pi\tau} + \kappa\psi_{\pi\tau}\right)\frac{\delta H_{\alpha\beta,\nu}}{\delta\phi_{\pi\tau}}\right)c^\nu \\ &+ \bar{c}_\mu F^{\mu,\alpha\beta}H_{\alpha\beta,\nu}\gamma^\nu + \bar{\gamma}_\mu F^{\mu,\alpha\beta}H_{\alpha\beta,\nu}c^\nu\Bigg] + \sqrt{\bar{g}}(\phi_{\mu\nu} + \psi_{\mu\nu})j^{\mu\nu}\Bigg\}. \end{aligned} \tag{38}$$

Again using Eq. (21), we see that the ghost contribution to Eq. (38) is $\det^2(F^{\mu,\alpha\beta}H_{\alpha\beta,\nu})$ which is the square of the Faddeev-Popov contribution arising when considering the Einstein-Hilbert action alone. The Lagrange multiplier fields associated with the quantum fields $\phi_{\mu\nu}$, c_μ and d_μ are $\psi_{\mu\nu}$, ξ_μ and ζ_μ respectively.

We now can make the usual choice of background field metric $\bar{g}_{\mu\nu} = \delta_{\mu\nu}$ (flat space). Since the vertices containing N external fields $(\phi_{\mu\nu}, \psi_{\mu\nu})$ can be obtained from Eq. (38) by expanding $\sqrt{g}G^{\mu\nu}$ up to $\mathcal{O}(\phi^{N-1})$ in $\phi_{\mu\nu}$, the derivation of the Feynman rules needed for calculations of one loop Green's functions following from Eq. (38) is simpler than in the usual approach following just from S_{2EH}. Using this approach we have generated the vertices up to the four external fields $\phi_{\mu\nu}$ and explicitly verified that $\langle\phi\psi\rangle = \langle\phi\phi\rangle$, $\langle\phi\phi\psi\rangle = \langle\phi\phi\phi\rangle$ and $\langle\phi\phi\phi\psi\rangle = \langle\phi\phi\phi\phi\rangle$ in agreement with general expressions which follow from Eq. (7). This, together with the fact that the combinatorial factors of loop diagrams with mixed propagators are twice the ones in the usual theory and also that there are two ghost fields in Eq. (38), is sufficient to demonstrate that the results for all the one-loop diagrams will be twice the corresponding results in the usual formulation of quantum gravity. We note that vertices in the 2EH action become simpler if we were to use the first order (Palatini) action 1EH [15, 16].

When using background field quantization then both dimensional arguments and explicit calculation show that all the one loop divergences for the 2EH effective action alone are of the form [1, 2]

$$\int dx \left[\sigma_1\ R^2 + \sigma_2 R^2_{\mu\nu} + \sigma_3 R^2_{\mu\nu\lambda\sigma}\right]. \tag{39}$$

By the Gauss-Bonnet theorem, $R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma} = $ (surface term), the expression in the bracket is a surface term, so that $R^2_{\mu\nu\lambda\sigma}$ can be expressed in terms of R^2 and $R^2_{\mu\nu}$ and the one loop divergences in $n = 4 - \epsilon$ dimensions $(\sqrt{\bar{g}}/(8\pi^2\epsilon)(\frac{1}{120}\bar{R}^2 + \frac{7}{20}\bar{R}^2_{\mu\nu}))$ are proportional to terms that vanish when the equations of motion are satisfied. This means that they can be removed by a field redefinition [1, 2] when working with the S_{2EH} alone or by rescaling the field $\lambda_{\mu\nu}$ in Eq. (38). The two loop divergences that arise using S_{2EH} alone can only be removed if a new term appears in the classical action; introduction of the Lagrange multiplier field circumvents this problem. This approach preserves the structure of the conventional theory to one-loop order but suppresses higher loop contributions where non renormalizable (by power counting) divergences arise. In this way, pure gravity effectively becomes renormalizable.

There are differences between the divergences of Eq. (39) arising when examining one-loop corrections to the Einstein-Hilbert action, and those appearing in the Yang-Mills theory. With Yang-Mills theory and the Dyson procedure, the fields and couplings appearing in the original classical action can be rescaled in order to absorb divergences as all divergences appear in terms that are of the same functional form as ones in the original classical action. This feature is also present when the Yang-Mills action is supplemented by a Lagrange multiplier term that eliminates higher loop corrections [10]. However, when considering the Einstein-Hilbert action alone where divergences at one-loop order are of the form of Eq. (39), this is no longer the case as neither R^2, $R_{\mu\nu}R^{\mu\nu}$ nor $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ appear in the action of Eq. (22). But since Eq. (39) is proportional to $G_{\mu\nu}$ (once the Gauss-Bonnet theorem is used), these divergences can be absorbed by shifting the metric as its equation of motion is $G_{\mu\nu} = 0$ (from Eq. (32)). The introduction of the Lagrange multiplier field makes it possible to eliminate divergences by an alternate shift; instead of shifting the metric as in refs. [1, 2], it is now possible to shift the Lagrange multiplier field in order to absorb one-loop divergences. This will prove possible even when matter fields are present.

In more detail, we see that from Eq. (38) we have the contribution

$$S_{\bar{\lambda}} = -\frac{1}{\kappa^2}\int d^4x\sqrt{\bar{g}}\bar{\lambda}^{\mu\nu}\bar{G}_{\mu\nu} \tag{40}$$

in the effective action, as well as the divergent piece

$$S_{div} = \frac{2}{8\pi^2\epsilon}\int d^4x\sqrt{\bar{g}}\left(\frac{1}{120}\bar{R}^2 + \frac{7}{20}\bar{R}^2_{\mu\nu}\right). \tag{41a}$$

From the definition of $G_{\mu\nu}$ in Eq. (48), this becomes

$$= \frac{1}{4\pi^2\epsilon}\int d^4x\sqrt{\bar{g}}\left(\frac{7}{20}\bar{G}^{\mu\nu} + \frac{1}{120}\bar{G}\bar{g}^{\mu\nu}\right)\bar{G}_{\mu\nu}. \tag{41b}$$

The divergence in the effective action can be removed by shifting $\bar{\lambda}_{\mu\nu}$ to $\bar{\lambda}^R_{\mu\nu}$ where

$$\bar{\lambda}^R_{\mu\nu} = \bar{\lambda}_{\mu\nu} - \frac{\kappa^2}{4\pi^2\epsilon}\left(\frac{7}{20}\bar{G}^{\mu\nu} + \frac{1}{120}\bar{G}\bar{g}^{\mu\nu}\right). \tag{42}$$

There is no need to renormalize $\bar{g}_{\mu\nu}$ or κ^2 and no further divergences can arise since no radiative effects occur beyond one-loop order.

The mass dimension of $g_{\mu\nu}$ is $[\text{mass}]^0$ in n spatial dimensions, and R, $R_{\mu\nu}$, G and $G_{\mu\nu}$ are $[\text{mass}]^2$. Consequently, κ^2 and $\lambda_{\mu\nu}$ have mass dimensions $[\text{mass}]^{\epsilon-2}$ and $[\text{mass}]^0$. As a result, in $n = 4 - \epsilon$ dimensions, κ^2 incorporates an arbitrary mass parameter μ^2 so that

$$\kappa^2 = 16\pi G_N \mu^\epsilon. \tag{43}$$

From Eqs. (41b,42) we see that as $\epsilon \to 0$, the effective action will contain an arbitrary term proportional to $\ln \mu$ once the renormalization of Eq. (42) is taken into account. This arbitrariness is compensated by an arbitrariness in $\bar{\lambda}^R_{\mu\nu}$ ($\bar{g}_{\mu\nu}$ and κ^2 are not altered by changes in μ). We see that together Eqs. (42, 43) result in

$$\bar{\lambda}^R_{\mu\nu} = -\frac{4}{\pi} G_N \left(\frac{7}{20} \bar{G}^{\mu\nu} + \frac{1}{120} \bar{G} \bar{g}^{\mu\nu} \right) \ln\left(\frac{\mu}{\Lambda}\right). \tag{44}$$

where μ/Λ is fixed by experiment. Together Eqs. (40, 41b, 42, 44) imply that $S_{\bar{\lambda}} + S_{div}$ give a contribution to the effective action of

$$S_{new} = \frac{1}{4\pi^2} \ln\left(\frac{\mu}{\Lambda}\right) \int d^4x \sqrt{\bar{g}} \left(\frac{1}{120} \bar{R}^2 + \frac{7}{20} \bar{R}_{\mu\nu} \bar{R}^{\mu\nu} \right). \tag{45}$$

The consequence of renormalization is thus to give to the effective action a contribution quadratic in $\bar{R}$ and $\bar{R}_{\mu\nu}$ with undetermined strength $\frac{1}{4\pi^2} \ln\left(\frac{\mu}{\Lambda}\right)$.

We now examine how the Lagrange multiplier field can be used when the metric couples to a self interacting scalar field so that renormalizability is retained while all higher loop contributions involving internal scalar lines still contribute to the effective action. Such higher loop contributions must be included if our approach were to be applied to a fully covariant version of the Standard Model that is consistent with experiments.

IV. A COVARIANT ACTION WITH A SELF INTERACTING SCALAR FIELD

The action we will consider is of the form $S = S_{2EH} + S_\lambda + S_\phi$ where

$$S_{2EH} = \frac{1}{\kappa^2} \int d^4x \sqrt{g} R(g_{\mu\nu}) \tag{46}$$

$$S_\lambda = -\frac{1}{\kappa^2} \int d^4x \sqrt{g} \lambda_{\mu\nu} G^{\mu\nu}(g_{\mu\nu}) \tag{47}$$

$$S_\phi = \int d^4x \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2}(m^2 - \kappa R)\phi^2 - \frac{1}{4!} \lambda \phi^4 + \Lambda \right). \tag{48}$$

The contribution $S_{2EH} + S_\lambda$ by itself was examined in the preceding section; there it was shown that the presence of the field $\lambda_{\mu\nu}$ restricts the radiative corrections to one loop order and these one-loop corrections are twice those arising from S_{2EH} alone. Adding S_ϕ does not change this conclusion. The new diagrams arising upon including S_ϕ all have internal propagators arising from the scalar field ϕ and either the metric or the scalar field on the external legs. Those diagrams with internal lines coming from the scalar propagator occur at arbitrarily high order in the loop expansion.

The divergences that arise due to quantum effects in the model of a scalar field in the presence of a background metric are analyzed in ref. [17] (see ch. 3 and references there in). It is shown that divergences can either be absorbed by renormalizing the parameters and fields occurring in S_ϕ itself, or arise due to vacuum effects when the background space-time is curved. In this later case, the divergences can be absorbed either by renormalizing $\sqrt{g}R$ in S_{2EH}, by renormalizing the cosmological constant term $\sqrt{g}\Lambda$ in S_ϕ, or are of the form of Eq. (39) in which case they can be absorbed by renormalizing the Lagrange multiplier field $\lambda_{\mu\nu}$ as are the divergences arising from $S_{2EH} + S_\lambda$ which are discussed in the preceding section. It thus proves possible to eliminate all divergences arising from $S_{2EH} + S_\lambda + S_\phi$.

The same conclusion can be reached if in addition to scalar fields, there are also spinor and vector fields present. As a result, it should be possible, using the Lagrange multiplier field, to have a renormalizable Standard Model that is fully covariant. It should also be possible to use a Lagrange multiplier to eliminate radiative effects beyond one-loop order in Supergravity Models.

There are some interesting consequences to having introduced this Lagrange multiplier. Its presence eliminates all higher loop effects (which are known to give rise to non-renormalizable divergences), while at the same time leaves the theory unitary. The classical consequences of the Einstein-Hilbert action are all retained. Matter fields can be coupled to the metric without affecting these features, and the matter fields themselves are coupled only to a background metric (which may have consequences when considering Hawking radiation). One-loop correction to the Einstein-Hilbert action are all in principle computable. Just as introducing the Higgs makes quantizing the Standard Model viable, we may consider the Lagrange multiplier as a possible candidate for a mechanism to reconcile gravity and the quantum theory.

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