On robust stability of linear neutral systems with nonlinear parameter perturbations

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Abstract — **The robust stability of uncertain linear neutral systems with *time-varying* discrete and neutral delays is investigated. The uncertainties under consideration are nonlinear time-varying parameter perturbations and norm-bounded uncertainties, respectively. Both *delay-dependent* and *delay-derivative-dependent* stability criteria are proposed and are formulated in the form of linear matrix inequalities (LMIs). The results in this paper contain some existing results as their special cases. A numerical example is also given to indicate significant improvements over some existing results.**

I. Introduction

THE problem of stability of delay-differential systems of neutral type has received considerable attention in the last two decades; see for example, [1]. The practical examples of neutral systems include the distributed networks containing lossless transmission lines [2], and population ecology [3]. Depending on whether a stability criterion itself contains the information of delay or not, current stability criteria on this topic can be divided into two categories, namely, *delay-independent* stability criteria [4-5] and *delay-dependent* stability criteria [6-8]. However, the references mentioned above only consider the neutral systems with a *constant* neutral delay.

In recent years, the problem of robust stability of *retarded* systems with nonlinear parameter perturbations has also received considerable attention. In [9], for example, some delay-independent and delay-dependent stability criteria are obtained by using the properties of the matrix measure and comparison theorem. In [10], based on the matrix measure, the matrix norm and a decomposition technique, two stability criteria are derived. The results in [9-10] are very conservative since they required the matrix measure to be negative. In [11], a model transformation technique is used to transform the system with a discrete delay to a system with a distributed delay, and *delay-dependent* stability criteria are obtained by using a Lyapunov-Krasovskii functional approach. Although these results in [11] are less conservative than some existing ones, they are still conservative since the model transformation introduced additional dynamics discussed in [12]. In [13], based on a descriptor model transformation [8] and the decomposition technique of a discrete-delay term matrix, the robust stability of uncertain systems with a single time-varying discrete delay is investigated by applying an integral inequality that is introduced in [13] instead of applying bounding of the cross terms introduced in [14]. Numerical examples show that the results obtained in [13] are less conservative than some existing ones in the literature. To the author's best knowledge, up to now, the problem of robust stability of *neutral* systems with nonlinear parameter perturbations has not been addressed in the case of a *time-varying* neutral delay.

In this paper, based on the Lyapunov-Krasovskii functional approach, we will investigate the robust stability of uncertain neutral systems. We will consider both nonlinear parameter perturbations and the well-known norm-bounded uncertainties. The delays under considerations will include time-varying discrete and neutral delays. Then we will transform the robust stability problem of considered system into the *existence* of some symmetric positive-definite matrices. Both *delay-dependent* and *delay-derivative-dependent* stability criteria will be proposed and be formulated in the form of linear matrix inequalities (LMIs), which can be effectively solved by well-known interior-point optimization algorithms [15].

In this paper, a *delay-dependent* stability criterion for

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linear systems with a time-varying delay means that the criterion itself contains the information of both the bound and delay-derivative bound of the time-varying delay while a *delay-derivative-dependent* criterion only contains the information of delay-derivative bound of the time-varying delay. For the case of a *constant* time-delay, the *delay-derivative-dependent* criterion reduces to delay-independent one.

The purpose of this paper is to formulate some practically computable criteria to check the stability of system described by (1)~(3).

II. PROBLEM STATEMENT

Consider the following linear neutral system with time-varying discrete and neutral delays

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bx(t-r(t)) + C\dot{x}(t-\tau(t)) + f(x(t),t) \\ &\quad + g(x(t-r(t)),t) + h(\dot{x}(t-\tau(t)),t) \qquad (1)\end{aligned}$$

where $x(t) \in \mathbb{R}^n$ is the state, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{n\times n}$ are constant matrices. The time-varying vector -valued functions $f(x(t),t) \in \mathbb{R}^n$, $g(x(t-r(t)),t) \in \mathbb{R}^n$ and $h(\dot{x}(t-\tau(t)),t) \in \mathbb{R}^n$ are unknown and represent the parameter perturbations with respect to the current state $x(t)$ and delayed state $x(t-r(t))$ and $\dot{x}(t-\tau(t))$ of the system, respectively. They satisfy that $f(0,t)=0$, $g(0,t)=0$ and $h(0,t)=0$. The delay $r(t)$ is a time-varying discrete delay and $\tau(t)$ is a time-varying neutral delay, which satisfy

$$0 \le r(t) \le r_M,\ \dot{r}(t) \le r_d;\ 0 \le \tau(t) \le \tau_M,\ \dot{\tau}(t) \le \tau_d \qquad (2)$$

where r_M, r_d, τ_M and τ_d are constants, and $0 \le r_d < 1$ and $0 \le \tau_d < 1$.

The initial condition of system (1) is given by

$$\begin{aligned}& x(t_0+\theta) = \varphi(\theta),\ \dot{x}(t_0+\theta) = \dot{\varphi}(\theta) \\ & \forall\theta \in [-\max\{r_M, \tau_M\},\ 0] \qquad (3)\end{aligned}$$

where $\varphi(\cdot)$ is a vector-valued initial function.

III. NONLINEAR PARAMETER PERTURBATION

In this section, we assume that $f(x(t),t)$, $g(x(t-h(t)),t)$ and $h(\dot{x}(t-\tau(t)),t)$ represent the nonlinear parameter time-varying perturbations of system (1) which satisfy that

$$\|f(x(t),t)\| \le \alpha\|x(t)\| \qquad (4a)$$

$$\|g(x(t-r(t)),t)\| \le \beta\|x(t-r(t))\| \qquad (4b)$$

$$\|h(\dot{x}(t-\tau(t)),t)\| \le \gamma\|\dot{x}(t-\tau(t))\| \qquad (4c)$$

where $\alpha \ge 0$, $\beta \ge 0$ and $\gamma \ge 0$ are given constants.

Constraint (4) can be rewritten as

$$f^T(x(t),t)f(x(t),t) \le \alpha^2 x^T(t)x(t) \qquad (5a)$$

$$\begin{aligned}& g^T(x(t-r(t)),t)g(x(t-r(t)),t) \\ & \le \beta^2 x^T(t-r(t))x(t-r(t)) \qquad (5b)\end{aligned}$$

$$\begin{aligned}& h^T(\dot{x}(t-\tau(t)),t)h(\dot{x}(t-\tau(t)),t) \\ & \le \gamma^2 \dot{x}^T(t-\tau(t))\dot{x}(t-\tau(t)). \qquad (5c)\end{aligned}$$

For robust stability of system (1)~(3), with uncertainty (4), we have the following *delay-dependent* stability result.

Proposition 1: The system described by (1) to (3), with uncertainty described by (4) is asymptotically stable if $\|C\| + \gamma < 1$ and there exist a real matrix X, symmetric positive definite matrices *P, R, S, Y* and scalars $\varepsilon_1 \ge 0$, $\varepsilon_2 \ge 0$ and $\varepsilon_3 \ge 0$ such that the LMI (6) (at the bottom of the page) holds, where

$$(1,1) \triangleq (A+B)^T P + P(A+B) + R + X^T B + B^T X + \varepsilon_1\alpha^2 I$$

$$(2,2) \triangleq -(1-r_d)R + \varepsilon_2\beta^2 I,\ (3,3) \triangleq -(1-\tau_d)S + \varepsilon_3\gamma^2 I$$

$$\begin{pmatrix} (1,1) & -X^T B & PC & P & P & P & A^T S & A^T B^T Y & r_M(X^T+P) \\ -B^T X & (2,2) & 0 & 0 & 0 & 0 & B^T S & B^T B^T Y & 0 \\ C^T P & 0 & (3,3) & 0 & 0 & 0 & C^T S & C^T B^T Y & 0 \\ P & 0 & 0 & -\varepsilon_1 I & 0 & 0 & S & B^T Y & 0 \\ P & 0 & 0 & 0 & -\varepsilon_2 I & 0 & S & B^T Y & 0 \\ P & 0 & 0 & 0 & 0 & -\varepsilon_3 I & S & B^T Y & 0 \\ SA & SB & SC & S & S & S & -S & 0 & 0 \\ YBA & YBB & YBC & YB & YB & YB & 0 & -(1-r_d)Y & 0 \\ r_M(X+P) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Y \end{pmatrix} < 0 \qquad (6)$$

Choosing the Lyapunov-Krasovskii functional candidate for system (1) as

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t)$$

where

$$V_1(t) = x^T(t)Px(t)$$

$$V_2(t) = \frac{1}{1-r_d}\int_{t-r(t)}^{t}\left(r(t)-t+\xi\right)\dot{x}^T(\xi)B^TQB\dot{x}(\xi)d\xi$$

$$V_3(t) = \int_{t-r(t)}^{t} x^T(\xi)Rx(\xi)d\xi$$

$$V_4(t) = \int_{t-\tau(t)}^{t} \dot{x}^T(\xi)S\dot{x}(\xi)d\xi$$

where symmetric positive definite matrices *P*, *R*, *S*, $Y(=r_M Q)$ are solutions of (6), one can prove Proposition 1. For the detail, see the full version of the paper [16].

Remark 1: The condition $\|C\| + \gamma < 1$ in Proposition 1 guarantees that Lipschitz constant for the right hand of (1) with respect to $\dot{x}(t-\tau(t))$ is less than one.

If we choose the following Lyapunov-Krasovskii functional candidate for system (1) as

$$V(t) = x^T(t)Px(t) + \int_{t-r(t)}^{t} x^T(\xi)Rx(\xi)d\xi + \int_{t-\tau(t)}^{t} \dot{x}^T(\xi)S\dot{x}(\xi)d\xi$$

Similar to the proof of Proposition 1, we have the following *delay-derivative-dependent* stability result.

Proposition 2: The system described by (1) to (3), with uncertainty described by (4) is asymptotically stable if $\|C\| + \gamma < 1$ and there exist symmetric positive definite matrices *P*, *R*, *S*, and scalars $\varepsilon_1 \ge 0$, $\varepsilon_2 \ge 0$ and $\varepsilon_3 \ge 0$ such that the following LMI holds

$$\begin{pmatrix} (1,1) & PB & PC & P & P & P & A^TS \\ B^TP & (2,2) & 0 & 0 & 0 & 0 & B^TS \\ C^TP & 0 & (3,3) & 0 & 0 & 0 & C^TS \\ P & 0 & 0 & -\varepsilon_1 I & 0 & 0 & S \\ P & 0 & 0 & 0 & -\varepsilon_2 I & 0 & S \\ P & 0 & 0 & 0 & 0 & -\varepsilon_3 I & S \\ SA & SB & SC & S & S & S & -S \end{pmatrix} < 0 \quad (7)$$

where

$$(1,1) \triangleq A^TP + PA + R + \varepsilon_1\alpha^2 I$$

$$(2,2) \triangleq -(1-r_d)R + \varepsilon_2\beta^2 I$$

$$(3,3) \triangleq -(1-\tau_d)S + \varepsilon_3\gamma^2 I .$$

If $C \equiv 0$ and $h(\dot{x}(t-\tau(t)),t) \equiv 0$, then system (1) reduces to the following system

$$\dot{x}(t) = Ax(t) + Bx(t-r(t)) + f(x(t),t) + g(x(t-r(t)),t) \quad (8)$$

with initial condition

$$x(t_0+\theta) = \varphi(\theta),\ \forall\theta \in [-r_M,\ 0]. \quad (9)$$

According to Proposition 1, we have the following corollary for the *delay-dependent* stability of system (8)~(9).

Corollary 1: The system described by (8), (9), (3), with uncertainty described by (4a) and (4b) is asymptotically stable if there exist a real matrix *X*, symmetric positive definite matrices *P*, *R*, *Y* and scalars $\varepsilon_1 \ge 0$, $\varepsilon_2 \ge 0$ such that the following LMI holds

$$\begin{pmatrix} (1,1) & -X^TB & P & P & A^TB^TY & r_M(X^T+P) \\ -B^TX & (2,2) & 0 & 0 & B^TB^TY & 0 \\ P & 0 & -\varepsilon_1 I & 0 & B^TY & 0 \\ P & 0 & 0 & -\varepsilon_2 I & B^TY & 0 \\ YBA & YBB & YB & YB & -(1-r_d)Y & 0 \\ r_M(X+P) & 0 & 0 & 0 & 0 & -Y \end{pmatrix} < 0 \quad (10)$$

where

$$(1,1) \triangleq (A+B)^TP + P(A+B) + R + X^TB + B^TX + \varepsilon_1\alpha^2 I$$

$$(2,2) \triangleq -(1-r_d)R + \varepsilon_2\beta^2 I .$$

Remark 2: If $f(x(t),t) \equiv 0$, $g(x(t-h(t)),t) \equiv 0$, and $r(t) \equiv r$ (constant), system (8)~(9) becomes

$$\dot{x}(t) = Ax(t) + Bx(t-r) \quad (11)$$

with initial condition

$$x(t_0+\theta) = \varphi(\theta),\ \forall\theta \in [-r,\ 0]. \quad (12)$$

By Corollary 1, we can conclude that system (11)~(12) is asymptotically stable if there exist a real matrix *X*, symmetric positive definite matrices *P*, *R*, and *Y* such that

$$\begin{pmatrix} (1,1) & -X^TB & A^TB^TY & r(X^T+P) \\ -B^TX & -R & B^TB^TY & 0 \\ YBA & YBB & -Y & 0 \\ r(X+P) & 0 & 0 & -Y \end{pmatrix} < 0 \quad (13)$$

where

$$(1,1) \triangleq (A+B)^TP + P(A+B) + R + X^TB + B^TX .$$

Then the Theorem 1 in [16] is recovered.

By Proposition 2, the following corollary is easily obtained for the *delay-derivative-dependent* stability of system (8)~(9).

Corollary 2: The system described by (8), (9), (3), with uncertainty described by (4a) and (4b) is asymptotically stable if there exist symmetric positive definite matrices *P*, *R*, and scalars $\varepsilon_1 \geq 0$, $\varepsilon_2 \geq 0$ such that the following LMI holds

$$\begin{pmatrix} \begin{pmatrix} A^T P + PA + \\ R + \varepsilon_1 \alpha^2 I \end{pmatrix} & PB & P & P \\ B^T P & \begin{pmatrix} -(1-r_d)R \\ +\varepsilon_2 \beta^2 I \end{pmatrix} & 0 & 0 \\ P & 0 & -\varepsilon_1 I & 0 \\ P & 0 & 0 & -\varepsilon_2 I \end{pmatrix} < 0 . \quad (14)$$

Remark 3: Corollary 2 is the Theorem 1 in [13]. This means that Proposition 2 extends the result in [13] to neutral systems.

IV. Norm-bounded Uncertainty

In this section we will handle the case that $f(x(t),t)$, $g(x(t-h(t)),t)$ and $h(\dot{x}(t-\tau(t)),t)$ are norm-bounded uncertainties that are well known in robust control of uncertain systems [15]. Then system (1) becomes the following system

$$\dot{x}(t) = \left(A + LF(t)E_a\right)x(t) + \left(B + LF(t)E_b\right)x(t-r(t)) + \left(C + LF(t)E_c\right)\dot{x}(t-\tau(t)) \quad (15)$$

where $F(t) \in \mathbb{R}^{p\times q}$ is an unknown real and possibly time-varying matrix with Lebesgue measurable elements satisfying

$$F^T(t)F(t) \leq I \quad (16)$$

and L, E_a, E_b, and E_c are known real constant matrices which characterize how the uncertainty enters the nominal matrices *A, B* and *C*.

System (15) can be written as

$$\dot{x}(t) = Ax(t) + Bx(t-r(t)) + C\dot{x}(t-\tau(t)) + Lu \quad (17a)$$

$$y = E_a x(t) + E_b x(t-r(t)) + E_c \dot{x}(t-\tau(t)) \quad (17b)$$

with the constraint

$$u = F(t)y . \quad (18)$$

We further rewrite (17)~(18) as

$$\dot{x}(t) = Ax(t) + Bx(t-r(t)) + C\dot{x}(t-\tau(t)) + Lu \quad (19a)$$

$$u^T u \leq \left(E_a x(t) + E_b x(t-r(t)) + E_c \dot{x}(t-\tau(t))\right)^T \times \left(E_a x(t) + E_b x(t-r(t)) + E_c \dot{x}(t-\tau(t))\right) \quad (19b)$$

We now state and establish the following *delay-dependent* stability result.

Proposition 3: The system described by (15), (16), (2), (3) is asymptotically stable if there exist a scalar $\delta > 0$ satisfying $\delta I - L^T L > 0$, and a real matrix $\tilde{X}$, symmetric positive definite matrices $\tilde{P}$, $\tilde{R}$, $\tilde{S}$ and $\tilde{Y}$ such that the following LMI

$$\begin{pmatrix} C^T C - I + \delta E_c^T E_c & C^T L \\ LC & -(\delta I - L^T L) \end{pmatrix} < 0 \quad (20)$$

and the LMI (21) (at the bottom of the page) hold ,where

$$(1,1) \triangleq (A+B)^T \tilde{P} + \tilde{P}(A+B) + \tilde{R} + \tilde{X}^T B + B^T \tilde{X} .$$

Proof: See the full version of the paper [16].

Remark 4: Although norm-bounded uncertainties can be treated as a special case of nonlinear parameter perturbations, one can get a less conservative result using Proposition 3 than Proposition 1.

$$\begin{pmatrix} (1,1) & -\tilde{X}^T B & \tilde{P}C & \tilde{P}L & A^T\tilde{S} & A^T B^T \tilde{Y} & r_M(\tilde{X}^T + \tilde{P}) & E_a \\ -B^T\tilde{X} & -(1-r_d)\tilde{R} & 0 & 0 & B^T\tilde{S} & B^T B^T\tilde{Y} & 0 & E_b \\ C^T\tilde{P} & 0 & -(1-\tau_d)\tilde{S} & 0 & C^T\tilde{S} & C^T B^T\tilde{Y} & 0 & E_c \\ L^T\tilde{P} & 0 & 0 & -I & L^T\tilde{S} & L^T B^T\tilde{Y} & 0 & 0 \\ \tilde{S}A & \tilde{S}B & \tilde{S}C & \tilde{S}L & -\tilde{S} & 0 & 0 & 0 \\ \tilde{Y}BA & \tilde{Y}BB & \tilde{Y}BC & \tilde{Y}BL & 0 & -(1-r_d)\tilde{Y} & 0 & 0 \\ r_M(\tilde{X}+\tilde{P}) & 0 & 0 & 0 & 0 & 0 & -\tilde{Y} & 0 \\ E_a^T & E_b^T & E_c^T & 0 & 0 & 0 & 0 & -I \end{pmatrix} < 0 \quad (21)$$

Similar to Proposition 2, we have the following *delay-derivative-dependent* stability result.

Proposition 4: The system described by (15), (16), (2), (3) is asymptotically stable if there exist a scalar $\delta > 0$ satisfying $\delta I - L^T L > 0$, and symmetric positive definite matrices $\tilde{P}$, $\tilde{R}$, $\tilde{S}$, such that (20) and the following LMI are satisfied,

$$\begin{pmatrix} (1,1) & \tilde{P}B & \tilde{P}C & \tilde{P}L & A^T\tilde{S} & E_a \\ B^T\tilde{P} & (2,2) & 0 & 0 & B^T\tilde{S} & E_b \\ C^T\tilde{P} & 0 & (3,3) & 0 & C^T\tilde{S} & E_c \\ L^T\tilde{P} & 0 & 0 & -I & L^T\tilde{S} & 0 \\ \tilde{S}A & \tilde{S}B & \tilde{S}C & \tilde{S}L & -\tilde{S} & 0 \\ E_a^T & E_b^T & E_c^T & 0 & 0 & -I \end{pmatrix} < 0 \quad (22)$$

where

$$(1,1) \triangleq A^T\tilde{P} + \tilde{P}A + \tilde{R}, \ (2,2) \triangleq -(1-r_d)\tilde{R}$$
$$(3,3) \triangleq -(1-\tau_d)\tilde{S}$$

If $C \equiv 0$ and $E_c \equiv 0$, then system (15) reduces to the following system

$$\dot{x}(t) = \left(A + LF(t)E_a\right)x(t) + \left(B + LF(t)E_b\right)x(t - r(t)) \quad (23)$$

with initial condition

$$x(t_0 + \theta) = \varphi(\theta), \ \forall \theta \in [-r_M, \ 0]. \quad (24)$$

For the stability of system (23) to (24), in light of Propositions 3 and 4, we have the following corollaries.

Corollary 3 (Delay-dependent stability): The system described by (23), (24), (16), (2) is asymptotically stable dependence if there exist a real matrix $\tilde{X}$, symmetric positive definite matrices $\tilde{P}$, $\tilde{R}$, and $\tilde{Y}$ such that the LMI (25) (at the bottom of the page) holds, where

$$(1,1) \triangleq (A+B)^T P + P(A+B) + R + X^T B + B^T X .$$

Corollary 4 (Delay-derivative-dependent stability): The system described by (23), (24), (16), (2) is asymptotically stable if there exist symmetric positive definite matrices $\tilde{P}$ and $\tilde{R}$ such that the LMI (26) (at the bottom of the page) is satisfied.

V. An Example

Consider system (1) with

$$A = \begin{pmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{pmatrix}, \ B = \begin{pmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{pmatrix}, \ C = \begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix},$$
$$\|f(x(t),t)\| \le \alpha \|x(t)\|,$$
$$\|g(x(t-r(t)),t)\| \le \beta \|x(t-r(t))\|,$$
$$\|h(\dot{x}(t-\tau(t)),t)\| \le \gamma \|\dot{x}(t-\tau(t))\|$$

where $0 \le |c| < 1$, $\alpha \ge 0$, $\beta \ge 0$, $\gamma \ge 0$.

Case I: For $c \equiv 0$ and $h(\dot{x}(t-\tau(t)),t) \equiv 0$, the system under consideration reduces to the system studied in [13]. Applying the criteria in [13], [15] and in this paper, the maximum value of r_M for stability of system is listed in Table 1. It is easy to see that the stability criterion in this paper gives a much less conservative result than one in [13] and [15]. Other results surveyed in [13] are even more conservative.

	$\alpha = 0$, $\beta = 0.1$		$\alpha = 0.1$, $\beta = 0.1$	
	$r_d = 0$	$r_d = 0.5$	$r_d = 0$	$r_d = 0.5$
[13]	0.6811	0.5467	0.6129	0.4950
[15]	1.3279	0.6743	1.2503	0.5716
This paper	2.7427	0.8036	1.8753	0.7037

Table 1. Bound r_M for $c \equiv 0$ and $h(\dot{x}(t-\tau(t)),t) \equiv 0$

Case II: For $h(\dot{x}(t-\tau(t)),t) \equiv 0$ and $\tau_d = 0$, the maximum value r_M is listed in Table 2 for different *c*. As $|c|$ increases, r_M decreases.

$$\begin{pmatrix} (1,1) & -\tilde{X}^T B & \tilde{P}L & A^T B^T \tilde{Y} & r_M(\tilde{X}^T + \tilde{P}) & E_a \\ -B^T\tilde{X} & -(1-r_d)\tilde{R} & 0 & B^T B^T \tilde{Y} & 0 & E_b \\ L^T\tilde{P} & 0 & -I & L^T B^T \tilde{Y} & 0 & 0 \\ \tilde{Y}BA & \tilde{Y}BB & \tilde{Y}BL & -(1-r_d)\tilde{Y} & 0 & 0 \\ r_M(\tilde{X}+\tilde{P}) & 0 & 0 & 0 & -\tilde{Y} & 0 \\ E_a^T & E_b^T & 0 & 0 & 0 & -I \end{pmatrix} < 0 \quad (25);$$

$$\begin{pmatrix} A^T\tilde{P} + \tilde{P}A + \tilde{R} & \tilde{P}B & \tilde{P}L & E_a \\ B^T\tilde{P} & -(1-r_d)\tilde{R} & 0 & E_b \\ L^T\tilde{P} & 0 & -I & 0 \\ E_a^T & E_b^T & 0 & -I \end{pmatrix} < 0 \quad (26)$$

	$\alpha = 0$, $\beta = 0.1$		$\alpha = 0.1$, $\beta = 0.1$	
	$r_d = 0$	$r_d = 0.5$	$r_d = 0$	$r_d = 0.5$
$\lvert c \rvert = 0.1$	2.0366	0.6596	1.4753	0.5762
$\lvert c \rvert = 0.3$	1.0924	0.4016	0.8587	0.3463
$\lvert c \rvert = 0.5$	0.5314	0.1888	0.4312	0.1547
$\lvert c \rvert = 0.7$	0.1765	0.0265	0.1336	0.0064

Table 2. Bound r_M for $h(\dot{x}(t-\tau(t)),t) \equiv 0$, $\tau_d = 0$ and different c

Case III: For $c = 0.1$ and/or $\tau_d = 0$ ($\tau_d = 0.5$), we now consider the effect of uncertainty bound γ on the maximum value r_M. Tables 3 and 4 illustrates the numerical results for different γ, $\tau_d \equiv 0$ and $\tau_d = 0.5$, respectively. We can see that r_M decreases as γ increases.

	$\alpha = 0$, $\beta = 0.1$		$\alpha = 0.1$, $\beta = 0.1$	
	$r_d = 0$	$r_d = 0.5$	$r_d = 0$	$r_d = 0.5$
$\gamma = 0.0$	2.0366	0.6596	1.4753	0.5762
$\gamma = 0.1$	1.4937	0.5234	1.1356	0.4553
$\gamma = 0.2$	1.0838	0.3986	0.8451	0.3440
$\gamma = 0.3$	0.7697	0.2858	0.6215	0.2428

Table 3. Bound r_M for $c = 0.1$ and $\tau_d = 0$

	$\alpha = 0$, $\beta = 0.1$		$\alpha = 0.1$, $\beta = 0.1$	
	$r_d = 0$	$r_d = 0.5$	$r_d = 0$	$r_d = 0.5$
$\gamma = 0.0$	1.7967	0.6028	1.3287	0.5257
$\gamma = 0.1$	1.1481	0.4197	0.8995	0.3628
$\gamma = 0.2$	0.7054	0.2606	0.5718	0.2200
$\gamma = 0.3$	0.3923	0.1269	0.3166	0.0988

Table 4. Bound r_M for $c = 0.1$ and $\tau_d = 0.5$

VI. CONCLUSION

The robust stability problem of uncertain linear systems with time-varying discrete and neutral delays has been studied. Some practically computable stability criteria have been obtained. The results have included some existing results as their special cases. An example has also been given to show significant improvements over the existing results in the literature.

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