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On Soft Preopen Sets and Soft Pre Separation Axioms

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ABSTRACT

Arockiarani and Lancy [7], defined soft pre-open (closed) sets on soft topology. In this paper, we are continue investigating the properties of soft pre-open (closed) sets and define soft preclosure and soft preinterior in soft topological spaces. We are also introduce and research basic properties of the concepts of soft pre-regular spaces, soft P_3-spaces, soft pre-normal spaces and soft P_4-spaces in soft topological spaces, which are basic for further research on soft topology and will fortify the footing of the theory of soft topological space.

Keywords:

1. INTRODUCTION

Molodtsov [2] introduced the concept of a soft set in order to solve complicated problems in the economics, engineering, and environmental areas, because no mathematical tools can successfully deal with the various kinds of uncertainties in these problems. In the recent years in development in the fields of soft set theory and its application has been taking place in a rapid pace. Of late many authors [6,8,9,21,22,23] have studied various properties of soft topological spaces. This is because of the general nature of parameterization expressed by a soft set. Shabir and Naz [4] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Later, Akdag et al.[8], Aygunoglu et al [6] and Hussain et al are continued to study the properties of soft topological space. They got many important results in soft topological spaces. Weak forms of soft open sets were first studied by Chen [5]. He investigated soft semi-open sets in soft topological spaces and studied some properties of it. Arockiarani and Lancy are defined soft β-open sets and continued to study weak forms of soft open sets in soft topological space. Akdag and Ozkan [20], defined soft α-open and soft α-closed sets in soft topological spaces and studied many important results and some properties of it.

Pre-open sets were introduced by Mashhour et al. [10] and since then different topological properties have been defined in terms of pre-open sets and investigated by many researchers [11,12,13,14,15,16,17,18,19]. In this paper we extend the notion of preopenness to soft sets [9]. We define soft preinteriors and preclosures and investigate their properties. Most attention is paid to the extension of separation notions to soft topological spaces with the help of soft pre-open sets. We obtain interesting characterization of soft pre separation axioms.

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2. PRELIMINARIES

Definition 1. [2] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F,A) is called a soft set over X, where F is a mapping given by $F\colon A \to P(X)$.
In other words, a soft set over X is a parameterized family of subsets of the universe X. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F,A).
Definition 2. [2] Let (F,A) and (G,B) be two soft sets over a common universe X. $(F,A) \tilde{\subseteq} (G,B)$, if $A \subset B$, and $F(e) \subset G(e)$ for all $e \in A$.
Definition 3. [3] Two soft sets (F,A) and (G,B) over a common universe X are said to be soft equal if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A).
Definition 4. [3] A soft set (F,A) over X is called a null soft set, denoted by $\emptyset$, if $e \in A$, $F(e) = \emptyset$.
Definition 5. [3] A soft set (F,A) over X is called an absolute soft set, denoted by $\tilde{A}$, if $e \in A$, $F(e) = X$.
Definition 6. [2] The union of two soft sets of (F,A) and (G,B) over the common universe X is the soft set (H,C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} \mathrm{F(e)}, & \text{if } \mathrm{e \in A - B} \\ \mathrm{G(e)}, & \text{if } \mathrm{e \in B - A} \\ \mathrm{F(e) \cup G(e)}, & \text{if } \mathrm{e \in (A \cap B)} \end{cases}$$

We write $(F,A) \tilde{\cup} (G,B) = (H,C)$.

Definition 7. [2] The intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe X, denoted $(F,A) \tilde{\cap} (G,B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.
Definition 8. [1] For a soft set (F,A) over X the relative complement of (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c,A)$, where $F^c\colon A \to P(X)$ is a mapping given by $F^c(\alpha) = X - F(\alpha)$ for all $\alpha \in A$.
Definition 9. [4] Let (F,E) be a soft set over X and $x \in X$. We say that $x \in (F,E)$ read as x belongs to the soft set (F,E), whenever $x \in F(\alpha)$ for all $\alpha \in E$.
Note that for $x \in X$, $x \notin (F,E)$ if $x \notin F(\alpha)$ for some $\alpha \in E$.
Definition 10. [4] Let $x \in X$; then (x,E) denotes the soft set over X for which $x(\alpha) = \{x\}$, for all $\alpha \in E$.
Lemma 1. [22] Let (F,E) be a soft set over X and $x \in X$. Then:
(1) $x \in (F,E)$ iff $(x,E) \tilde{\subseteq} (F,E)$;
(2) if $(x,E) \tilde{\cap} (F,E) = \Phi$ then $x \notin (F,E)$.
Definition 11. [4] Let (F,E) be a soft set over X and Y be a non-empty subset of X. Then the subsoft set of (F,E) over Y denoted by $({}^{Y}F,E)$, is defined as follows
${}^{Y}F(\alpha) = Y \cap F(\alpha)$, for all $\alpha \in E$.
In other words $({}^{Y}F,E) = \tilde{Y} \tilde{\cap} (F,E)$.
Definition 12. [4] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if satisfies the following axioms.
(1) $\emptyset, X$ belong to τ,
(2) the union of any number of soft sets in τ belongs to τ,
(3) the intersection of any two soft sets in τ belongsto τ.
The triplet (X,τ,E) is called a soft topological space over X. Let (X,τ,E) be a soft topological space over X, then the members of τ are said to be soft open sets in X. Let (X,τ,E) be a soft topological space over X. A soft set (F,E) over X is said to be a soft closed set in X, if its relative complement $(F,E)^c$ belongs to τ. If (X,τ,E) is a a soft topological space with $\tau = \{\emptyset, X\}$, then τ is called the soft indiscrete topology on X and (X,τ,E) is said to be a soft indiscrete topological space. If (X,τ,E) is a a soft topological space with τ is the collection of all soft sets which can be defined over X, then τ is called the soft discrete topology on X and (X,τ,E) is said to be a soft discrete topological space.
Throughout the paper, the space X and Y (or (X,τ,E) and (Y,σ,E)) stand for soft topological spaces assumed unless otherwise stated.
Let (F,A) be a subset of a soft topological space X. The closure and interior of (F,A) are denoted by $cl((F,A))$ and $int((F,A))$, respectively.
Definition 13. [4] Let (X,τ,E) be a soft topological space over X, (G,E) be a soft set over X and $x \in X$. Then x is said to be soft interior point of (G,E) if there exists a soft open set (F,E) such that $x \in (F,E) \tilde{\subseteq} (G,E)$.
Definition 14. [4] Let (X,τ,E) be a soft topological space over X, (G,E) be a soft set over X and $x \in X$. Then (G,E) is said to be a soft neighborhood of x if there exists a soft open set (F,E) such that $x \in (F,E) \tilde{\subseteq} (G,E)$.
Definition 15. [5] Let (X,τ,E) be a soft topological space over X and (F,A) be a soft set over X.
(1) The soft interior of (F,A) is the soft set

$$int((F,A)) = \tilde{\cup} \{(O,E) : (O,E) \text{ is a soft open and } (O,E) \subset (F,A)\}$$

(2) The soft closure of (F,A) is the soft set

$$cl((F,A)) = \tilde{\cap} \{(F,E) : (F,E) \text{ is a soft closed and } (F,A) \subset (F,E)\}.$$

3. SOME PROPERTIES OF SOFT PRE-OPEN SETS AND PRE-CLOSED SETS

Definition 16. [7] Let (F,A) be any soft set of a soft topological space (X,τ,E). (F,A) is called
(1) (F,A) soft pre-open set of X if $(F,A)\,\tilde{\subseteq}\, int(cl((F,A)))$, and
(2) (F,A) soft pre-closed set of X if $(F,A)\,\tilde{\supseteq}\, cl(int((F,A)))$.
Theorem 1. (1) Arbitrary union of soft pre-open sets is a soft pre-open sets, and
(2) Arbitrary intersection of soft pre-closed sets is a soft pre-closed set.
Proof (1) Let $\{(F_\alpha,A):\alpha\in\Lambda\}$ be a collection of soft pre-open sets. Then, for each $\alpha\in\Lambda$,
$(F_\alpha,A)\,\tilde{\subseteq}\, int(cl((F_\alpha,A)))$. Now
$\tilde{\cup}\,(F_\alpha,A)\,\tilde{\subseteq}\tilde{\cup}\, int(cl((F_\alpha,A)))\,\tilde{\subseteq}\, int\,\tilde{\cup}\,(cl((F_\alpha,A)))=int(cl(\tilde{\cup}\,((F_\alpha,A)))$.
Hence $\tilde{\cup}\,(F_\alpha,A)$ is a soft pre-open set.
(2) Follows easily by taking complements.
Remark 1. It is obvious that every soft open (closed) set is a soft pre-open (pre-closed) set. That the converse is false, is shown by following Example.
Example 1. Let $X=\{x_1,x_2,x_3,x_4\}$, $E=\{e_1,e_2,e_3\}$ and
$\tau=\{\Phi,X,(F_1,E),(F_2,E),(F_3,E),\dots(F_{15},E)\}$ where
$(F_1,E),(F_2,E),(F_3,E),\dots(F_{15},E)$ are soft sets over X, defined as follows
$(F_1,E)=\{(e_1,\{x_1\}),(e_2,\{x_2,x_3\}),(e_3,\{x_1,x_4\})\}$,
$(F_2,E)=\{(e_1,\{x_2,x_4\}),(e_2,\{x_1,x_3,x_4\}),(e_3,\{x_1,x_2,x_3\})\}$,
$(F_3,E)=\{(e_2,\{x_3\}),(e_3,\{x_1\})\}$,
$(F_4,E)=\{(e_1,\{x_1,x_2,x_4\}),(e_2,X),(e_3,X)\}$,
$(F_5,E)=\{(e_1,\{x_1,x_3\}),(e_2,\{x_2,x_4\}),(e_3,\{x_2\})\}$,
$(F_6,E)=\{(e_1,\{x_1\}),(e_2,\{x_2\})\}$,
$(F_7,E)=\{(e_1,\{x_1,x_3\}),(e_2,\{x_2,x_3,x_4\}),(e_3,\{x_1,x_2,x_4\})\}$,
$(F_8,E)=\{(e_2,\{x_4\}),(e_3,\{x_2\})\}$,
$(F_9,E)=\{(e_1,X),(e_2,X),(e_3,\{x_1,x_2,x_3\})\}$,
$(F_{10},E)=\{(e_1,\{x_1,x_3\}),(e_2,\{x_2,x_3,x_4\}),(e_3,\{x_1,x_2\})\}$,
$(F_{11},E)=\{(e_1,\{x_1,x_2,x_4\}),(e_2,X),(e_3,\{x_1,x_2,x_3\})\}$,
$(F_{12},E)=\{(e_1,\{x_1\}),(e_2,\{x_2,x_3,x_4\}),(e_3,\{x_1,x_2,x_4\})\}$,
$(F_{13},E)=\{(e_1,\{x_1\}),(e_2,\{x_2,x_4\}),(e_3,\{x_2\})\}$,
$(F_{14},E)=\{(e_2,\{x_3,x_4\}),(e_3,\{x_1,x_2\})\}$,
$(F_{15},E)=\{(e_1,\{x_1\}),(e_2,\{x_2,x_3\}),(e_3,\{x_1\})\}$.
Then τ defines a soft topology on X and thus (X,τ,E) is a soft topological space over X.
Clearly the soft closed sets are $\tilde{X},\Phi,(F_1,E)^c,(F_2,E)^c,(F_3,E)^c,\dots,(F_{15},E)^c$.
Then, let us take $(G,E)=\{(e_3,\{x_2\})\}$ then $cl((G,E))=(F_1,E)^c$,
$int(cl((G,A)))=(F_8,E)$, and so $(G,E)\,\tilde{\subseteq}\, int(cl((G,E)))$, hence (G,E) is soft pre-open but not soft open.
Let (F,A) be a subset of a soft topological space X. The soft preclosure and preinterior of (F,A) are denoted by $spcl((F,A))$ and $spint((F,A))$, respectively.
Definition 17. Let (X,τ,E) be a soft topological space over X and (F,A) be a soft set over X. Then:
(1) $spint((F,A))=\tilde{\cup}\,\{(O,E):(O,E)\ is\ a\ soft\ pre-open\ and\ (O,E)\,\tilde{\subseteq}\,(F,A)\}$ is called preinterior.
(2) $spcl((F,A))=\tilde{\cap}\,\{(F,E):(F,E)\ is\ a\ soft\ pre-closed\ and\ (F,A)\,\tilde{\subseteq}\,(F,E)\}$ is called preclosure.
Clearly $spint((F,A))$ is the largest soft pre-open set over X which is contained in (F,A) and $spcl((F,A))$ is the smallest soft pre-closed set over X which contains (F,A).
Theorem 2. Let (F,A) be any soft set in a soft space (X,τ,E). Then,
(1) $spcl((F,A)^c)=X-spint((F,A))$ and
(2) $spint((F,A)^c)=X-spcl((F,A))$.
Proof (1) We see that a soft pre-open set $(G,B)\,\tilde{\subseteq}\,(F,A)$ is precisely the complement of a pre-closed set $(H,C)\,\tilde{\supseteq}\,(F,A)^c$. Thus

$$spint((F,A))=\tilde{\cup}\,\{(H,C)^c:(H,C)\ is\ soft\ pre-closed\ and\ (H,C)\,\tilde{\supseteq}\,(F,A)^c\}$$
$$=X-\tilde{\cap}\,\{(H,C):(H,C)\ is\ soft\ pre-closed\ and\ (H,C)\,\tilde{\supseteq}\,(F,A)^c\}$$

$=X-spcl((F,A)^c)$, whence
$spcl((F,A)^c)=X-spint((F,A))$.
(2) Next let (O,E) be any soft pre-open set. Then for a soft pre-closed set $(U,E)\,\tilde{\supseteq}\,(F,A)$, $(O,E)=(U,E)^c\,\tilde{\subseteq}\,(F,A)^c$.

$$spcl((F,A))=\tilde{\cap}\,\{(O,E)^c:(O,E)\ is\ soft\ pre-open\ and\ (O,E)\,\tilde{\subseteq}\,(F,A)^c\}$$
$$=X-\tilde{\cup}\,\{(O,E):(O,E)\ is\ soft\ pre-open\ and(O,E)\,\tilde{\subseteq}\,(F,A)^c\}$$

$=X-spint((F,A)^c)$. Thus
$spint((F,A)^c)=X-spcl((F,A))$.
Theorem 3. In a soft topological space (X,τ,E), a soft set (F,A) is soft pre-closed (pre-open) iff $(F,A)=spcl(F,A)$ $((F,A)=spint(F,A))$.
Proof Let (F,A) be soft pre-closed in (X,τ,E). Since $(F,A)\,\tilde{\subseteq}\,(F,A)$ and (F,A) is soft pre-closed.
$(F,A)\in\{(F,E):(F,E)\ is\ soft\ pre-closed\ and\ (F,A)\,\tilde{\subseteq}\,(F,E)\}$,and
$(F,A)\,\tilde{\subseteq}\,(F,E)$ implies that
$(F,A)=\tilde{\cap}\,\{(F,E):(F,E)\ is\ soft\ pre-closed\ and\ (F,A)\,\tilde{\subseteq}\,(F,E)\}$, i.e.
$(F,A)=spcl((F,A))$.
Conversely suppose that $(F,A)=spcl((F,A))$, i.e.

$(F,A) = \tilde{\cap} \{(F,E): (F,E)\ is\ soft\ pre-closed\ and\ (F,A) \tilde{\subset} (F,E)\}$.
This implies that $(F,A) \in \{(F,E): (F,E)\ is\ soft\ pre-closed\ and\ (F,A) \tilde{\subset} (F,E)\}$.
Hence (F,A) is soft pre-closed. For the soft preopenness case use the definition of soft preinteriors.
Proposition 1. In a soft space (X,τ,E), the folowing hold for soft preclosure:
(1) $spcl(\emptyset) = \emptyset$,
(2) $spcl((F,A))$ is soft pre-closed in (X,τ,E) for each soft subset (F,A) of X,
(3) $spcl((F,A)) \tilde{\subset} spcl((G,B))$, if $(F,A) \tilde{\subset} (G,B)$,
(4) $spcl(spcl((F,A))) = spcl((F,A))$.
Proof Easy.
Theorem 4. In a soft topological space the following relations hold:
(1) $spcl((F,A) \tilde{\cup} (G,B)) \tilde{\supset} spcl((F,A)) \tilde{\cup} spcl((G,B))$,
(2) $spcl((F,A) \tilde{\cap} (G,B)) \tilde{\subset} spcl((F,A)) \tilde{\cap} spcl((G,B))$.
Proof Easy.
Remark 2. Similar results hold for soft preinteriors.
Definition 18. [4] Let (X,τ,E) be a soft topological space over X and Y be a non-empty subset of X. Then

$$\tau_Y = \{({}^Y F,E) | (F,E) \in \tau\}$$

is said to be the soft relative topology on Y and (Y,τ_Y,E) is called a soft subspace of (X,τ,E).
We can easily verify that τ_Y is, in fact, a soft topology on Y.
Theorem 5. Let $(F,A) \subset Y \subset X$, where (X,τ,E) is a soft topological space and (Y,τ_Y,E) is a soft pre-open subspace of (X,τ,E). Then (F,A) is soft pre-open in X iff it is soft pre-open in Y.
Proof Since (F,A) is soft pre-open in X, $(F,A) \tilde{\subset} int(cl((F,A)))$. But $(F,A) \subset Y$ implies that $(F,A) \tilde{\cap} Y = (F,A)$, so that $((F,A) \tilde{\cap} Y) \tilde{\subset} int(cl((F,A) \tilde{\cap} Y))$. Therefore (F,A) is soft pre-open in Y.
Conversely let (F,A) be soft pre-open in Y. Then $(F,A) \tilde{\subset} int_Y\left(cl_Y\big((F,A)\big)\right)$ which is soft pre-open in Y. There exists an open soft set (O,E) in X such that $(O,E) \tilde{\cap} \tilde{Y} = int_Y\left(cl_Y\big((F,A)\big)\right)$. Thus $(F,A) \tilde{\subset} (O,E) \tilde{\cap} \tilde{Y}$. But Y is soft pre-open in X. Thus, $Y \tilde{\subset} int(cl(Y))$.
Hence we have $(F,A) \tilde{\subset} (O,E) \tilde{\cap} int(cl(Y)) \tilde{\subset} int(cl((O,E) \tilde{\cap} \tilde{Y})))$

$$= int\left(cl\left(int_Y\left(cl_Y\big((F,A)\big)\right)\right)\right) \tilde{\subset} int\left(cl\left(cl_Y\big((F,A)\big)\right)\right) \tilde{\subset} int\left(cl\left(cl\big((F,A)\big)\right)\right) = int\left(cl\big((F,A)\big)\right).$$

Hence (F,A) is soft pre-open in X.

4. SOME PROPERTIES OF SOFT PRE-SEPARATIONS AXIOMS

Arokia and Arockiarani [24], defined some notions of soft pre-separation axioms. The this section, we introduce and study the new concepts of soft pre-separation axioms and investigated basic properties of these concepts in soft topolgical space.
Definition 19. [24] Let (X,τ,E) be a soft topological space over X and $x,y \in X$ such that $x \neq y$. Then a soft topological space (X,τ,E) is said to be soft P_0-space if there exist soft pre-open sets (F,E) and (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ or $y \in (G,E)$ and $x \notin (G,E)$.
Example 2. A discrete soft topological space is a soft P_0-space since every $x_1 \in X$ is a soft pre-open set in the discrete space.
A characterization for soft P_0-space is the following.
Theorem 6. A soft topological space (X,τ,E) is soft P_0-space iff soft preclosures of any two different soft singletons are distinct
Proof Let (X,τ,E) be soft P_0-space, and (x_1,E), (x_2,E) be two soft singletons, where $x_1 \neq x_2$. (X,τ,E) being soft P_0-space, there exists a soft pre-open set (U,E) such that $(x_1,E) \tilde{\subset} (U,E) \tilde{\subset} (x_2,E)^c$.
This implies $(x_2,E) \tilde{\subset} spcl((x_2,E)) \tilde{\subset} (U,E)^c$. Since $(x_1,E) \tilde{\not\subset} (U,E)^c$, $(x_1,E) \tilde{\not\subset} spcl((x_2,E))$. But $(x_1,E) \tilde{\not\subset} spcl((x_1,E))$. Hence $spcl((x_1,E)) \neq spcl((x_2,E))$.
Conversely, let (x_1,E) and (x_2,E) be any two soft singletons such that $x_1 \neq x_2$.
By hypothesis $(x_1,E) \notin spcl((x_2,E))$ or $(x_2,E) \notin spcl((x_1,E))$.
Then since $(x_1,E) \notin spcl((x_2,E))$, $(pcl((x_2,E)))^c$ is a soft pre-open set such that $x_1 \in (spcl((x_2,E)))^c$ or since $(x_2,E) \notin spcl((x_1,E))$, $(pcl((x_1,E)))^c$ is a soft pre-open set such that $x_2 \in (pcl((x_1,E)))^c$.
Thus (X,τ,E) is soft P_0-space.
Theorem 7. A soft subspace of a soft P_0-space is soft P_0.
Proof Let Y be a soft subspace of a soft P_0-space X and let y_1, y_2 be two distinct soft points of Y. Since these soft points are also in X, then at least one a soft pre-open set (H,E) containing one soft point but not the other. Then $(H,E) \tilde{\cap} \tilde{Y}$ is a soft pre-open set containing one soft point but not the other.
Definition 20. [24] Let (X,τ,E) be a soft topological space over X and $x,y \in X$ such that $x \neq y$. Then a soft topological space (X,τ,E) is said to be soft P_1-space if there exist soft pre-open sets (F,E) and (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ and $y \in (G,E)$ and $x \notin (G,E)$.
Example 3. Let $X = \{x_1,x_2\}$, $E = \{e_1,e_2\}$ and $\tau = \{\Phi, X, (F_1,E), (F_2,E)\}$ where
$(F_1,E) = \{(e_1,\{x_1\})\}$, $(F_2,E) = \{(e_1,\{x_1,x_2\}), (e_2,\{x_2\})\}$. Then τ defines a soft topology on X and thus (X,τ,E) is a soft topological space over X.
For $x_1,x_2 \in X$, since $x_1 \neq x_2$, we can take soft pre-open sets (F_1,E) and (F_2,E) satisfyig $x_1 \in (F_1,E)$ and $x_2 \notin (F_1,E)$; and $x_2 \in (F_2,E)$ and $x_1 \notin (F_2,E)$. Hence the soft topological (X,τ,E) is a soft P_1-space.

Remark 3. Obviously every soft P_1-space is soft P_0-space but the converse is not always true, as shown in the next example.
Example 4. Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E)\}$ where
$(F_1, E) = \{(e_1, \{x_1\})\}$, $(F_2, E) = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}$. Then τ defines a soft topology on X and thus (X, τ, E) is a soft topological space over X.
For $x_1, x_2 \in X$, since $x_1 \neq x_2$, we can take soft pre-open sets (F_1, E) and (F_2, E) satisfyig $x_1 \in (F_1, E)$ and $x_2 \notin (F_1, E)$. Thus the soft topological (X, τ, E) is a soft P_0-space but not soft P_1-space.
Theorem 8. A soft subspace of a soft P_1-space is soft P_1.
Proof Let Y be a soft subspace of a soft P_0-space X and let y_1, y_2 be two distinct soft points of Y. Since these soft points are also in X, then there exist soft pre-open sets (F, E) and (G, E) such that $y_1 \in (F, E)$ and $y_2 \notin (F, E)$; and $y_2 \in (G, E)$ and $y_1 \notin (G, E)$. Then $(F, E) \tilde{\cap} \tilde{Y}$ and $(G, E) \tilde{\cap} \tilde{Y}$ are soft pre-open set containing one soft point but not the other in Y. Therefore Y is a soft P_1-space.
Theorem 9. Let (X, τ, E) be a soft topological space over X. If every soft point of a soft topological space (X, τ, E) is a soft pre-closed set, then (X, τ, E) is a soft P_1-space.
Proof Let x_1 be a soft point of X which is a soft pre-closed set then $\{x_1\}^c$ is a soft pre-open set. Then for distinct soft points x_1, x_2, we have $\{x_1\}^c$, $\{x_2\}^c$ are soft pre-open sets such that $x_2 \in \{x_1\}^c$ and $x_2 \notin \{x_1\}$; $x_1 \in \{x_2\}^c$ and $x_1 \notin \{x_2\}$.
Definition 21. [24] Let (X, τ, E) be a soft topological space over X and $x, y \in X$ such that $x \neq y$. Then a soft topological space (X, τ, E) is said to be soft P_2-space if there exist soft pre-open sets (F, E) and (G, E) such that $x \in (F, E)$, $y \in (G, E)$ and $(F, E) \tilde{\cap} (G, E) = \Phi$.
Example 5. Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, X, (F_1, E), (F_2, E)\}$ where
$(F_1, E) = \{(e_1, \{x_1\}), (e_2, \{x_1\})\}$, $(F_2, E) = \{(e_1, \{x_2\}), (e_2, \{x_2\})\}$. Then τ defines a soft topology on X and thus (X, τ, E) is a soft topological space over X.
For $x_1, x_2 \in X$, since $x_1 \neq x_2$, we can take soft pre-open sets (F_1, E) and (F_2, E) satisfyig $x_1 \in (F_1, E)$, $x_2 \in (F_2, E)$ and $(F_1, E) \tilde{\cap} (F_2, E) = \Phi$. Therefore the soft topological (X, τ, E) is a soft P_2-space.
Remark 4. Obviously every soft P_2-space is soft P_1-space but, the converse is not always true, as shown in the next example.
Example 6. It is shown by Example 3 because of $(F_1, E) \tilde{\cap} (F_2, E) \neq \Phi$.
Theorem 10. A soft subspace of a soft P_2-space is soft P_2.
Proof Let (X, τ, E) be a soft P_2-space and Y be a soft subspace of X. Let y_1 and y_2 be two distinct soft points of Y. Since X is soft P_2-space, there exist two disjoint soft pre-open sets (H, E) and (G, E) such that $y_1 \in (H, E)$, $y_2 \in (G, E)$. Then $(H, E) \cap Y$ and $(G, E) \cap Y$ are soft pre-open sets satisfying the requirements for Y to be a soft P_2-space.
Theorem 11. Let (X, τ, E) be a soft topological space over X and $x \in X$. If X is a soft P_2-space, then $(x, E) = \tilde{\cap} (F, E)$ for each soft pre-open set (F, E) with $x \in (F, E)$.
Proof Suppose there exists $z \in X$ such that $x \neq z$ and $z \in \cap F(\alpha)$ for some $\alpha \in E$. Since X is soft P_2-space, there exist soft pre-open sets (H, E) and (G, E) such that $x \in (H, E)$ and $z \in (G, E)$ and $(H, E) \tilde{\cap} (G, E) = \Phi$ and so $(H, E) \cap (z, E) = \Phi$ and $H(\alpha) \cap z(\alpha) = \emptyset$. This contradicts the fact that $z \in \cap F(\alpha)$ for some. This completes the proof.
Corollary 1. Let (X, τ, E) be a soft topological space over X and $x \in X$. If X and E are finite, and if X is a soft P_2-space, then (x, E) is a soft pre-open set for $x \in X$.
Definition 22. Let (X, τ, E) be a soft topological space over X, and let (G, E) be a soft pre-closed set in X and $x \in X$ such that $x \notin (G, E)$. If there exist soft pre-open sets (F_1, E) and (F_2, E) such that $x \in (F_1, E)$, $(G, E) \tilde{\subseteq} (F_2, E)$ and $(F_1, E) \tilde{\cap} (F_2, E) = \Phi$, then (X, τ, E) is called a soft pre-regular space.
Proposition 2. Let (X, τ, E) be a soft topological space over X. If every soft pre-open set of X is closed, then X is soft pre-regular space.
Proof Let every soft pre-open set in X is closed, and let (K, E) be a soft pre-closed set in X and $x \in X$ such that $x \in (K, E)^c$. Then (K, E) and $(K, E)^c$ are soft pre-open sets, which containing (K, E) and x, respectively. Viz $x \in (K, E)^c$, $(K, E) \tilde{\subseteq} (K, E)$ and $(K, E)^c \tilde{\cap} (K, E) = \Phi$, then X is a soft pre-regular space.
Example 7. Since every soft pre-open set in discrete topology on X is closed, X is soft pre-regular.
Lemma 2. Let (X, τ, E) be a soft topological space over X, and let (G, E) be a soft pre-closed set in X and $x \in X$ such that $x \notin (G, E)$. If (X, τ, E) is a soft pre-regular space, then there exist soft pre-open sets (F, E) such that $x \in (F, E)$ and $(F, E) \tilde{\cap} (G, E) = \Phi$.
Theorem 12. Let (X, τ, E) be a soft topological space over X and $x \in X$. If X is a soft pre-regular space, then:
(1) For a soft pre-closed set (G, E), $x \notin (G, E)$ iff $(x, E) \tilde{\cap} (G, E) = \Phi$.
(2) For a soft pre-open set (F, E), $x \notin (F, E)$ iff $(x, E) \tilde{\cap} (F, E) = \Phi$.
Proof (1) Let $x \notin (G, E)$. By Lemma 2, there exists a soft pre-open set (F, E) such that $x \in (F, E)$ and $(F, E) \tilde{\cap} (G, E) = \Phi$. Since $(x, E) \tilde{\subseteq} (F, E)$, we have $(x, E) \tilde{\cap} (G, E) = \Phi$.
The converse is obtained by Lemma 1(2).
(2) Let $x \notin (F, E)$. Then there are two cases:
(i) $x \notin F(\alpha)$ for all α∈E and
(ii) $x \notin F(\alpha)$ and $x \in F(\beta)$ for some $\alpha, \beta \in E$.
In case (i) it is obvious that $(x, E) \tilde{\cap} (F, E) = \Phi$. In the other case, $x \in F^c(\alpha)$ and $x \notin F^c(\beta)$ for some $\alpha, \beta \in E$ and so $(F, E)^c$ is a soft pre-closed set such that $x \notin (F, E)^c$, by (1), $(x, E) \tilde{\cap} (F, E)^c = \Phi$. So $(x, E) \tilde{\subseteq} (F, E)$ but this contradicts $x \notin F(\alpha)$ for some $\alpha \in E$. Consequently, we have $(x, E) \tilde{\cap} (F, E) = \Phi$.
The converse is obvious.
Theorem 13. Let (X, τ, E) be a soft topological space over X and $x \in X$. Then the following are equivalent:
(1) (X, τ, E) is a soft pre-regular space.

(2) For each soft pre-closed set (G,E) such that $(x,E)\tilde{\cap}(G,E)=\Phi$, there exist soft pre-open sets (F_1,E) and (F_2,E) such that $(x,E)\tilde{\subseteq}(F_1,E)$, $(G,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E)=\Phi$.

Proof The proof is obvious, obtained by Theorem 12(1) and Lemma 1(1).

Theorem 14. Let (X,τ,E) be a soft topological space over X and $x\in X$. If X is a soft pre-regular space, then:

(1) For a soft pre-open set (F,E), $x\in(F,E)$ iff $x\in F(\alpha)$ for some $\alpha\in E$.

(2) For a soft pre-open set (F,E), $(F,E)=\tilde{\cup}\{(x,E): x\in F(\alpha)$ for some $\alpha\in E\}$.

(3) For each $\alpha,\beta\in E$, $\tau_\alpha=\tau_\beta$.

Proof (1) Assume that for some $\alpha\in E$, $x\in F(\alpha)$ and $x\notin(F,E)$. Then by Theorem 12(2), $(x,E)\tilde{\cap}(F,E)=\Phi$. By the assumption, this is a contradiction and so $x\in(F,E)$.

The converse is obvious.

(2) It follows from (1) and $x\in(F,E)$ iff $(x,E)\tilde{\subseteq}(F,E)$.

(3) It is obtained from (2).

Theorem 15. Let (X,τ,E) be a soft topological space over X. If (X,τ,E) is a soft pre-regular space, then the following are equivalent:

(1) (X,τ,E) is a soft P_1-space.

(2) For $x,y\in X$ with $x\neq y$, there exist soft pre-open sets (F,E) and (G,E) such that $(x,E)\tilde{\subseteq}(F,E)$ and $(y,E)\tilde{\cap}(F,E)=\Phi$, and $(y,E)\tilde{\subseteq}(G,E)$ and $(x,E)\tilde{\cap}(G,E)=\Phi$.

Proof It is obvious that $x\in(F,E)$ iff $(x,E)\tilde{\subseteq}(F,E)$, and by Theorem 12, $x\notin(F,E)$ iff $(x,E)\tilde{\cap}(F,E)=\Phi$. Hence we have that the above statements are equivalent.

Definition 23. Let (X,τ,E) be a soft topological space over X. Then (X,τ,E) is said to be a soft P_3-space if it is a soft pre-regular and soft P_1-space.

Theorem 16. Let (X,τ,E) be a soft topological space over X. If (X,τ,E) is a soft P_3-space, then for each $x\in X$, (x,E) is soft pre-closed.

Proof We show that $(x,E)^c$ is soft pre-open. For each $y\in X-\{x\}$, since (X,τ,E) is a soft pre-regular and P_1-space, by Theorem 15, there exists a soft pre-open set (F_y,E)such that $(y,E)\tilde{\subseteq}(F_y,E)$ ve $(x,E)\,\widetilde{\cap}\,(F_y,E)=\Phi$.

So $\tilde{\cup}_{y\in X-\{x\}}(F_y,E)\tilde{\subseteq}(x,E)^c$. For the other inclusion, let $\tilde{\cup}_{y\in X-\{x\}}(F_y,E)=(H,E)$ where $H(\alpha)=\cup_{y\in X-\{x\}}.\ F_y(\alpha)$ for all $\alpha\in E$. Moreover, from the definition of the relative complement and Definition 10, we know that $(x,E)^c=(x^c,E)$, where $x^c(\alpha)=X-\{x\}$ for each $\alpha\in E$. Now, for each $y\in X-\{x\}$ and for each $\alpha\in E$, $x^c(\alpha)=X-\{x\}=\cup_{y\in X-\{x\}}\{y\}=\cup_{y\in X-\{x\}}y(\alpha)\subset\cup_{y\in X-\{x\}}F(\alpha)=H(\alpha)$. By Definition 2, this implies that $(x,E)^c\tilde{\subseteq}\tilde{\cup}_{y\in X-\{x\}}(F_y,E)$, and so $(x,E)^c=\tilde{\cup}_{y\in X-\{x\}}(F_y,E)$. Since (F,E) is soft pre-open for each $\in X-\{x\}\in X-\{x\}$, consequently, (x,E) is soft pre-closed.

Theorem 17. A soft P_3-space is soft P_2.

Proof Let (X,τ,E) be any soft P_3-space. For $x,y\in X$ with $x\neq y$, by the above Theorem 16, (y,E) is soft pre-closed and $x\notin(y,E)$. From the soft regularity, there exist soft pre-open sets (F_1,E) and (F_2,E) such that $x\in(F_1,E)$, $y\in(y,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E)=\Phi$. Thus (X,τ,E) is soft P_2.

Definition 24. A soft topological space (X,τ,E) is said to be a soft pre-normal space if for every pair of disjoint pre-closed soft sets (F,E) and (G,E), there exist two disjoint soft pre-open sets (F_1,E) and (F_2,E) such that $(F,E)\tilde{\subseteq}(F_1,E)$ and $(G,E)\tilde{\subseteq}(F_2,E)$.

Remark 5. We give an example to show that every soft pre-normal space does not have to be both soft pre-regular and soft P_1-space.

Example 8. Let $X=\{x_1,x_2,x_3\}$, $E=\{e_1,e_2\}$ and
$\tau=\{\Phi,X,(F_1,E),(F_2,E),(F_3,E)\}$ where
$(F_1,E)=\{(e_1,\{x_1\}),(e_2,\{x_1\})\}$,
$(F_2,E)=\{(e_1,\{x_2\}),(e_2,\{x_2\})\}$,
$(F_3,E)=\{(e_1,\{x_1,x_2\}),(e_2,\{x_1,x_2\})\}$.

Then τ defines a soft topology on X and therefore (X,τ,E) is a soft topological space over X. Also (X,τ,E) soft pre-normal space over X, but neither soft pre-regular nor soft P_1-space.

Theorem 18. A soft topological space (X,τ,E) is soft pre-normal iff for any soft pre-closed set (F,E) and soft pre-open set (G,E) containing (F,E), there exists a soft pre-openset (H,E) such that $(F,E)\tilde{\subseteq}(H,E)$ and $spcl((H,E))\tilde{\subseteq}(G,E)$.

Proof Let (X,τ,E) be soft pre-normal space and (F,E) be a soft pre-closed set and (G,E) be a soft pre-open set containing $(F,E)\Rightarrow(F,E)$ and $(G,E)^c$ are disjoint soft pre-closed sets $\Rightarrow\exists$ two disjoint soft pre-open sets (H,A), (H,B) such that $(F,E)\tilde{\subseteq}(H,A)$ and $(G,E)^c\tilde{\subseteq}(H,B)$.

Now $(H,A)\tilde{\subseteq}(H,B)^c\Rightarrow spcl((H,A))\tilde{\subseteq}spcl((H,B)^c)=(H,B)^c$.

Also, $(G,E)^c\tilde{\subseteq}(H,B)\Rightarrow(H,B)^c\tilde{\subseteq}(G,E)\Rightarrow spcl((H,A))\tilde{\subseteq}(G,E)$.

Conversely, let (L,E) and (K,E) be any disjoint pair soft pre-closed sets $\Rightarrow(L,E)\tilde{\subseteq}(K,E)^c$, then by hypothesis there exists a soft pre-open set (H,E) such that $(L,E)\tilde{\subseteq}(H,E)$ and
$spcl((H,E))\tilde{\subseteq}(K,E)^c\Rightarrow((K,E))\tilde{\subseteq}(spcl((H,E)))^c\Rightarrow(H,E)$ and $(spcl((H,E)))^c$ are disjoint soft pre-open sets such that $(L,E)\tilde{\subseteq}(H,E)$ and $(K,E)\tilde{\subseteq}(spcl((H,E)))^c$.

Definition 25. A soft pre-normal P_1-space is called a soft P_4-space.

Example 9. Let $X=\{x_1,x_2\}$, $E=\{e_1,e_2\}$ and
$\tau=\{\Phi,X,(F_1,E),(F_2,E)\}$ where
$(F_1,E)=\{(e_1,\{x_1\}),(e_2,\{x_1\})\}$,
$(F_2,E)=\{(e_1,\{x_2\}),(e_2,\{x_2\})\}$.

Then τ defines a soft topology on X and hence (X, τ, E) is a soft topological space over X. Moreover (X, τ, E) is a soft P_4-space over X.

Theorem 19. Every soft P_4-space is soft P_3.

Proof Let X be a soft P_4-space. Since X is also the soft P_1-space, only soft pre-regular is enough to show that space. Let (K, E) be a soft pre-closed set in X and $x \notin (K, E)$. Since X is soft P_1-space, $\{x\}$ is soft pre-closed. Then $(K, E) \tilde{\cap} \{x\} = \Phi$, and since X is soft pre-regular, there exist soft pre-open sets (F_1, E) and (F_2, E) in X such that $\{x\} \tilde{\subseteq} (F_1, E)$ and $(K, E) \tilde{\subseteq} (F_2, E)$ and $(F_1, E) \tilde{\cap} (F_2, E) = \Phi$. Thus X is soft pre-regular, and so soft P_3-space.

Remark 6. Every soft P_4-space is soft P_3-space, every soft P_3-space is soft P_2-space, every soft P_2-space is soft P_1-space and every soft P_1-space is soft P_0-space.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

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