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On Some Advancements within Certain Multicriteria Decision Making Support Methodology

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Deriving true priority vectors from intuitive pairwise comparison matrices (PCMs) and consistency measurement of decision makers judgments about their genuine weights are crucial issues within the multicriteria decision making support methodology called Analytic Hierarchy Process (AHP). The most popular procedure in the ranking process, constitutes the Right Eigenvector Method (REV). The inventor of the AHP convinces that as long as inconsistent PCMs are allowed in the AHP none of the other existing procedures qualify and the REV provides the only right solution in this process. The objective of this scientific paper is to examine if the former opinion can be considered as experimentally confirmed. For this purpose it was decided to apply Monte Carlo methodology. However, rather than simulate and analyze simulations results for a single PCM, as it has been done so far by many other authors, we decided to design and analyze computer simulations results for a singular model of the AHP framework. Our findings lead to inevitable conclusion that the REV cannot longer be perceived as a dominant procedure within the AHP methodology, especially when nonreciprocal PCMs are considered. It was verified empirically in our research that in the situation when nonreciprocal PCMs are considered the REV impoverishes the entire AHP methodology by its lack of PCMs inconsistency measure in such cases. Moreover, it provides less accurate rankings for a particular decision in comparison to other presented methods. It was also unequivocally verified that the enforced reciprocity of PCM leads directly to worse estimates of priorities weights. Altogether, it seems very important from the perspective of methodology supporting multicriteria decision making, the crucial process embedded in most of management activity. In the consequence, because the REV recedes other prioritization procedures available for the AHP methodology, it is advised to consider them instead, especially under some circumstances of an important and very tight managerial decisions.

Keywords: Analytic hierarchy process, right eigenvector, prioritization, ranking, constrained optimization, consistency measurement

Introduction

Probably all of us at certain point of our life were involved in a decision where the numbers told us to do one thing but our intuition told us something else. As an individual we have that luxury to dismiss the numbers and trust our intuition. Obviously, a corporate decision-making group or a government agency should not and cannot proceed in this way. Besides, there are plenty examples that intuition-based decision making can lead to fallacious conclusions. This phenomena is probably the fundamental reason why scientists continuously deal with explanation and modeling of decisional problems in the way that common human being can comprehend them. This research paper focuses on a decision making support methodology called Analytic Hierarchy Process (AHP).

Judging by the number of articles devoted to the AHP it seems that it is the most widely used decision making approach in the world today, as well most validated methodology – thousands of actual applications in which the AHP results were accepted and used by the competent decision markets, see e.g. Kazibudzki (2012) and references in there, as well (Grzybowski, 2012; Ishizaka & Labib, 2011; Ho, 2008; Vaidya & Kumar, 2006). However, despite of its popularity it has been also criticized mainly for the mathematical analysis which it applies, i.e. the right eigenvector method (REV), what constitutes the main point of reference for this paper. It is so because the REV is supposed to operate only with reciprocal pairwise comparisons matrices (reciprocal PCMs), otherwise it is not possible to measure consistency of decision makers judgments (Saaty's consistency index is inexplicable for nonreciprocal PCMs). That entails a limited range of applications and increase estimation errors. As a result, in practice reciprocity of PCMs is a very popular requirement, although many authors argue that it is an artificial condition which impoverish the PCMs about information concerning the unknown priority vector that otherwise could be revealed (Grzybowski, 2012; Basak, 1998; Budescu, Zwick & Rapoport, 1986; Hovanov, Kolari & Sokolov, 2008; Lipovetsky & Tishler, 1997; Zahedi, 1986). Thus, taking above into consideration, we argue that

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it is justifiable to search for other methods that could operate within the Analytic Hierarchy Process in order to face the critique, and in the consequence eliminate some flaws of the methodology and the concept itself, which in our opinion is very applicable. That is why we decided to reintroduce the concept of Logarithmic Utility Approach to the REV (Kazibudzki, 2012) and compare its performance with the REV and three other, rated best procedures (Choo & Wedley, 2004; Lin, 2007; Grzybowski, 2012) available for the AHP. For this purpose Monte Carlo methodology was applied and validation studies were designed accordingly. Although, we refined simulation frameworks already proposed in the literature, e.g. (Basak, 1998; Choo & Wedley, 2004; Lin, 2007; Grzybowski, 2012; Zahedi, 1986), our simulation scenario and research focus have never been taken into consideration yet. For instance in the simulation framework described in (Choo & Wedley, 2004) it was proposed to study two types of PCMs: the PCMs that contain many small errors and the PCMs that contain one large error. In the revised framework described in (Lin, 2007) author proposed to consider additionally PCMs with many large errors as well as with many small and one large error. However, the simulations described in (Choo & Wedley, 2004; Lin, 2007) were based only on one known priority vector which was not normalized and thus the observed average errors are not comparable with errors corresponding to other vectors having different dimensions and priority values.

Furthermore, we also took into account rounding errors as in Grzybowski (2012) because randomly disturbed ratios were rounded to the closest values from a particular scale in order to make the simulation scenario truly realistic. Thus, it seems to reflect real AHP procedure that assumes that decision makers must express their opinions on a given scale. Certainly, this prerequisite leads directly and inevitably to additional source of errors that should be taken into account, see e.g. (Dong, Xu, Li & Dai, 2008). Moreover, in order to make the results of our research more representative we examined a performance of the entire AHP framework (not single PCM) with different number of criteria and different number of alternatives.

Thus, we compared results of random normalized priority vectors within the most simple AHP framework, comprising three levels: goal, criteria and alternatives. In this way we adapted the scenario procedure described in Kazibudzki (2012 & 2013) but as opposite to the research described in there we focused our attention on the existing information gap that was not covered yet, i.e. nonreciprocal PCMs performance within singular AHP frameworks and related consistency measurement in these cases.

Notations and Principles of the Analytic Hierarchy Process

The AHP is grounded on the well-defined mathematical structure of consistent matrices and their associated right-eigenvector's ability to generate true or approximate weights (Kazibudzki, 2012; Grzybowski, 2012; Merkin, 1979; Saaty, 1990). Fundamentally a problem of deriving priority weights from so called pairwise comparison matrix (PCM) denoted as $A(\boldsymbol{a})=[a_{ij}]_{nxn}$ with elements $a_{ij}=a_i/a_j$ is to estimate a priority vector (PV) $\boldsymbol{w}=[w_1, w_2, w_3,\ldots, w_n]^T$ on the bases of the matrix $A(\boldsymbol{a})$ which comprises a decision maker (DM) pairwise comparisons judgments concerning the importance of a given binary set of alternatives. Commonly the priority weights w_i , i=1,…,n, are chosen to be positive and normalized to unity: $\sum_i^n w_i = 1$, and the elements a_{ij} of the matrix $A(\boldsymbol{a})$ are then the DM judgments about the priority ratios w_i / w_j , i,j=1,…,n, where n is the number of all alternatives being considered. In a perfect judgment case then, we have:

$$A \cdot \boldsymbol{w} = n \cdot \boldsymbol{w} \qquad [1]$$

and in this situation the PV $\boldsymbol{w}$ can be computed by solving the eigenvector equation. It is so because the number n in the perfect case (matrix $A(\boldsymbol{a})$ is consistent) is the principal eigenvalue of $A(\boldsymbol{a})$, i.e. the largest eigenvalue obtained from the solution of the characteristic equation:

$$\det(A - lambda \cdot I) \qquad [2]$$

where I denotes identity matrix of order n. In this case it is also the only nonzero eigenvalue. On the other hand, when the case is not perfect (matrix $A(\boldsymbol{a})$ is inconsistent) an estimate of the true PV is normalized right eigenvector (REV) associated with the largest eigenvalue. Thus, in order to obtain the estimate we need to solve the general eigenvector equation:

$$A \cdot \boldsymbol{w} = lambda_{max} \cdot \boldsymbol{w} \qquad [3]$$

where $lambda_{max}$ denotes the principal eigenvalue which is then always bigger than n, is simple and its existence is guaranteed by Perron–Frobenius theorem, while other lambdas are close to zero. If the elements of a matrix $A(\boldsymbol{a})$ satisfy the condition $a_{ij}=1/a_{ji}$ for all i,j=1,…,n then the matrix $A(\boldsymbol{a})$ is said to be reciprocal (RPCM). If its elements satisfy the condition $a_{ik}a_{kj}=a_{ij}$ for all i,j,k=1,…,n and the matrix is reciprocal, then it is called consistent. Finally, the matrix $A(\boldsymbol{a})$ is said to be transitive (TPCM) if the following conditions hold: (i) if for any l=1,…,n, an element a_{lj} is not less than an element a_{lk} then $a_{ij} \geq a_{ik}$ for i=1,…,n, and (ii) if for any l=1,…,n, an element a_{jl} is not less than an element a_{kl} then $a_{ji} \geq a_{ki}$ for i=1,…,n. Certainly, in the case of the reciprocal PCM the two conditions (i) and (ii) are equivalent. It is important to notice that the methodology of capturing the degree of PCM inconsistency in order to derive a vector of priorities is a central point of the AHP and a crucial issue for the whole priority theory. Obviously, significant violations of the consistency may mislead about the value of information obtained making the methodology simply useless. As far it is concerned, the most popular measure of PCM inconsistency belongs to Saaty (1980) and is strictly related to the REV, what makes it especially attractive. The point here is that there is quite few propositions in the literature for consistency measures, see e.g. Grzybowski (2012) or references in Kazibudzki (2012). However, it is argued that so long as inconsistency is tolerated, the REV is the basic theoretical concept for deriving a scale and no other methods qualify, what entails the concern about the usefulness of inconsistency measures derived from them. Although, during the last three decades plenty of other methods have been proposed, see for instance Grzybowski (2012) or/and Kazibudzki (2012) and references in there.

Novel Prioritization Method and Related Consistency Index

There is a well known principle in mathematics saying that a necessary condition for a credible problem solving procedure is "that if it produces desired results, and we perturb the variables of the problem in some small sense, it gives us results that are 'close' to the original ones" (Saaty, 1990, p. 18). It should be noticed that the procedure described in this paper possesses this property. We start from the following theorems (Saaty, 1986, 2006):

Theorem 1: A positive n by n matrix has the ratio form $A(\mathbf{w})=(w_i/w_j)$, $i,j=1,\ldots,n$, if, and only if, it is consistent.

Theorem 2: The matrix of ratios $A(\mathbf{w})=(w_i/w_j)$ is consistent if and only if n is its principal eigenvalue and $A\cdot \mathbf{w} = n\cdot \mathbf{w}$. Further, $\mathbf{w}$ with positive entries is unique to within a multiplicative constant.

Obviously, in the real business situations, when the task is to derive w from pairwise comparisons of elements w_i and w_j on the bases of managerial intuitive judgments, we cannot even expect to have a consistent outcome. Instead of having $A(\mathbf{w})$, we have only its estimate $A(\mathbf{a})$ containing managerial intuitive judgments, more or less close to $A(\mathbf{w})$ in accordance to their judgmental skills and experience. So, when the consistency property does not hold, the relation between the elements of $A(\mathbf{a})$ and $A(\mathbf{w})$ can be expressed in the following form:

$$a_{ij}=e_{ij}\cdot w_i/w_j \qquad [4]$$

where e_{ij} is a perturbation factor which is expected to be near 1, e.g. (Kazibudzki, 2012; Grzybowski, 2012; Saaty, 2003; Sun & Greenberg, 2006; Ishizaka & Labib, 2011). In a statistical approach and many simulation studies the perturbation factor is interpreted as a realization of a random variable, e.g. (Grzybowski, 2012; Kazibudzki, 2013; Zahedi, 1986). This fact however does not hinder to describe the procedure for deriving $\mathbf{w}$ from $A(\mathbf{a})$ in the case of perfect consistency (e_{ij}=1) which relies on the second theorem and fundamental mathematical constrained optimization guidelines (Grzybowski, 2012; Kazibudzki, 2012):

$$\min LUA=\sum_{i=1}^{n}\left(\ln\left(\sum_{j=1}^{n}\frac{a_{ij}\times w_j}{n\times w_i}\right)\right)^2 \qquad [5]$$

subject to:

$$\sum_{j=1}^{n} w_j = 1, \qquad w_i > 0, \qquad i,j=1,\ldots,n$$

Because the utility of the method decreases logarithmically, inversely proportionally to the inconsistency growth, it seems natural to identify it as the Logarithmic Utility Approach (LUA) to the Eigenvector Method (REV). Basically, the concept of the method is to search for the vector $\boldsymbol{w}$ that the multiplication result $A\cdot \mathbf{w}\approx n\cdot \mathbf{w}$ (Grzybowski, 2012). Thus, in the consistent case the solution of the procedure is exactly the same as that given by the REV (theorem 2), whereas in the inconsistent case ($e_{ij}\neq 1$) the solution results with the PV which best fits (from the perspective of criterion [5]) to that one which delivers consistent PCM.

Recalling the fact, that the methodology of capturing the degree of PCM inconsistency is a central point of the AHP and a crucial issue for the whole prioritization theory, we must underline that LUA provides such a inconsistency measure (actually the LUA itself constitutes the measure) what enables decision makers acceptance or rejection of the PV estimate. If we denote the minimum value of the LUA in the relation [5] as MLUA, then we can express our consistency index in the following form:

$$CI(n)=\frac{1}{n}\sqrt{MLUA} \qquad [6]$$

The index can be interpreted as the average deviation from ideal judgment about priorities in the case of perfect consistency i.e. $CI(n)$=0. In the case of unsatisfactory consistency level, the attempt to improve consistency is suggested together with a new $\boldsymbol{w}$ derivation. After each iteration, we assume that the new matrix $A(\mathbf{a})$ is a perturbation of $A(\mathbf{w})$ and its derived PV is a perturbation of $\mathbf{w}$. In the literature we find five conditions for good approximations (Saaty, 2006): reciprocity, homogeneity (the elements being compared must be of the same order of magnitude), independency (judgments about, or the priorities of, the elements in a hierarchy cannot depend on lower level elements), near consistency and uniform continuity (elements w_i, $i=1,\ldots,n$ should be relatively insensitive to small changes in the elements a_{ij}, only then good approximations to the a_{ij} remain w_i/w_j ratios). Beside the reciprocity condition we find them applicable also for the LUA.

Validation Studies on the Bases of Novel Methodical Framework

Description of the simulation scenario

The intent of this section is to evaluate the performance of the LUA on the background of performance of other chosen methods available for the AHP. In order to achieve this objective we are going to proceed with simulations but not such commonly known from literature, i.e. concerned with only one single PCM. Our simulation scenario will involve the entire AHP framework which is supposed to reflect the hypothetical decisional problem (Kazibudzki, 2012 & 2013). For this purpose the following methods will be considered:

– (GM) i.e. geometric mean procedure (Crawford & Williams, 1985) given by the formula:

$$w_i = \left(\prod_{j=1}^{n} a_{ij}\right)^{1/n} \Bigg/ \sum_{i=1}^{n}\left(\prod_{j=1}^{n} a_{ij}\right)^{1/n} \qquad [7]$$

– (REV) i.e. principal right eigenvector method (Saaty, 1980), already described in this paper,

– (LUA) i.e. logarithmic utility approach, earlier introduced and described in this paper,

– (SRDM) i.e. sum of squared relative differences (Grzybowski, 2012), given by the formula:

$$\min SRDM\,(\mathrm{w}) = \sum_{i=1}^{n}\left(\frac{1}{n w_i}\sum_{j=1}^{n} a_{ij} w_j - 1\right)^2 \qquad [8]$$

subject to:

$$\sum_{j=1}^{n} w_j = 1, \qquad w_i > 0, \qquad i, j = 1, \ldots, n$$

– SNCS i.e. simple normalized column sum procedure (Choo & Wedley, 2004), given by the formula:

$$w_i = \frac{1}{n}\sum_{j=1}^{n}\left(a_{ij} \Big/ \sum_{k=1}^{n} a_{kj}\right) \qquad [9]$$

The performance evaluation study of above chosen methods available for the AHP, rated as dominant among others (Choo & Wedley, 2004; Lin, 2007; Grzybowski, 2012) is going to be based on the following seminal assumptions (Kazibudzki, 2012). We assume, the hierarchy consist of three levels: goal, criteria and alternatives, which is supposed to reflect the hypothetic case of real decisional problem. Then, in order to compare the accuracy of the estimations obtained by chosen methods we simulate different situations related to various sources of the PCM inconsistency (Grzybowski, 2012). Fundamentally, the inconsistency commonly results from errors caused by the nature of human judgments and errors due to the technical realization of the comparison procedure i.e. rounding errors and errors resulting from the forced reciprocity requirement. Nature of human judgments can be represented as the realization of some random process in accordance with the formula [4]. Probability distributions of the perturbation factor e_{ij} mainly involve uniform and gamma, as well truncated normal or log-normal (Kazibudzki, 2013; Basak, 1998; Choo & Wedley, 2004; Lin, 2007; Zahedi, 1986). The rounding errors, on the other hand, are related to the numerical ratio scale whose values should be used by prospective decision makers in order to express somehow their judgments (Grzybowski, 2012; Dong, Xu, Li & Dai, 2008; Lipovetsky & Tishler, 1997; Lipovetsky & Conklin, 2002). In conventional AHP applications the most popular is Saaty's numerical scale which comprises the integers from 1 to 9 and their reciprocals but there exist also others i.e. geometric scale and numerical scale. The first one usually consists of the numbers computed in accordance with the formula $\left(\sqrt{2}\right)^n$ where n comprises the integers from minus 8 to 8. The latter involves arbitrary integers from 1 to n and their reciprocals.

Basically, in order to run the validation studies of the above presented methods we refined simulation frameworks already proposed in the literature, e.g. (Basak, 1998; Choo & Wedley, 2004; Lin, 2007; Grzybowski, 2012; Zahedi, 1986). For instance in the simulation framework described in (Choo & Wedley, 2004) it was proposed to study two types of PCMs: the PCMs that contain many small errors and the PCMs that contain one large error. In the revised framework described in (Lin, 2007) author proposed to consider additionally PCMs with many large errors as well as with many small and one large error. In our simulations we adapted these frameworks in order to make them more representative and realistic. Firstly, the simulations described in (Choo & Wedley, 2004; Lin, 2007) were based only on one known priority vector. Moreover, the vector was not normalized and thus the observed average errors cannot be compared with errors corresponding to other vectors having different dimensions and priority values. Secondly, we also took into account the rounding errors. Therefore the randomly disturbed ratios were rounded to the closest values from particular scale (Grzybowski, 2012; Dong, Xu, Li & Dai, 2008). However, the simulations described in Grzybowski (2012) were designed exclusively for performance measurement of procedures operating within different but single priority vectors. In order to make the results more representative in our simulation program we used the entire AHP framework with different number of criteria and different number of alternatives comprising many random normalized priority vectors– in the most simple AHP framework considered, with three levels: goal, criteria and alternatives (Kazibudzki, 2012 & 2013). Thus, our simulation scenario realizes the steps proposed in Kazibudzki (2012), but the difference is we focus more on nonreciprocal cases and related inconsistency measurement in such situations. Certainly, some performance statistics are calculated as the scenario is iterated prescribed number of times in order to obtain such mean values of performance measures like: the Pearson correlation coefficient (PCC), Spearman rank correlation coefficient (SRCC) (Grzybowski, 2012; Moy, Lam & Choo, 1997; Budescu, Zwick & Rapoport, 1986), and mean absolute deviation (Kazibudzki, 2012; Choo & Wedley, 2004; Lin, 2007; Dong, Xu, Li & Dai, 2008). There are considered two approximation options within the simulation scenario: with and without forced reciprocity. When forced reciprocity condition is executed, the perturbed PCM inputs are taken only from above its diagonal elements, and the remaining ones are entered as the inverses of the corresponding symmetric units in relation to its diagonal elements (Kazibudzki, 2012; Grzybowski, 2012). Matrices with forced reciprocity condition applied we denote as FRPCMs, whereas the other ones as APCMs (arbitrary pairwise comparisons matrices).

Exemplification of the simulation framework

In order to clarify the methodology lying behind the scenario introduced in former subchapter of this section we are going to present the simplified example now. For the illustration purpose we take into consideration only technical perturbation of PCMs resulting from rounding errors (Saaty's scale) and forced reciprocity. Let us consider the following ideal model of the AHP framework with three levels (four criteria and four alternatives):

With respect to the GOAL:

$$\begin{array}{c} \\ c1 \\ c2 \\ c3 \\ c4 \end{array} \begin{array}{c} \begin{array}{cccc} c1 & c2 & c3 & c4 \end{array} \\ \begin{bmatrix} 1 & 1.4 & 3.5 & 1.16667 \\ 0.714286 & 1 & 2.5 & 0.833333 \\ 0.285714 & 0.4 & 1 & 0.333333 \\ 0.857143 & 1.2 & 3 & 1 \end{bmatrix} \end{array} \quad \begin{array}{c} \mathrm{IPPV_c} \\ \begin{bmatrix} 0.35 \\ 0.25 \\ 0.10 \\ 0.30 \end{bmatrix} \end{array}$$

with respect to criteria c1–c2:

$$\begin{array}{c} \\ a1 \\ a2 \\ a3 \\ a4 \end{array} \begin{array}{c} \begin{array}{cccc} a1 & a2 & a3 & a4 \end{array} \\ \begin{bmatrix} 1 & 1.4 & 2.33333 & 1.4 \\ 0.714286 & 1 & 1.66667 & 1 \\ 0.428571 & 0.6 & 1 & 0.6 \\ 0.714286 & 1 & 1.66667 & 1 \end{bmatrix} \end{array} \quad \begin{array}{c} \mathrm{IPPV_a} \\ \begin{bmatrix} 0.35 \\ 0.25 \\ 0.15 \\ 0.25 \end{bmatrix} \end{array}$$

with respect to criteria c3–c4:

$$\begin{array}{c} \\ a1 \\ a2 \\ a3 \\ a4 \end{array} \begin{array}{c} \begin{array}{cccc} a1 & a2 & a3 & a4 \end{array} \\ \begin{bmatrix} 1 & 0.666667 & 0.285714 & 0.25 \\ 1.5 & 1 & 0.428571 & 0.375 \\ 3.5 & 2.33333 & 1 & 0.875 \\ 4 & 2.66667 & 1.14286 & 1 \end{bmatrix} \end{array} \quad \begin{array}{c} \mathrm{IPPV_a} \\ \begin{bmatrix} 0.10 \\ 0.15 \\ 0.35 \\ 0.40 \end{bmatrix} \end{array}$$

After synthesis, the following result is obtained ITPV=[0.25, 0.21, 0.23, 0.31]^T. In accordance with the simulation scenario we can now perturb every PCM in the presented framework. For the purpose of scenario illustration only, we apply two kinds of perturbation error consecutively. We round each element of the particular PCM to Saaty's numerical scale and force its reciprocity. Then, on the bases of such PCMs we compute their respective priority vectors (PPV), in our example with the application of the REV, and finally calculate the TPV for the illustrative model of the AHP framework. Thus, after all these transformations our model could be presented as follows:

with respect to the GOAL:

$$\begin{array}{c} \\ c1 \\ c2 \\ c3 \\ c4 \end{array} \begin{array}{c} \begin{array}{cccc} c1 & c2 & c3 & c4 \end{array} \\ \begin{bmatrix} 1 & 1 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 1/3 & 1/2 & 1 & 1/3 \\ 1 & 1 & 3 & 1 \end{bmatrix} \end{array} \quad \begin{array}{c} \mathrm{REV_c} \\ \begin{bmatrix} 0.304999 \\ 0.276859 \\ 0.113143 \\ 0.304999 \end{bmatrix} \end{array}$$

with respect to criteria c1–c2:

$$\begin{array}{c} \\ a1 \\ a2 \\ a3 \\ a4 \end{array} \begin{array}{c} \begin{array}{cccc} a1 & a2 & a3 & a4 \end{array} \\ \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 2 & 1 \\ 1/2 & 1/2 & 1 & 1/2 \\ 1 & 1 & 2 & 1 \end{bmatrix} \end{array} \quad \begin{array}{c} \mathrm{REV_a} \\ \begin{bmatrix} 0.285714 \\ 0.285714 \\ 0.142857 \\ 0.285714 \end{bmatrix} \end{array}$$

with respect to criteria c3–c4:

$$\begin{array}{c} \\ a1 \\ a2 \\ a3 \\ a4 \end{array} \begin{array}{c} \begin{array}{cccc} a1 & a2 & a3 & a4 \end{array} \\ \begin{bmatrix} 1 & 1/2 & 1/4 & 1/4 \\ 2 & 1 & 1/2 & 1/3 \\ 4 & 2 & 1 & 1 \\ 4 & 3 & 1 & 1 \end{bmatrix} \end{array} \quad \begin{array}{c} \mathrm{REV_a} \\ \begin{bmatrix} 0.0887547 \\ 0.1611320 \\ 0.3550190 \\ 0.3950950 \end{bmatrix} \end{array}$$

After synthesis, the following result is obtained TPV_{REV}= [0.2034, 0.2336, 0.2316, 0.3315]^T and we are ready to compute earlier mentioned performance measures between ITPV and TPV, i.e. SRCC, PCC and MAD. For this particular example we have SRCC=0.2, PCC=0.8142, MAD= 0.023325. As we can see only with the application of rounding errors and forced reciprocity of PCMs we have the situation which let us experience the rank reversal phenomena for the entire AHP framework as from the perspective of ITPV we have ranks {2, 4, 3, 1} and the resulting TPV_{REV} provides ranks {4, 2, 3, 1}. Basing only on this very simple illustrative scenario we can realize that it is quite reasonable to search for other methods that can successfully operate within the Analytic Hierarchy Process. Certainly, it is very reasonable to make the effort and at least strive to reduce the technical errors within the AHP. Obviously, we cannot avoid the application of the particular scale which entails rounding errors because we have to enable decision makers to express somehow their judgments but certainly we can reduce technical errors caused by the PCM forced reciprocity requirement. Then however, we need to look for other prioritization procedures which not only can operate with nonreciprocal PCMs but above all they are able to provide in these circumstances meaningful consistency indices of decision makers judgments.

Preliminary simulation results

Taking into account the research of Saaty and Hu (1998) illustrating the case where variability in ranks does not occur for each individual judgment matrix, but still occurs in the overall ranking of the final alternatives due to the multicriteria process itself we decided as in Kazibudzki (2012) to examine the results of adequately designed simulations in order to analyze the same three levels AHP framework as in the cited example, i.e. goal, criteria and alternatives. Thus, we simulated 1000 such AHP frameworks in order to evaluate the performance of arbitrarily chosen methods under the scenario assuming application of rounding errors only, and 50 such AHP frameworks making them inconsistent 100 times in each case. In the latter scenario the inconsistency was executed exclusively due to perturbation factor (e_{ij}) drawn uniformly from the interval $e_{ij} \in [0.01, 1.99]$. In both scenarios the number of criteria and alternatives in each AHP framework was drawn uniformly from the interval {5, 6, 7, …, 15}. Thus, tables 1 and 2 present the results of average performances of chosen methods for either one thousand uniformly random AHP frameworks or five thousands cases of AHP frameworks (fifty uniformly random AHP frameworks, each perturbed one hundred times in accordance with the given scenario).

Table 1. Performance evaluations of arbitrarily chosen five different methods for 1,000 AHP frameworks (n_k, $n_a \in$ {5, 15}) exclusively with the application of rounding errors.

Method	FRPCM			APCM		
	MAD	SRCC	PCC	MAD	SRCC	PCC
GM	0.00806526	0.972270	0.996995	0.00678213	0.978178	0.997865
REV	0.00724835	0.972379	0.997704	0.00609367	0.978513	0.998381
LUA	0.00736441	0.972510	0.997620	0.00617176	0.978416	0.998317
SRDM	0.00735463	0.972399	0.997628	0.00616422	0.978304	0.998321
SNCS	0.00788974	0.971815	0.997441	0.00658964	0.977771	0.998175

Table 2. Performance evaluations of arbitrarily chosen five different methods for 5,000 cases of AHP frameworks (n_k, $n_a \in$ {5, 15}) exclusively with the application of perturbation factor drawn uniformly from the interval $e_{ij} \in [0.01, 1.99]$.

Method	FRPCM			APCM		
	MAD	SRCC	PCC	MAD	SRCC	PCC
GM	0.0193661	0.773331	0.915057	0.01053150	0.865141	0.980755
REV	0.0275356	0.659897	0.799391	0.00702442	0.917224	0.991594
LUA	0.0248149	0.696302	0.844117	0.00713523	0.916219	0.991291
SRDM	0.0265010	0.685005	0.831165	0.01217000	0.915188	0.991085
SNCS	0.0215227	0.733349	0.895638	0.00803085	0.911193	0.989905

Discussion with principal simulation outcomes

Basing on performance evaluations of arbitrary chosen methods presented in tables 1&2 we can notice that from the perspective of technical realization of the comparison procedure (table 1) all chosen methods perform very similar in all scenarios, i.e. FRPCM and APCM. Obviously, judging their performance on SRCC as the indicator of ‘true’ ranks estimation ability, all of them perform slightly better in nonreciprocal cases. However, taking account of the methods’ performance when human judgments errors are introduced to the simulation’s scenario (table 2) we can observe the significant volatility of the outcome, which fluctuation is visible especially in conditions when forced reciprocity to PCMs is executed. Certainly, all methods again perform better (this time significantly) and their performance scores are more or less similar in the situation when nonreciprocal PCMs are accepted within the AHP.

To elaborate on this further we examine then the average performance results for 2,500 cases (50 AHP frameworks, each perturbed 50 times) but this time only for arbitrary PCMs (APCMs) and with the application of different scales available (Saaty's, geometric and numerical for n =50). The inconsistency will be imposed now due to rounding errors combined with perturbation factor (e_{ij}) drawn uniformly, log-normally, truncated normally or gamma from imposed different intervals. The number of alternatives and criteria for each framework also as previously will be drawn uniformly from different imposed intervals. The simulations results are presented in the consecutive Tables 3–6.

Table 3. Performance evaluations of arbitrarily chosen five different methods for 2,500 cases of different uniformly drawn AHP frameworks with the application of both: perturbation factor drawn **uniformly** from the given interval and rounding errors connected with the assigned scale executed without forced reciprocity.

Scenario details			Average	GM	REV	LUA	SRDM	SNCS
Saaty's scale	$e_{ij}\in[0.75,1.25]$	$n_k,n_a\in\{3,7\}$	MAD	0.0130458	0.0126586	0.0127309	0.0127362	0.0138414
			SRCC	0.957823	0.958874	0.958446	0.958514	0.957331
			PCC	0.997353	0.997439	0.997404	0.997397	0.997236
		$n_k,n_a\in\{8,12\}$	MAD	0.0057758	0.0052214	0.0052839	0.0052783	0.0055759
			SRCC	**0.974015**	**0.973975**	0.973820	0.973795	0.973532
			PCC	0.998227	0.998570	0.998526	0.998527	0.998505
	$e_{ij}\in[0.05,1.95]$	$n_k,n_a\in\{3,7\}$	MAD	0.0274188	0.0235090	0.0239460	0.0242531	0.0272002
			SRCC	0.868120	0.892791	0.891103	0.890089	0.886706
			PCC	0.972679	0.981205	0.979872	0.979237	0.979101
		$n_k,n_a\in\{8,12\}$	MAD	0.0113223	0.0095430	0.0095948	0.0096054	0.0111978
			SRCC	0.896199	0.926508	0.925951	0.925639	0.919981
			PCC	0.977721	0.985330	0.985042	0.984970	0.982725
Geometric scale	$e_{ij}\in[0.75,1.25]$	$n_k,n_a\in\{3,7\}$	MAD	0.0111407	0.0104963	0.0106354	0.0106286	0.0119955
			SRCC	0.974489	0.975137	0.975023	0.974726	0.975043
			PCC	0.998478	0.998665	0.998631	0.998631	0.998434
		$n_k,n_a\in\{8,12\}$	MAD	0.0043648	0.0038646	0.0039199	0.0039159	0.0042695
			SRCC	0.976335	**0.977594**	**0.977618**	**0.977607**	0.976452
			PCC	0.998430	0.998728	0.998697	0.998699	0.998567
	$e_{ij}\in[0.05,1.95]$	$n_k,n_a\in\{3,7\}$	MAD	0.0266659	0.0222967	0.0229416	0.0234542	0.0257579
			SRCC	0.863123	0.891477	0.890226	0.890037	0.887520
			PCC	0.978324	0.985570	0.984183	0.983383	0.983837
		$n_k,n_a\in\{8,12\}$	MAD	0.0090704	0.0071004	0.0071529	0.0071726	0.0085273
			SRCC	0.890638	0.923771	0.923058	0.922473	0.918694
			PCC	0.981531	0.989661	0.989484	0.989394	0.987821
Numerical scale	$e_{ij}\in[0.75,1.25]$	$n_k,n_a\in\{3,7\}$	MAD	0.0071042	0.0069933	0.0071099	0.0071289	0.0070565
			SRCC	0.967414	0.967589	0.966834	0.966414	0.967731
			PCC	0.998296	0.998322	0.998271	0.998264	0.998310
		$n_k,n_a\in\{8,12\}$	MAD	0.0024932	0.0024018	0.0024392	0.0024409	0.0024663
			SRCC	0.979277	0.979581	0.979270	0.979218	0.979392
			PCC	0.999113	0.999154	0.999130	0.999128	0.999139
	$e_{ij}\in[0.05,1.95]$	$n_k,n_a\in\{3,7\}$	MAD	0.0280545	0.0227402	0.0237604	0.0244559	0.0255492
			SRCC	0.878420	0.902011	0.898563	0.895694	0.896043
			PCC	0.977389	0.985240	0.983307	0.981900	0.982503
		$n_k,n_a\in\{8,12\}$	MAD	0.0085972	0.0063969	0.0065323	0.0065894	0.0071323
			SRCC	0.891820	0.925288	0.923506	0.922891	0.920081
			PCC	0.982247	0.990260	0.989799	0.989649	0.988908

Table 4. Performance evaluations of arbitrarily chosen five different methods for 2,500 cases of different uniformly drawn AHP frameworks with the application of both: perturbation factor drawn **log-normally** from the given interval and rounding errors connected with the assigned scale executed without forced reciprocity

Scenario details			Average	GM	REV	LUA	SRDM	SNCS
Saaty's scale	$e_{ij}\in[0.75,1.25]$	$n_k,n_a\in\{3,7\}$	MAD	0.0142571	0.0139780	0.0141020	0.0141339	0.0149231
			SRCC	**0.966206**	**0.965297**	0.965097	0.965134	**0.965543**
			PCC	0.993392	0.993622	0.993470	0.993439	0.993265
		$n_k,n_a\in\{8,12\}$	MAD	0.0072657	0.0067926	0.0068701	0.0068614	0.0073447
			SRCC	0.957104	0.957484	0.957012	0.956976	0.957140
			PCC	0.996865	0.997209	0.997137	0.997139	0.997083
	$e_{ij}\in[0.05,1.95]$	$n_k,n_a\in\{3,7\}$	MAD	0.0305822	0.0330802	0.0340382	0.0357697	0.0339215
			SRCC	**0.862386**	**0.851629**	0.845357	0.837306	**0.860937**
			PCC	0.969425	0.962036	0.958860	0.954328	0.967904
		$n_k,n_a\in\{8,12\}$	MAD	0.0114709	0.0127733	0.0128175	0.0129585	0.0134907
			SRCC	**0.887247**	**0.870413**	0.867313	0.864747	**0.880653**
			PCC	0.981927	0.976832	0.976042	0.974809	0.979869

(Continued)

Table 4. Continued.

Geometric scale	$e_{ij}\in[0.75,1.25]$	$n_k,n_a\in\{3,7\}$	MAD	0.0128633	0.0127354	0.0127982	0.0128182	0.0136260
			SRCC	0.962902	**0.963300**	0.962649	0.962597	**0.963434**
			PCC	0.997118	0.997062	0.997043	0.997023	0.996975
		$n_k,n_a\in\{8,12\}$	MAD	0.0052568	0.0047566	0.0048145	0.0048111	0.0053568
			SRCC	**0.973069**	**0.972397**	**0.972507**	**0.972501**	0.972262
			PCC	0.997709	0.997934	0.997917	0.997915	0.997779
	$e_{ij}\in[0.05,1.95]$	$n_k,n_a\in\{3,7\}$	MAD	0.0290892	0.0313247	0.0328331	0.0351722	0.0310231
			SRCC	**0.881520**	**0.868474**	0.864397	0.852783	**0.879223**
			PCC	0.971459	0.964668	0.961850	0.955243	0.971532
		$n_k,n_a\in\{8,12\}$	MAD	0.0091140	0.0102803	0.0104189	0.0106102	0.0107429
			SRCC	**0.892613**	**0.873998**	0.870582	0.868336	**0.885740**
			PCC	0.984618	0.979775	0.979003	0.977936	0.982673
Numerical scale	$e_{ij}\in[0.75,1.25]$	$n_k,n_a\in\{3,7\}$	MAD	0.0100140	0.0100800	0.0102147	0.0102610	0.0100918
			SRCC	**0.966029**	**0.964814**	0.964289	0.963960	**0.965237**
			PCC	0.996761	0.996739	0.996651	0.996627	0.996722
		$n_k,n_a\in\{8,12\}$	MAD	0.0033806	0.0033383	0.0033975	0.0034025	0.0034114
			SRCC	**0.964389**	**0.963772**	0.963189	0.963122	0.963729
			PCC	0.998224	0.998224	0.998165	0.998159	0.998248
	$e_{ij}\in[0.05,1.95]$	$n_k,n_a\in\{3,7\}$	MAD	0.0297726	0.0325399	0.0341693	0.0365031	0.0301814
			SRCC	**0.866574**	**0.854169**	0.845574	0.833017	**0.867431**
			PCC	0.970203	0.961648	0.958120	0.952032	0.969562
		$n_k,n_a\in\{8,12\}$	MAD	0.0096297	0.0111019	0.0113577	0.0116762	0.0106907
			SRCC	**0.876516**	**0.855135**	0.851045	0.846685	**0.871440**
			PCC	0.983556	0.977749	0.976788	0.975329	0.981996

Table 5. Performance evaluations of arbitrarily chosen five different methods for 2,500 cases of different uniformly drawn AHP frameworks with the application of both: perturbation factor drawn ***truncated-normally*** from the given interval and rounding errors connected with the assigned scale executed without forced reciprocity.

Scenario details			Average	GM	REV	LUA	SRDM	SNCS
Saaty's scale	$e_{ij}\in[0.75,1.25]$	$n_k,n_a\in\{3,7\}$	MAD	0.0145333	0.0140003	0.0141235	0.0141195	0.0156128
			SRCC	0.960671	0.963691	0.963443	0.963414	0.958597
			PCC	0.997628	0.997906	0.997854	0.997858	0.997433
		$n_k,n_a\in\{8,12\}$	MAD	0.0059563	0.0053326	0.0053878	0.0053827	0.0057465
			SRCC	0.980945	0.981171	0.981071	0.981072	0.980543
			PCC	0.998031	0.998327	0.998293	0.998294	0.998187
	$e_{ij}\in[0.05,1.95]$	$n_k,n_a\in\{3,7\}$	MAD	0.0181351	0.0172557	0.0173967	0.0174646	0.0191873
			SRCC	0.948574	0.952197	0.951343	0.950920	0.951034
			PCC	0.994288	0.995008	0.994811	0.994725	0.994664
		$n_k,n_a\in\{8,12\}$	MAD	0.0071966	0.0066036	0.0066401	0.0066366	0.0073019
			SRCC	0.939509	0.945351	0.944769	0.944689	0.943666
			PCC	0.995910	0.996758	0.996712	0.996705	0.996516
Geometric scale	$e_{ij}\in[0.75,1.25]$	$n_k,n_a\in\{3,7\}$	MAD	0.0086117	0.0081317	0.0081932	0.0081889	0.0091745
			SRCC	**0.973211**	**0.972989**	**0.973063**	0.972954	0.970631
			PCC	0.998195	0.998402	0.998373	0.998375	0.997933
		$n_k,n_a\in\{8,12\}$	MAD	0.0041155	0.0035110	0.0035652	0.0035615	0.0038870
			SRCC	0.983408	**0.984299**	**0.984299**	0.984276	0.983558
			PCC	0.999089	0.999355	0.999339	0.999340	0.999242
	$e_{ij}\in[0.05,1.95]$	$n_k,n_a\in\{3,7\}$	MAD	0.0135980	0.0124764	0.0126109	0.0126553	0.0138182
			SRCC	0.954928	0.959803	0.959120	0.958817	0.958397
			PCC	0.993321	0.994541	0.994359	0.994291	0.994045
		$n_k,n_a\in\{8,12\}$	MAD	0.0058222	0.0050881	0.0051309	0.0051306	0.0057823
			SRCC	0.951047	0.958394	0.957960	0.957925	0.956669
			PCC	0.994974	0.996193	0.996143	0.996138	0.995726
Numerical scale	$e_{ij}\in[0.75,1.25]$	$n_k,n_a\in\{3,7\}$	MAD	0.0063357	0.0060781	0.0061920	0.0062025	0.0063208
			SRCC	**0.985169**	**0.983154**	**0.983663**	**0.983583**	0.981983
			PCC	0.998710	0.998817	0.998783	0.998780	0.998747
		$n_k,n_a\in\{8,12\}$	MAD	0.0022199	0.0020856	0.0021232	0.0021229	0.0021720
			SRCC	0.983636	0.984306	0.984155	0.984151	0.983929
			PCC	0.999190	0.999261	0.999241	0.999240	0.999230
	$e_{ij}\in[0.05,1.95]$	$n_k,n_a\in\{3,7\}$	MAD	0.0140668	0.0128562	0.0131446	0.0132621	0.0133971
			SRCC	0.935714	0.942317	0.941031	0.940760	0.940303
			PCC	0.989812	0.991289	0.990926	0.990784	0.990937
		$n_k,n_a\in\{8,12\}$	MAD	0.0046522	0.0041408	0.0042081	0.0042182	0.0043255
			SRCC	0.948664	0.954517	0.953535	0.953312	0.953732
			PCC	0.995803	0.996697	0.996586	0.996570	0.996545

Table 6. Performance evaluations of arbitrarily chosen five different methods for 2,500 cases of different uniformly drawn AHP frameworks with the application of both: perturbation factor drawn ***gamma*** from the given interval and rounding errors connected with the assigned scale executed without forced reciprocity.

Scenario details			Average	GM	REV	LUA	SRDM	SNCS
Saaty's scale	$e_{ij}\in[0.75,1.25]$	$n_k,n_a\in\{3,7\}$	MAD	0.0146236	0.0144099	0.0145395	0.0145660	0.0154600
			SRCC	0.954497	0.955880	0.955560	0.955480	0.953957
			PCC	0.994635	0.994807	0.994687	0.994657	0.994598
		$n_k,n_a\in\{8,12\}$	MAD	0.0062636	0.0058828	0.0059213	0.0059173	0.0063711
			SRCC	0.958675	0.959156	0.958923	0.958930	0.958267
			PCC	0.997164	0.997442	0.997397	0.997395	0.997332
	$e_{ij}\in[0.05,1.95]$	$n_k,n_a\in\{3,7\}$	MAD	0.0415948	0.0388801	0.0407574	0.0428978	0.0427987
			SRCC	0.836706	**0.849291**	0.840320	0.827494	**0.851737**
			PCC	0.944911	0.951084	0.943874	0.934234	0.954396
		$n_k,n_a\in\{8,12\}$	MAD	0.0144223	0.0135374	0.0136279	0.0137550	0.0151896
			SRCC	0.832827	**0.851094**	0.846411	0.843572	**0.856371**
			PCC	0.971288	0.975553	0.974462	0.973539	0.975290
Geometric scale	$e_{ij}\in[0.75,1.25]$	$n_k,n_a\in\{3,7\}$	MAD	0.0129840	0.0128303	0.0129180	0.0129530	0.0136092
			SRCC	**0.968149**	**0.968140**	0.967506	0.967240	0.968066
			PCC	0.996598	0.996773	0.996738	0.996720	0.996596
		$n_k,n_a\in\{8,12\}$	MAD	0.0047593	0.0043302	0.0043737	0.0043717	0.0048051
			SRCC	0.970217	**0.970243**	**0.970287**	**0.970248**	0.969779
			PCC	0.997520	0.997777	0.997759	0.997757	0.997625
	$e_{ij}\in[0.05,1.95]$	$n_k,n_a\in\{3,7\}$	MAD	0.0376882	0.0346883	0.0367135	0.0385173	0.0367414
			SRCC	0.832483	**0.843137**	0.838009	0.828203	**0.853026**
			PCC	0.946212	0.955356	0.948289	0.942639	0.958049
		$n_k,n_a\in\{8,12\}$	MAD	0.0129163	0.0116866	0.0118396	0.0120637	0.0132075
			SRCC	0.848394	**0.865567**	0.862649	0.859490	**0.871440**
			PCC	0.963923	0.971706	0.970677	0.969011	0.971404
Numerical scale	$e_{ij}\in[0.75,1.25]$	$n_k,n_a\in\{3,7\}$	MAD	0.0099081	0.0098137	0.0099730	0.0100189	0.0097824
			SRCC	0.954260	**0.954866**	0.954614	0.954414	**0.956194**
			PCC	0.995830	0.995876	0.995720	0.995684	0.995947
		$n_k,n_a\in\{8,12\}$	MAD	0.0034102	0.0033330	0.0033891	0.0033947	0.0034160
			SRCC	**0.966000**	**0.965888**	0.965327	0.965266	**0.965969**
			PCC	0.997939	0.998010	0.997952	0.997945	0.997971
	$e_{ij}\in[0.05,1.95]$	$n_k,n_a\in\{3,7\}$	MAD	0.0414851	0.0387284	0.0415120	0.0441710	0.0383818
			SRCC	0.826497	**0.834877**	0.824840	0.812126	**0.850540**
			PCC	0.936588	0.944530	0.934647	0.924594	0.952789
		$n_k,n_a\in\{8,12\}$	MAD	0.0121159	0.0106528	0.0109481	0.0117477	0.0113674
			SRCC	0.827889	**0.854658**	0.850964	0.846019	**0.859897**
			PCC	0.968161	0.975174	0.973771	0.971879	0.975017

As we can notice all arbitrarily chosen methods perform very steady and similarly under all scenarios being studied. However, there exist also some discrepancies among their performance that should be disclosed here and discussed in more detail. First of all, as the rule of thumb, all methods arbitrarily chosen for the analysis perform better (judging on the SRCC) when geometric scale is applied as opposite to Saaty's and numerical scales. Secondly, what was bolded in the tables no. 3–6, the REV (Saaty's approach) is not a dominant method under all scenario being studied but is very often dominated at least by one of the method considered in the simulations (mostly by the GM, LUA and SNCS).

The very important thing here is that, as opposite to the REV, the first two methods provide meaningful inconsistency measures of human judgments which operate with both reciprocal and nonreciprocal PCMs. Analyzing then the performance measures of these two methods (GM and LUA) we can realize that the latter's performance is basically less vulnerable to change as the result of different scenarios applied during the research plan.

Consistency Measurement and Inconsistency Level Acceptance

Apart from deriving priority vectors, very crucial issue connected with the AHP is how to measure the degree of inconsistency for the given PCM and in consequence for the entire AHP framework. Obviously, significant violations of the consistency can lead to vague results, not necessary reflecting the real priorities. This is why it is indispensable to control inconsistency of PCMs in order to be able to refine them during successive iterations of the weighting process. Certainly, the best way to control PCMs consistency is to measure their inconsistency.

However, as so far the only widely accepted procedure of PCM inconsistency measure belongs to Saaty and is closely related to the REV. Supposedly, that makes the latter particularly attractive although it works only with reciprocal PCMs. According to this concept (Saaty, 1980) the inconsistency of the data is measured as follows. First an inconsistency index INC(n) is computed as an average of difference between $lambda_{max}$ and n for all eigenvalues except the principal one. Next, the value of the index is compared with an average random inconsistency index RINC(n) obtained from a sample of 500 randomly generated reciprocal PCMs of order n. Finally, it is proposed to use so-called consistency ratio CR(n)=INC(n)/ RINC(n) for testing whether the information contained in the PCM is consistent enough to be acceptable. Unfortunately, this index is interpretable only for reciprocal PCMs and what was recently revealed (Grzybowski, 2012) it is improperly constructed. As was suggested in novel revelations of Grzybowski (2012) we should measure the inconsistency of our judgments by their comparison with random but either only transitive matrices or transitive and reciprocal as opposite to purely random reciprocal ones. Fortunately, the inconsistency measure proposed in this paper reflected by the formula [6] perfectly fits these ideas.

As it has been already noticed the objective function minimum [5] itself constitutes the inconsistency measure. In the case when PCM is entirely consistent the value of the function is equal to zero. Only inconsistent PCMs lead to higher values. Of course, as the rule of the thumb we can assume, that closer the function is to zero, the better and more precise the outcome in the form of PV. But such an approach seems to be not precise enough. That is why we have proceeded with simulations in order to provide certain point of reference about the scale of inconsistency.

We simulated randomly (uniform distribution) one thousand transitive and both reciprocal and transitive PCMs of different size from n=3 to n=10. For each PCM we calculated then a random consistency index (denoted as RCI(n)), i.e. square root of the LUA objective function minimum for a given PCM divided by n. During the simulations we strived to capture fundamental statistical characteristics of RCI(n) empirical distribution. Thus, we found its mean, maximum and minimum value, together with few fundamental quantiles of order p. These findings are presented in tables 7–8, where ARCI(n), denotes an average value of our random consistency index for transitive, and both transitive and reciprocal PCMs of given size, respectively.

Table 7. Statistical characteristics of RCI(n) uniform empirical distribution for 1000 random transitive PCMs.

Empirical distribution characteristics	Number of alternatives (n)			
	3	4	5	6
ARCI(n)	0.154126	0.120249	0.0919235	0.0768787
MAX	0.508844	0.408991	0.3647490	0.3202210
MIN	0	1.30328x10^{-8}	0.0000820799	0.000259083
Quantiles(p)				
p=0.50	0.13709900	0.10673200	0.08146770	0.067284200
p=0.25	0.06179450	0.05021620	0.04119980	0.033547500
p=0.10	0.02333830	0.02033520	0.01520760	0.013061900
p=0.05	0.01202070	0.00998503	0.00852620	0.006296580
p=0.01	0.00188312	0.00168073	0.00141884	0.001139629
Empirical distribution characteristics	Number of alternatives (n)			
	7	8	9	10
ARCI(n)	0.0649968	0.0560421	0.0529448	0.0454035
MAX	0.2864140	0.2503580	0.2177790	0.2435240
MIN	0.000154599	0.000194726	0.0000456564	0.0000293881
Quantiles(p)				
p=0.50	0.05723690	0.046621300	0.046588000	0.038270800
p=0.25	0.02816140	0.022476900	0.022511700	0.018412800
p=0.10	0.01099060	0.008868580	0.008853040	0.007443720
p=0.05	0.00623333	0.004343320	0.004128230	0.003554770
p=0.01	0.00111583	0.000530469	0.000687716	0.000508242

Following the standard AHP approach to inconsistency proposed by Saaty we could proceed similarly. Taking into account ARCI(n) we have the index which reflects the average inconsistency for random PCMs of the certain size and type (either transitive or both transitive and reciprocal). In this way, we may establish the point of reference. Taking the quotient of CI(n) for regarded PCM with DM judgments and ARCI(n) we obtain the ratio of consistency, denoted as RC(n). It can be provided in the form of the following formula:

$$RC(n) = [CI(n)/ARCI(n)] \times 100\% \qquad [10]$$

However, presented approach fits only the situations when purely transitive PCMs are considered as the point of reference although in our opinion it is still statistically vague. But, in the situations when we would need the point of reference for transitive and reciprocal PCMs, it seems totally irrational. Let us assume that for example we accept only such transitive and reciprocal PCMs with DM judgments for which RC(n)<10% (Saaty's suggestion). As we may notice from table 8, for n>5 the minimum value of RCI(n) is higher than 0.1 x ARCI(n) what means that all purely random transitive and reciprocal PCMs have higher inconsistency measures than the point of reference. Other words the acceptance threshold is set too low in these cases because even if the DM judgments represented by his or her PCM would have higher RC(n) values than 10% it is still okay, for example for n=9, even RC(n)=16% still guarantees that this particular PCM is very consistent.

Table 8. Statistical characteristics of RCI(n) uniform empirical distribution for 1000 random transitive and reciprocal PCMs.

Empirical distribution characteristics	Number of alternatives (n)			
	3	4	5	6
ARCI(n)	0.0261667	0.029777	0.030853	0.0294431
MAX	0.0989477	0.136671	0.118688	0.1123730
MIN	0	$1.01876\text{x}10^{-8}$	0.00147941	0.00342439
Quantiles(p)				
p=0.50	0.02039090	0.02406540	0.02564450	0.02494560
p=0.25	0.00556640	0.01131720	0.01430590	0.01450020
p=0.10	0.00106425	0.00569450	0.00825254	0.00873334
p=0.05	0.00029661	0.00388420	0.00610716	0.00672822
p=0.01	$6.17051\text{x}10^{-9}$	0.00129368	0.00402920	0.00409556
Empirical distribution characteristics	Number of alternatives (n)			
	7	8	9	10
ARCI(n)	0.0265863	0.0254875	0.0244738	0.0229575
MAX	0.10082	0.111298	0.0912381	0.0833758
MIN	0.00287502	0.0039654	0.00396829	0.00359805
Quantiles(p)				
p=0.50	0.02235100	0.02052160	0.02118710	0.01952190
p=0.25	0.01347480	0.01282920	0.01260140	0.01185330
p=0.10	0.00841233	0.00849714	0.00867249	0.00818474
p=0.05	0.00690568	0.00672296	0.00715629	0.00692282
p=0.01	0.00444787	0.00535900	0.00557264	0.00516794

That is why we propose here different approach for inconsistency measurement. Our approach fundamentally resembles standard statistical methodology. If we decide to apply only reciprocal PCMs within the AHP process, in order to evaluate consistency of the particular PCM we have to compare its CI(n) with adequate quantile of order p and given n for transitive and reciprocal PCMs distribution. When we accept the opinion that only nonreciprocal PCMs provide better estimations of 'true' DM priorities, in the same way better accuracy of DMs judgments, in order to evaluate consistency of the particular PCM we have to compare its CI(n) with respective quantile of order p and given n for purely transitive PCMs distribution.

Anyway, we have the information then how much the particular judgment comprises the element of chance variation. Generally, closer the value of CI(n) is to zero, better the approximation becomes of 'true' PV under the assigned level of significance (given by the accepted order of quantile). We accept DM judgments then and only then when CI(n)<Quantile(p). Otherwise, we reject the judgments. As suggested in Grzybowski (2012), it is crucial to compare CI(n) for a given PCM with DM judgments to quantiles of RCI(n) empirical distribution for random transitive or both transitive and reciprocal PCMs, because only then we have the information about DMs judgment consistency from the perspective of their prioritization accuracy as opposed to the information about their priorities order (the case when we compare CI(n) for a given PCM with DM judgments to quantiles of RCI(n) empirical distribution for random not transitive but purely reciprocal PCMs).

Conclusions

Deriving true priority vectors from intuitive pairwise comparison matrices (PCMs) and consistency measurement of decision makers judgments about their genuine weights are a crucial issue within the multicriteria decision making support methodology called the Analytic Hierarchy Process (AHP). The most popular procedure in the ranking process, commonly applied in the AHP, constitutes the Right Eigenvector Method (REV) conceived together with the AHP methodology by Thomas Saaty. This procedure however, has serious drawbacks and flaws which from a very long time constitute the main theme of its opponent's critique. Still, as long as inconsistent

PCMs are allowed in the AHP, although other procedures exist, the inventor of the AHP convinces that none of them qualifies and the REV provides the only right solution in this process. In this scientific quest we examined some fundamental issues within this field thanks to computer simulations for the entire AHP framework (as opposed to a single PCM simulation research). Our findings verify the statement that the REV cannot longer be perceived as the dominant procedure within the AHP especially when nonreciprocal PCMs are considered. It is so mainly because it impoverishes the AHP methodology by its lack of PCMs consistency measure in such cases, which is an indispensable element of the entire AHP concept. Thus, especially in multicriteria decision making problems embedded in management processes we advise the application (together with the AHP) of other available methods like for example presented in this research which perform quite steady, more accurately, contrary to the REV they allow to introduce additional constraints that enable order preservation of weights and most of all provide valid and meaningful inconsistency measure for both reciprocal and nonreciprocal PCMs. The fact especially important because the simulation performance results of different methods presented in this research indicate unequivocally that the enforced reciprocity of PCM leads directly to worse estimates of priorities weights. Thus, the crucial implication of this study for the entire AHP methodology is such, that if we care for betterment of its prospective applicative results, the performance of the AHP can and should be improved by the application of presented here novel scientific findings.

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