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ON SOME CLASS OF QUASILINEAR HYPERBOLIC SYSTEMS OF PARTIAL DIFFERENTIAL-FUNCTIONAL EQUATIONS OF THE FIRST ORDER

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1. Introduction. We consider quasilinear hyperbolic systems of differential-functional equations in the Schauder canonical form

$$(1) \quad \sum_{j=1}^{n} A_{ij}(x, y, z(x, y)) \left[\frac{\partial z_j(x, y)}{\partial x} + \sum_{k=1}^{m} \varrho_{ik}(x, y, z(x, y), (Vz)(x, y)) \frac{\partial z_j(x, y)}{\partial y_k} \right] = $$
$$= f_i(x, y, z(x, y), (Vz)(x, y)),$$

$(x, y) \in D_a = I_a \times R^m$, $i = 1, \ldots, n$, with initial data

$$(2) \qquad z(x, y) = \gamma(x, y) \quad \text{for} \quad (x, y) \in D_\tau^0 = I_\tau^0 \times R^m,$$

where $I_t = [0, t]$, $I_t^0 = [-t, 0]$, $t \geqq 0$, $y = (y_1, \ldots, y_m) \in R^m$, $m \geqq 1$, $z(x, y) = (z_1(x, y), \ldots, z_n(x, y))$, $(Vz)(x, y) = ((V_1 z)(x, y), \ldots, (V_l z)(x, y))$, $\gamma(z, y) = (\gamma_1(x, y), \ldots, \gamma_n(x, y))$.

In this paper we shall consider the existence and uniqueness for local generalized solutions of problem (1), (2) in the sense "almost everywhere" (that is, the solution possesses partial derivatives a.e. and satisfies system (1) a.e.).

Generalized solutions of quasilinear equations were first investigated by Hopf [11]. In papers [5], [6], [10], [11], [14] and [16] by a solution of quasilinear equations a function satisfying a certain integral identity is understood. This kind of definition made it possible to get a global solution of initial problems by difference or small parameter methods.

Generalized solutions of nonlinear partial differential equations of the first order in the class of Lipschitz continuous functions were considered by Kružkov [15].

If the functions ϱ_{ik} and f_i in (1) do not depend on the last variable then system (1) reduces to a quasilinear hyperbolic system in the "second canonical" form which has been studied in a large number of papers by various authors. We refer here in particular to the papers by L. Cesari [7], [8], P. Bassanini [1]–[3] and M. Cinquini-Cibrario [9]. Quasilinear hyperbolic systems in the "first canonical" form (see book [17] with rich bibliography) are particular cases of system (1). A system of differential equations with a retarded argument (cf. [13]) and a few kinds of integrodifferential systems (cf. for instance P. Bassanini, M. C. Salvatori [4]) can be obtained from system (1) by specializing the operator V (see Section 6).

Nonlinear hyperbolic differential-functional equations in the C^1 class were considered by Z. Kamont [12].

The method used in this paper is based on the Banach fixed point theorem and it is close to that used in [7] (see also [1]).

2. Preliminaries and assumptions. We denote by $\|y\|_m = \max_{1 \leqq k \leqq m} |y_k|$ the norm of y in R^m and by $\|z\|_n = \max_{1 \leqq i \leqq n} |z_i|$ the norm of z in R^n. If $B = [b_{ij}]$, $i = 1, \ldots, n$, $j = 1, \ldots, m$, is an $n \times m$ matrix then $B_i = (b_{i1}, \ldots, b_{im})$. Let $\bar{\Omega}$ denote the interval $[-\Omega, \Omega]^n \subset R^n$, $\Omega > 0$, and let a_0 be a given positive constant.

Let J denote the class of all continuous functions $\gamma\colon D^0_\tau \to R^n$ for which there are constants ω, Λ, $0 \leqq \omega < \Omega$, $\Lambda \geqq 0$, such that for all $(x, y), (x, \bar{y}) \in D^0_\tau$ we have

$$\|\gamma(x, y)\|_n \leqq \omega\,, \quad \|\gamma(x, y) - \gamma(x, \bar{y})\|_n \leqq \Lambda \|y - \bar{y}\|_m\,.$$

For every $\gamma \in J$ let us consider the set K_γ of all continuous bounded functions $z\colon \tilde{D}_a = (I^0_\tau \cup I_a) \times R^m \to R^n$ satisfying the following conditions:

(i) $z(x, y) = \gamma(x, y)$ for $(x, y) \in D^0_\tau$;

(ii) there are a constant $Q > 0$ and a function $\mu\colon I_{a_0} \to R_+ = [0, \infty)$, $\mu \in L_1[0, a_0]$, such that for all $(x, y), (x, \bar{y}), (\bar{x}, y) \in D_a$ we have

$$\|z(x, y)\|_n \leqq \Omega\,,$$
$$\|z(x, y) - z(x, \bar{y})\|_n \leqq Q\|y - \bar{y}\|_m\,,$$
$$\|z(x, y) - z(\bar{x}, y)\|_n \leqq \left|\int_x^{\bar{x}} \mu(t)\, dt\right|\,,$$

where the constant Q and the function μ will be defined by (4), (5).

Note that K_γ is a closed (convex) subset of the Banach space $(C(\tilde{D}_a) \cap L_\infty(\tilde{D}_a))^n$ with the norm $\|z\|_a = \sup_{\tilde{D}_a} \|z(x, y)\|_n$.

We denote by K the set of all functions $z\colon D_a \to R^n$ satisfying the following conditions:

(i) $z(\cdot, y)\colon I_{a_0} \to R^n$ is measurable for every $y \in R^m$;

(ii) $z(x, \cdot)\colon R^m \to R^n$ is continuous for a.e. $x \in I_{a_0}$;

(iii) $\|z(x, y)\|_n \leqq \Omega$, $(x, y) \in D_a$.

Assumption H_1. Suppose that

1° $V_j\colon K_\gamma \to K$, $j = 1, \ldots, l$;

2° there are constants q_j, $e_j > 0$, $j = 1, \ldots, l$, such that for all $z \in K_\gamma$ and a.e.

in I_{a_0} we have

$$[\![(V_j)(x, \cdot)]\!] \leqq q_j[\![z(x, \cdot)]\!] + e_j\,, \quad j = 1, \ldots, l\,,$$

where

$$[\![z(x, \cdot)]\!] = \sup_{y,\bar{y} \in R^m} \frac{\|z(x, y) - z(x, \bar{y})\|_n}{\|y - \bar{y}\|_m}\,, \quad x \in I_{a_0}\,;$$

3° there are constants $M_j > 0$, such that for all $z, \bar{z} \in K_\gamma$, $y \in R^m$ and a.e. $x \in I_{a_0}$, we have

$$(3) \qquad \|(V_j z)(x, y) - (V_j \bar{z})(x, y)\|_n \leqq M_j \|z - \bar{z}\|_x\,, \quad j = 1, \ldots, l\,,$$

where $\|z\|_x = \sup_{\tilde{D}_x} \|z(x, y)\|_n$, $\tilde{D}_x = (I_\tau^0 \cup I_x) \times R^m$.

Remark. It follows from (3) that V_j satisfies the following Volterra condition: if $z, \bar{z} \in K_\gamma$ and $z(t, y) = \bar{z}(t, y)$ for $t \in [-\tau, x]$, $y \in R^m$, then $(V_j z)(x, y) = (V_j \bar{z})(x, y)$, $j = 1, \ldots, l$.

Assumption H_2. Suppose that

1° the matrix function $\varrho(\cdot, y, z, U) = [\varrho_{ik}(\cdot, y, z, U)]: I_{a_0} \to R^{nm}$, $i = 1, \ldots, n$, $k = 1, \ldots, m$, is measurable for every $(y, z, U) \in R^m \times \bar{\Omega} \times \bar{\Omega}^l$, where $U = (u_1, \ldots, u_l)$;

2° $\varrho(x, \cdot): R^m \times \bar{\Omega} \times \bar{\Omega}^l \to R^{nm}$ is continuous for a.e. $x \in I_{a_0}$;

3° there are functions $b, l: I_{a_0} \to R_+$, $b, l \in L_1[0, a_0]$, such that for all (y, z, U), $(\bar{y}, \bar{z}, \bar{U}) \in R^m \times \bar{\Omega} \times \bar{\Omega}^l$, $i = 1, \ldots, n$ and a.e. in I_{a_0}, we have

$$\|\varrho_i(x, y, z, U)\|_m \leqq b(x)\,,$$

$$\|\varrho_i(x, y, z, U) - \varrho_i(x, \bar{y}, \bar{z}, \bar{U})\|_m \leqq$$

$$\leqq l(x)\,[\|y - \bar{y}\|_m + \|z - \bar{z}\|_n + \sum_{j=1}^{l} \|u_j - \bar{u}_j\|_n]\,,$$

$$i = 1, \ldots, n,\ \bar{U} = (\bar{u}_1, \ldots, \bar{u}_l).$$

3. Bicharacteristics. Let $\bar{K}_0$ be the set of all systems $h = [h_{ik}]$, $i = 1, \ldots, n$, $k = 1, \ldots, m$, of continuous functions $h_{ik}: \Delta_a = I_a \times I_a \times R^m \to R$, for which there is p, $0 < p < 1$, such that

$$h(x, x, y) = 0\,, \quad (x, y) \in D_a\,,$$

$$\|h_i(\xi, x, y) - h_i(\bar{\xi}, x, y)\|_m \leqq \left|\int_\xi^{\bar{\xi}} b(t)\,\mathrm{d}t\right|\,,$$

$$\|h_i(\xi, x, y) - h_i(\xi, x, \bar{y})\|_m \leqq p\|y - \bar{y}\|_m$$

for all (ξ, x, y), $(\bar{\xi}, x, y)$, $(\xi, x, \bar{y}) \in \Delta_a$, $i = 1, \ldots, n$.

The function h is uniformly bounded in Δ_a, since

$$\|h_i(\xi, x, y)\|_m = \|h_i(\xi, x, y) - h_i(x, x, y)\|_m \leqq B_a = \int_0^a b(x)\,\mathrm{d}x\,, \quad i = 1, \ldots, n\,.$$

We denote by K_0 the set of all systems $g = [g_{ik}, i = 1, \ldots, n, k = 1, \ldots, m,]$ defined by $g_{ik}(\xi, x, y) = h_{ik}(\xi, x, y) + y_k$, $i = 1, \ldots, n$, $k = 1, \ldots, m$.

Thus, for all $(\xi, x, y), (\xi, x, \bar{y}) \in \Delta_a$ we have

$$\|g_i(\xi, x, y) - g_i(\xi, x, \bar{y})\|_m \leqq (1 + p) \|y - \bar{y}\|_m, \quad i = 1, \ldots, n.$$

Note that $\bar{K}_0$ is a closed (convex) subset of the Banach space $(C(\Delta_a) \cap L_\infty(\Delta_a))^{nm}$ with the norm $\|h\|_a = \max_{1 \leqq i \leqq n} \sup_{\Delta_a} \|h_i(\xi, x, y)\|_m$.

Further properties of h and g are reported in [7], [1].

Let us define constants

$$q = \sum_{j=1}^{l} (Qq_j + e_j), \quad M = \sum_{j=1}^{l} M_j, \quad L_a = \int_0^a l(x)\,\mathrm{d}x, \quad \lambda = [1 - L_a(1 + Q + q)]^{-1}.$$

Lemma 1. *If Assumptions* H_1 *and* H_2 *are satisfied and* a, $0 < a \leqq a_0$, *is sufficiently small and such that*

$$L_a(1 + p)(1 + Q + q) \leqq p \quad \text{and} \quad L_a(1 + Q + q) \leqq k < 1,$$

then for every fixed $z \in K_\gamma$ *the transformation* $T_z = (T_z^1, \ldots, T_z^n): \bar{K}_0 \to \bar{K}_0$ *defined by*

$$(T_z^i h_i)(\xi, x, y) = -\int_\xi^x \varrho_i(t, g_i(t, x, y), z(t, g_i(t, x, y)), (Vz)(t, g_i(t, x, y)))\,\mathrm{d}t$$

$(\xi, x, y) \in \Delta_a$, $i = 1, \ldots, n$, *has a unique fixed point* $h[z] \in \bar{K}_0$. *Furthermore, for all* $z, \bar{z} \in K_\gamma$ *we have*

$$\|g[z] - g[\bar{z}]\|_a = \|h[z] - h[\bar{z}]\|_a \leqq \lambda L_a(1 + M) \|z - \bar{z}\|_a.$$

It means that $z \to h[z]$ $(z \to g[z])$ *is a continuous map of* K_γ *into* $\bar{K}_0$ $(K_\gamma \to K_0)$.

The proof of this lemma is similar to that of Lemma 1 [13] (cf. also [7]); we omit the details.

4. Further assumptions and lemmas. If $D = [d_{ij}]$, $i, j = 1, \ldots, n$, is an $n \times n$ matrix then $\|D\| = \max_{1 \leqq i,j \leqq n} |d_{ij}|$.

Assumption H_3. Suppose that

1° $A = [A_{ij}]: I_{a_0} \times R^m \times \bar{\Omega} \to R^{n^2}$, $i, j = 1, \ldots, n$, is continuous;

2° $\det A \geqq \varkappa > 0$ in $I_{a_0} \times R^m \times \bar{\Omega}$ for some constant $\varkappa$;

3° there are constants $H > 0$, $C \geqq 0$ and a function $p: I_{a_0} \to R_+$, $p \in L_1[0, a_0]$, such that for all $(x, y, z), (x, \bar{y}, \bar{z}), (\bar{x}, y, z) \in I_{a_0} \times R^m \times \bar{\Omega}$ we have

$$\|A(x, y, z)\| \leqq H,$$

$$\|A(x, y, z) - A(x, \bar{y}, \bar{z})\| \leqq C[\|y - \bar{y}\|_m + \|z - \bar{z}\|_n],$$

$$\|A(x, y, z) - A(\bar{x}, y, z)\| \leqq \left|\int_x^{\bar{x}} p(t)\,\mathrm{d}t\right|.$$

We denote by α_{ij} the cofactor of A_{ij} in the matrix $A = [A_{ij}]$ divided by $\det A$, or $\alpha_{ij} = (A^{-1})_{ji}$. Since $\det A \geqq \bar{A} > 0$, relations 3° of Assumption H_2 yield analogous relations for the matrix $\alpha = [\alpha_{ij}]$. Thus, there are constants H', C' and a function $p': I_{a_0} \to R_+$, $p' \in L_1[0, a_0]$, such that for all (x, y, z), $(x, \bar{y}, \bar{z})$, $(\bar{x}, y, z) \in I_{a_0} \times R^m \times \bar{\Omega}$ we have

$$\|\alpha(x, y, z)\| \leqq H',$$

$$\|\alpha(x, y, z) - \alpha(x, \bar{y}, \bar{z})\| \leqq C'[\|y - \bar{y}\|_m + \|z - \bar{z}\|_n],$$

$$\|\alpha(x, y, z) - \alpha(\bar{x}, y, z)\| \leqq \left|\int_x^{\bar{x}} p'(t)\, dt\right|.$$

Assumption H_4. Suppose that

1° $f(\cdot, y, z, U) = (f_1(\cdot, y, z, U), \ldots, f_n(\cdot, y, z, U)): I_{a_0} \to R^n$ is measurable for every $(y, z, U) \in R^m \times \bar{\Omega} \times \bar{\Omega}^l$;

2° $f(x, \cdot): R^m \times \bar{\Omega} \times \bar{\Omega}^l \to R^n$ is continuous for a.e. $x \in I_{a_0}$;

3° there are functions $n, l_1: I_{a_0} \to R_+$, $n, l_1 \in L_1[0. a_0]$, such that for all (y, z, U), $(\bar{y}, \bar{z}, \bar{U}) \in R^m \times \bar{\Omega} \times \bar{\Omega}^l$ and a.e. in I_{a_0} we have

$$\|f(x, y, z, U)\|_n \leqq n(x),$$

$$\|f(x, y, z, U) - f(x, \bar{y}, \bar{z}, \bar{U})\|_n \leqq l_1(x)\,[\|y - \bar{y}\|_m + \|z - \bar{z}\|_n + \sum_{j=1}^{l} \|u_j - \bar{u}_j\|_n],$$

where $\bar{U} = (\bar{u}_1, \ldots, \bar{u}_1)$;

4° the vector function $\gamma: D_\tau^0 \to R^n$ belongs to J.

Now we consider the transformation F defined by

$$(Fz)(x, y) = \begin{cases} \gamma(0, y) + [\Delta_1(x, y) + \Delta_2(x, y) + \Delta_3(x, y)]\, \alpha(x, y, z(x, y)), & (x, y) \in D_a, \\ \gamma(x, y), \ (x, y) \in D_\tau^0, \end{cases}$$

where $\alpha = [\alpha_{ij}]$, $i, j = 1, \ldots, n$, $\Delta_j = (\Delta_{1j}, \ldots, \Delta_{nj})$, $j = 1, 2, 3$, and

$$\Delta_{s1}(x, y) = \int_0^x f_s(t, g_s(t, x, y), z(t, g_s(t, x, y)), (Vz)(t, g_s(t, x, y)))\, dt,$$

$$\Delta_{s2}(x, y) = \sum_{k=1}^{n} A_{sk}(0, g_s(0, x, y), z(0, g_s(0, x, y)))\,[\gamma_k(0, g_s(0, x, y)) - \gamma_k(0, g_s(x, x, y))],$$

$$\Delta_{s3}(x, y) = \int_0^x \sum_{k=1}^{n} (dA_{sk}(t, g_s(t, x, y), z(t, g_s(t, x, y)))/dt)\,[z_k(t, g_s(t, x, y)) - \gamma_k(0, g_s(x, x, y))]\, dt, \quad s = 1, \ldots, n, \quad (x, y) \in D_a,$$

and $g = g[z]$ is defined in Section 3 by the fixed points of T_z^i, $z \in K_\gamma$.

Lemma 2. *If Assumptions* $H_1 - H_4$ *are satisfied then for sufficiently small* a, $0 < a \leqq a_0$, *the transformation* F *maps* K_γ *into itself.*

Proof. By using the estimates (cf. [7])

$$\int_0^x \|\mathrm{d}A_s(t, g_s(t, x, y), z(t, g_s(t, x, y)))/\mathrm{d}t\|_n \,\mathrm{d}t \leqq P_a + mC(1 + nQ)\, B_a + nC\theta_a\,,$$

$$\|\mathrm{d}z(t, g_s(t, x, y))/\mathrm{d}t\|_n \leqq \mu(t) + mQ\, b(t)\,,$$

$$\|z(t, g_s(t, x, y)) - \gamma(0, g_s(x, x, y))\|_n \leqq \theta_a + QB_a\,, \quad s = 1, \ldots, n\,,$$

where

$$P_a = \int_0^a p(x)\,\mathrm{d}x\,, \quad \theta_a = \int_0^a \mu(x)\,\mathrm{d}x\,, \quad L_{1a} = \int_0^a l_1(x)\,\mathrm{d}x\,,$$

we get

$$\|\Delta_1(x, y)\|_n \leqq \int_0^a n(x)\,\mathrm{d}x = N_a\,,$$

$$\|\Delta_2(x, y)\|_n \leqq nH\Lambda B_a\,,$$

$$\|\Delta_3(x, y)\|_n \leqq n(P_a + mC(1 + nQ)\, B_a + nC\theta_a)(\theta_a + QB_a) = S_a\,, \quad (x, y) \in D_a\,.$$

Hence

$$\|(Fz)(x, y)\|_n \leqq \omega + nH'(N_a + nH\Lambda B_a + S_a) \leqq \omega + (\Omega - \omega) = \Omega\,,$$

provided a is assumed sufficiently small in order that

$$nH'(N_a + nH\Lambda B_a + S_a) \leqq \Omega - \omega\,.$$

For any two points $(x, y), (x, \bar{y}) \in D_a$ we can evaluate the difference $(Fz)(x, y) - (Fz)(x, \bar{y})$ term by term as follows:

$$\|\gamma(0, y) - \gamma(0, \bar{y})\|_n \leqq \Lambda \|y - \bar{y}\|_m\,,$$

$$\|[\Delta_1(x, y) + \Delta_2(x, y) + \Delta_3(x, y)]\,[\alpha(x, y, z(x, y)) - \alpha(x, \bar{y}, z(x, \bar{y}))]\|_n \leqq$$
$$\leqq nC'(1 + Q)(N_a + nH\Lambda B_a + S_a)\,\|y - \bar{y}\|_m\,,$$

$$\|[\Delta_1(x, y) - \Delta_1(x, \bar{y})]\,\alpha(x, \bar{y}, z(x, \bar{y}))\|_n \leqq nH'(1 + p)(1 + Q + q)\,\|y - \bar{y}\|_m\,,$$

$$\|[\Delta_2(x, y) - \Delta_2(x, \bar{y})]\,\alpha(x, \bar{y}, z(x, \bar{y}))\|_n \leqq$$
$$\leqq n^2H'[H\Lambda(2 + p) + C\Lambda(1 + Q)(1 + p)\,B_a]\,\|y - \bar{y}\|_m\,,$$

$$\|[\Delta_3(x, y) - \Delta_3(x, \bar{y})]\,\alpha(x, \bar{y}, z(x, \bar{y}))\|_n \leqq$$
$$\leqq n^2H'[C(1 + Q)\,\theta_a + CQ(1 + Q)(1 + p)\,B_a +$$
$$+ C(1 + Q)(1 + p)(\theta_a + mQB_a) +$$
$$+ (P_a + mC(1 + nQ)\,B_a + nC\theta_a)(Q(1 + p) + \Lambda)]\,\|y - \bar{y}\|_m\,,$$

and finally

$$\|(Fz)(x, y) - (Fz)(x, \bar{y})\|_n \leqq$$
$$\leqq [\Lambda(1 + n^2HH'(2 + p)) + \beta_1 N_a + \beta_2 P_a + \beta_3 L_{1a} + \beta_4 B_a + \beta_5 \theta_a]\,\|y - \bar{y}\|_m\,,$$

where

$\beta_1 = nC'(1 + Q)$,

$\beta_2 = n^2[C'(1 + Q)(\theta_a + QB_a) + H'(Q(1 + p) + \Lambda)]$,

$\beta_3 = nH'(1 + Q + q)(1 + p)$,

$\beta_4 = n^2[C'\Lambda H(1 + Q) + mCC'(1 + Q)(1 + nQ)(\theta_a + QB_a) + H'C\Lambda(1 + Q)(1 + p) + (m + 1) H'CQ(1 + Q)(1 + p) + mH'C(1 + nQ)(Q(1 + p) + \Lambda)]$,

$\beta_5 = n^2[nCC'(1 + Q)(\theta_a + QB_a) + H'C(1 + Q)(2 + p) + nH'C(Q(1 + p) + \Lambda)]$.

Let us choose the constant Q so that

$$Q > \Lambda(1 + n^2HH'(2 + p)). \tag{4}$$

If we assume a sufficiently small so that

$$\beta_1N_a + \beta_2P_a + \beta_3L_{1a} + \beta_4B_a + \beta_5\theta_a \leqq Q - \Lambda(1 + n^2HH'(2 + p)),$$

then we have for all $(x, y), (x, \bar{y}) \in D_a$

$$\|(Fz)(x, y) - (Fz)(x, \bar{y})\|_n \leqq Q\|y - \bar{y}\|_m.$$

By using the estimate (cf. [5])

$$\|g_s(\xi, x, y) - g_s(\xi, \bar{x}, y)\|_m \leqq \lambda \left|\int_x^{\bar{x}} b(t)\,dt\right|,$$

we can evaluate the difference $(Fz)(x, y) - (Fz)(\bar{x}, y)$ term by term as follows:

$$\|[\Delta_1(x, y) + \Delta_2(x, y) + \Delta_3(x, y)]\,[\alpha(x, y, z(x, y)) - \alpha(\bar{x}, y, z(\bar{x}, y))]\|_n \leqq$$
$$\leqq n(N_a + nH\Lambda B_a + S_a)\left(\left|\int_x^{\bar{x}} p(t)\,dt\right| + C'\left|\int_x^{\bar{x}} \mu(t)\,dt\right|\right),$$

$$\|[\Delta_1(x, y) - \Delta_1(\bar{x}, y)]\,\alpha(\bar{x}, y, z(\bar{x}, y))\|_n \leqq$$
$$\leqq nH'\left[(1 + Q + q)\,L_{1a}\lambda\left|\int_x^{\bar{x}} b(t)\,dt\right| + \left|\int_x^{\bar{x}} n(t)\,dt\right|\right],$$

$$\|[\Delta_2(x, y) - \Delta_2(\bar{x}, y)]\,\alpha(\bar{x}, y, z(\bar{x}, y))\|_n \leqq$$
$$\leqq n^2H'\left[H\Lambda\lambda\left|\int_x^{\bar{x}} b(t)\,dt\right| + C\Lambda\lambda(1 + Q)\,B_a\left|\int_x^{\bar{x}} b(t)\,dt\right|\right],$$

$$\|[\Delta_3(x, y) - \Delta_3(\bar{x}, y)]\,\alpha(\bar{x}, y, z(\bar{x}, y))\|_n \leqq$$
$$\leqq n^2H'\left[(\theta_a + QB_a)\left|\int_x^{\bar{x}} (p(t) + mC(1 + nQ)\,b(t) + nC\,\mu(t))\,dt\right| + \right.$$
$$+ 2C(1 + Q)\,\lambda(\theta_a + mQB_a)\left|\int_x^{\bar{x}} b(t)\,dt\right| + Q\lambda(P_a + mC(1 + nQ)\,B_a +$$
$$\left. + nC\theta_a)\left|\int_x^{\bar{x}} b(t)\,dt\right|\right],$$

and finally

$$\|(Fz)(x, y) - (Fz)(\bar{x}, y)\|_n \leqq nH' \left|\int_x^{\bar{x}} n(t)\, dt\right| + n^2 HH'\Lambda\lambda \left|\int_x^{\bar{x}} b(t)\, dt\right| +$$

$$+ \gamma_1 \left|\int_x^{\bar{x}} p(t)\, dt\right| + \gamma_2 \left|\int_x^{\bar{x}} p'(t)\, dt\right| + \gamma_3 \left|\int_x^{\bar{x}} b(t)\, dt\right| + \gamma_0 \left|\int_x^{\bar{x}} \mu(t)\, dt\right|,$$

where

$$\gamma_1 = n^2 H'(\theta_a + QM_a),$$

$$\gamma_2 = n(N_a + nH\Lambda B_a + S_a),$$

$$\gamma_3 = nH'(1 + Q + q)\lambda L_{1a} + n^2 H' C\Lambda\lambda(1 + Q) B_a +$$

$$+ mn^2 H' C(\theta_a + QB_a)(1 + nQ) + 2n^2 H' C(1 + Q)\lambda(\theta_a + mQB_a) +$$

$$+ n^2 H' Q\lambda(P_a + mC(1 + nQ) B_a + nC\theta_a),$$

$$\gamma_0 = nC'(N_a + nH\Lambda B_a + S_a) + n^3 H' C(\theta_a + QB_a).$$

Let us put

(5) $$\mu(x) = R_0\, n(x) + R_1\, p(x) + R_2\, p'(x) + R_3\, b(x), \quad x \in I_{a_0},$$

where

$$R_0 > nH', \quad R_1, R_2 > 0, \quad R_3 > n^2 HH'\Lambda(1 + k)^{-1}.$$

We shall take a so small that

$$\gamma_0 < 1 - R_0^{-1} nH', \quad \gamma_0 < 1 - R_3^{-1} n^2 HH'\Lambda\lambda, \quad \gamma_1 \leqq (1 - \gamma_0) R_1,$$

$$\gamma_2 < (1 - \gamma_0) R_2, \quad \gamma_3 \leqq (1 - \gamma_0) R_3 - n^2 HH'\Lambda\lambda.$$

Then $nH' + R_0\gamma_0 \leqq R_0$ and

$$\|(Fz)(x, y) - (Fz)(\bar{x}, y)\|_n \leqq nH' \left|\int_x^{\bar{x}} n(t)\, dt\right| + n^2 HH'\Lambda\lambda \left|\int_x^{\bar{x}} b(t)\, dt\right| +$$

$$+ (1 - \gamma_0) \left|\int_x^{\bar{x}} (R_1\, p(t) + R_2\, p'(t))\, dt\right| + [(1 - \gamma_0) R_3 + n^2 HH'\Lambda\lambda] \left|\int_x^{\bar{x}} b(t)\, dt\right| +$$

$$+ \gamma_0 \left|\int_x^{\bar{x}} (R_0\, n(t) + R_1\, p(t) + R_2\, p'(t) + R_3\, b(t))\, dt\right| \leqq \left|\int_x^{\bar{x}} \mu(t)\, dt\right|.$$

This concludes the proof.

Lemma 3. *If Assumption* $H_1 - H_4$ *are satisfied then for any two elements* $z \in K_\gamma$, $\bar{z} \in K_{\bar{\gamma}}$ *corresponding to* $g = g[z]$, $\bar{g} = g[\bar{z}] \in K_0$, *and any two elements* $\gamma, \bar{\gamma} \in J$, *the estimate*

(6) $$\|Fz - F\bar{z}\|_a \leqq \alpha\|\gamma - \bar{\gamma}\|_a + \beta\|z - \bar{z}\|_a$$

holds true, where

$$\alpha = 1 + 2n^2 HH' + n^2 H'(P_a + mC(1 + nQ) B_a + nC\theta_a),$$

$$\begin{aligned}\beta = {} & nC'(N_a + nH\Lambda B_a + S_a) + nH'L_{1a}(1 + M)\,[1 + (1 + Q + q)\,\lambda L_a] + \\ & + n^2H'\Lambda[2H\lambda L_a(1 + M) + CB_a(1 + (1 + Q)(1 + M)\,\lambda L_a)] + \\ & + n^2H'[C\theta_a + 2C(1 + (1 + Q)(1 + M)\,\lambda L_a)(\theta_a + mQB_a)/ + \\ & + (P_a + mC(1 + nQ)\,B_a + nC\theta_a)(1 + Q(1 + M)\,\lambda L_a)\,.\end{aligned}$$

Proof. Let $\gamma, \bar{\gamma}$ be any two elements of J, $z, \bar{z}$ any two elements of K_γ and $K_{\bar{\gamma}}$, respectively, and let $g, \bar{g}$ be the corresponding elements in K_0. Then we can derive

$$(Fz)(x, y) - (F\bar{z})(x, y) = \gamma(x, y) - \bar{\gamma}(x, y) + \sigma_1 + \sigma_2 + \sigma_3 + \sigma_4\,,$$

where

$$\begin{aligned}\|\sigma_1\|_n = {} & \|[\Delta_1(x, y) + \Delta_2(x, y) + \Delta_3(x, y)]\,[\alpha(x, y, z(x, y)) - \alpha(x, y, \bar{z}(x, y))]\|_n \leqq \\ & \leqq nC'(N_a + nH\Lambda B_a + S_a)\,\|z - \bar{z}\|_a\,,\end{aligned}$$

$$\begin{aligned}\|\sigma_2\|_n = {} & \|[\Delta_1(x, y) - \bar{\Delta}_1(x, y)]\,\alpha(x, y, \bar{z}(x, y))\|_n \leqq \\ & \leqq nH'L_{1a}(1 + M)\,[1 + (1 + Q + q)\,\lambda L_a]\,\|z - \bar{z}\|_a\,,\end{aligned}$$

$$\begin{aligned}\|\sigma_3\|_n = {} & \|[\Delta_2(x, y) - \bar{\Delta}_2(x, y)]\,\alpha(x, y, \bar{z}(x, y))\|_n \leqq 2n^2HH'\|\gamma - \bar{\gamma}\|_a + \\ & + n^2H'\Lambda[2H\lambda L_a(1 + M) + CB_a(1 + (1 + Q)(1 + M)\,\lambda L_a)]\,\|z - \bar{z}\|_a\,,\end{aligned}$$

$$\begin{aligned}\|\sigma_4\|_n = {} & \|[\Delta_3(x, y) - \bar{\Delta}_3(x, y)]\,\alpha(x, y, \bar{z}(x, y))\|_n \leqq \\ & \leqq n^2H'[C\theta_a + 2C(1 + (1 + Q)(1 + M)\,\lambda L_a)(\theta_a + mQB_a) + \\ & + (P_a + mC(1 + nQ)\,B_a + nC\theta_a)(1 + Q(1 + M)\,\lambda L_a)]\,\|z - \bar{z}\|_a + \\ & + n^2H'(P_a + mC(1 + nQ)\,B_a + nC\theta_a)\,\|\gamma - \bar{\gamma}\|_a\,.\end{aligned}$$

Here $\bar{\Delta}_j$, $j = 1, 2, 3$, can be obtained from Δ_j by replacing γ, z and g with $\bar{\gamma}$, $\bar{z}$ and $\bar{g}$, respectively. Combining the previous estimates we have

$$\|(Fz)(x, y) - (F\bar{z})(x, y)\|_n \leqq \alpha\|\gamma - \bar{\gamma}\|_a + \beta\|z - \bar{z}\|_a\,,$$

and finally

$$\|Fz - F\bar{z}\|_a \leqq \alpha\|\gamma - \bar{\gamma}\|_a + \beta\|z - \bar{z}\|_a\,.$$

Thus the proof of Lemma 3 is complete.

5. The main result. Theorem. *If Assumptions* $H_1 - H_4$ *are satisfied then for a sufficiently small,* $0 < a \leqq a_0$, *there is a vector function* $z\colon \tilde{D}_a \to R^n$, $z \in K_\gamma$, *which satisfies* (1) *a.e. in* D_a *and* (2) *everywhere in* D_τ^0. *Furthermore,* z *is unique in the class* K_γ *and depends continuously on* γ.

Proof. We have shown in Lemma 2 that the transformation F maps K_γ into itself. We now prove that the map $F\colon K_\gamma \to K_\gamma$ is a contraction. We shall take a so small that $\beta \leqq k < 1$. Then we find from (6) that for $\gamma \in J$ fixed and for every pair $z, \bar{z} \in K_\gamma$, corresponding to $g, \bar{g} \in K_0$ the following estimate holds:

$$\|Fz - F\bar{z}\|_a \leqq k\|z - \bar{z}\|_a\,,$$

where $k < 1$. Thus, the transformation F is a contraction mapping of K_γ into itself; and there exists a unique fixed point $z \in K_\gamma$, $Fz = z$, such that the following integral equations hold:

$$g_i(\xi, x, y) = y - (T_z^i g_i)(\xi, x, y), \quad (\xi, x, y) \in \Delta_a, \quad i = 1, \ldots, n,$$

$$z(x, y) = (Fz)(x, y), \quad (x, y) \in \tilde{D}_a.$$

We can show similarly as in [7] that the fixed point $z = z[\gamma]$ is the (unique in the class K_γ) solution of the Cauchy problem (1), (2).

Relation (6) now yields

$$\|z - \bar{z}\|_a = \|z[\gamma] - z[\bar{\gamma}]\|_a \leqq (1 - \beta)^{-1} \alpha \|\gamma - \bar{\gamma}\|_a,$$

that is, $z = z[\gamma]$ depends continuously on $\gamma \in J$.

Thus the proof of Theorem is complete.

6. Examples. We list below a few examples of systems which can be derived from (1) by specializing the operator V.

(i) As a particular case of (1), (2) we obtain the initial problem for the quasilinear hyperbolic system of partial differential equations with a retarded argument (cf. [13])

$$\sum_{j=1}^{n} A_{ij}(x, y, z(x, y)) \left[\frac{\partial z_j}{\partial x} + \sum_{k=1}^{m} \varrho_{ik}(x, y, z(x, y), z(\varphi(x), \psi(x, y))) \frac{\partial z_j}{\partial y_k} \right] =$$

$$= f_i(x, y, z(x, y), z(\varphi(x), \psi(x, y))), \quad (x, y) \in D_a,$$

$$z(x, y) = \gamma(x, y), \quad (x, y) \in D_\tau^0,$$

where $z(\varphi(x), \psi(x, y)) = (z(\varphi_1(x), \psi_1(x, y)), \ldots, z(\varphi_l(x), \psi_l(x, y)))$,

$$\psi_j = (\psi_{j1}, \ldots, \psi_{jm}), \quad j = 1, \ldots, l, \quad i = 1, \ldots, n.$$

Let us suppose that

1° $\varphi_j: I_{a_0} \to R$, $j = 1, \ldots, l$, are measurable, $-\tau \leqq \varphi_j(x) \leqq x$, $j = 1, \ldots, l$, a.e. in I_{a_0};

2° $\psi_j(\cdot, y): I_{a_0} \to R^m$, $j = 1, \ldots, l$, are measurable for every $y \in R^m$, and there are constants $r_j > 0$, such that for all $y, \bar{y} \in R^m$ and a.e. $x \in I_{a_0}$ we have

$$\|\psi_j(x, y) - \psi_j(x, \bar{y})\|_m \leqq r_j \|y - \bar{y}\|_m, \quad j = 1, \ldots, l.$$

Then Assumption H_1 is satisfied for

$$(V_j z)(x, y) = z(\varphi_j(x), \psi_j(x, y)), \quad j = 1, \ldots, l,$$

with $q_j = r_j$, $e_j = 0$ and $M_j = 1$, $j = 1, \ldots, l$.

(ii) Let

$$(V_j z)(x, y) = \int_{\varphi_j(x,y)}^{\psi_j(x,y)} z(s, t) K_j(s, t, x, y) \, ds \, dt, \quad j = 1, \ldots, l, \tag{7}$$

where K_j, $j = 1, \ldots, l$, are $n \times n$ matrix functions $K_j = [K_j^{ik}]$, $i, k = 1, \ldots, n$, $j = 1, \ldots, l$. Then problem (1), (2) reduces to the Cauchy problem for the system of partial integrodifferential equations

$$\sum_{j=1}^{n} A_{ij}(x, y, z(x, y)) \left[\frac{\partial z_j}{\partial x} + \sum_{k=1}^{m} \varrho_{ik}\left(x, y, z(x, y), \int_{\varphi(x,y)}^{\psi(x,y)} z(s, t)\, K(s, t, x, y)\, \mathrm{d}s\, \mathrm{d}t \right) \frac{\partial z_j}{\partial y_k} \right] =$$

$$= f_i\left(x, y, z(x, y), \int_{\varphi(x,y)}^{\psi(x,y)} z(s, t)\, K(s, t, x, y)\, \mathrm{d}s\, \mathrm{d}t \right), \quad (x, y) \in D_a,$$

$$z(x, y) = \gamma(x, y) \quad (x, y) \in D_\tau^0.$$

Let us assume

1° $\varphi_j(\cdot, y)$, $\psi_j(\cdot, y): I_{a0} \to R^{m+1}$, $j = 1, \ldots, l$, are measurable for every $y \in R^m$, $-\tau \leqq \varphi_{j1}(x, y) \leqq x$, $-\tau \leqq \psi_{j1}(x, y) \leqq x$, $(x, y) \in D_a$, and there are constants r_j, $\bar{r}_j > 0$, such that for all $y, \bar{y} \in R^m$ and a.e. in I_{a0} we have

$$\|\varphi_j(x, y) - \varphi_j(x, \bar{y})\|_{m+1} \leqq r_j \|y - \bar{y}\|_m^{1/m+1},$$

$$\|\psi_j(x, y) - \psi_j(x, \bar{y})\|_{m+1} \leqq \bar{r}_j \|y - \bar{y}\|_m^{1/m+1}, \quad j = 1, \ldots, l;$$

3° there are constants $d_j > 0$, such that for every $(x, y) \in D_a$ we have

$$\prod_{k=1}^{m+1} |\psi_{jk}(x, y) - \varphi_{jk}(x, y)| \leqq d_j, \quad j = 1, \ldots, l;$$

4° the matrix functions $K_j(\cdot, y) = [K_j^{ik}(\cdot, y)]: I_{a0} \times R^m \times I_{a0} \to R^{n^2}$, $i, k = 1, \ldots, n$, $j = 1, \ldots, l$, are measurable for every $y \in R^m$;

5° there are constants $c_j > 0$, such that for every $(s, t, x, y) \in I_{a0} \times R^m \times I_{a0} \times R^m$ we have $\|K_j(s, t, x, y)\| \leqq c_j$, $j = 1, \ldots, l$;

6° there are constants $\tilde{r}_j > 0$, such that for all $y, \bar{y} \in R^m$, $(s, t, x) \in I_{a0} \times R^m \times I_{a0}$ we have

$$\|K_j(s, t, x, y) - K_j(s, t, x, \bar{y})\| \leqq \tilde{r}_j \|y - \bar{y}\|_m, \quad j = 1, \ldots, l.$$

Then Assumption H_1 is satisfied for the operator V_j defined by (7) with $q_j = 0$, $e_j = \Omega(c_j(r_j^{m+1} + \bar{r}_j^{m+1}) + d_j \tilde{r}_j)$ and $M_j = d_j c_j$, $j = 1, \ldots, l$, provided $d_j c_j < 1$, $j = 1, \ldots, l$.

(iii) Let $(V_j z)(x, y) = \int_{-\infty}^{y} z(x, t)\, K_j(y - t)\, \mathrm{d}t$, $j = 1, \ldots, l$. Then system (1) is a system of integrodifferential equations, whose particular case ($l = 1$, $\varrho(x, y, z, u) = \bar{\varrho}(x, y, z)$ and $f_i(x, y, z, u) = \bar{f}_i(x, y, z) + u$, $i = 1, \ldots, n$) was considered by P. Bassanini, M. C. Salvatori [4], under slightly less restrictive assumptions.

Now Assumption H_1 is satisfied with $q_j = 0$, $e_j = \Omega(r_j + \sup_{R^m} \|K(y)\|)$ and $M_j = \|\int_0^{+\infty} K_j(t)\, \mathrm{d}t\|$, $j = 1, \ldots, l$, if we assume

1° the matrix functions $K_j(\cdot) = [K_j^{ik}(\cdot)]: R^m \to R^{n^2}$, $j = 1, \ldots, l$, are measurable and bounded;

2° there are constants $r_j > 0$, such that for all $y, \bar{y} \in R^m$ we have

$$\|K_j(y) - K_j(\bar{y})\| \leqq r_j \|y - \bar{y}\|_m, \quad j = 1, \ldots, l;$$

3° $\left\|\int_0^{+\infty} K_j(t)\,dt\right\| < 1$, $j = 1, \ldots, l$.

(iv) By A_m we denote the set of all elements $\mu = (\mu_0, \mu_1, \ldots, \mu_m)$, such that $\mu_i = 0$ or $\mu_i = 1$ for $i = 0, 1, \ldots, m$ and $1 \leqq |\mu| = \mu_0 + \ldots + \mu_m$. It is easy to see that the number of elements of A_m is equal to $2^{m+1} - 1$. Let $N_\mu = \{i: \mu_i = 1\}$. For $(s, t) \in D_a$ we define $\mu(s, t) = (\mu_0 s, \mu_1 t_1, \ldots, \mu_m t_m)$ (we shall often write $\mu(s, t)$ instead of $\mu(s, t))$. Let $1 - \mu = (1 - \mu_0, 1 - \mu_1, \ldots, 1 - \mu_m)$ and $(1 - \mu)(s, t) = ((1 - \mu_0)s, (1 - \mu_1)t_1, \ldots, (1 - \mu_m)t_m)$. Suppose that

$$\mu \, ds \, dt = \begin{cases} ds \, dt_{i_1} \ldots dt_{i_k} & \text{if } \; o \in N_\mu, \quad i_1, \ldots, i_k \in N_\mu, \\ dt_{i_0} \, dt_{i_1} \ldots dt_{i_k} & \text{if } \; o \overline{\in} N_\mu, \quad i_0, i_1, \ldots, i_k \in N_\mu, \quad k = 1, \ldots, m, \end{cases}$$

and $\varphi^{(\mu)}, \psi^{(\mu)}: D_a \to R^{|\mu|}$, where $\varphi^{(\mu)} = (\varphi_{i_0}^{(\mu)}, \ldots, \varphi_{i_k}^{(\mu)})$, $\psi^{(\mu)} = (\psi_{i_0}^{(\mu)}, \ldots, \psi_{i_k}^{(\mu)})$ and $0 \leqq i_0 < i_1 < \ldots < i_k \leqq m$, $i_0, i_1, \ldots, i_k \in N_\mu$, $k = 1, \ldots, m$.

We define the operator V_μ in the following way:

$$(8) \qquad (V_\mu z)(x, y) = \int_{\varphi^{(\mu)}(x,y)}^{\psi^{(\mu)}(x,y)} z(\mu(s, t) + (1 - \mu)(x, y)) \, \mu \, ds \, dt .$$

Here $\int \mu \, ds \, dt$ is the $|\mu|$-dimensional integral with respect to the variables $s, t_{i_1}, \ldots, t_{i_k}$ if $o \in N$, $i_1, \ldots, i_k \in N_\mu$, and it is the integral with respect to $t_{i_0}, \ldots, t_{i_k}$ if $o \overline{\in} N_\mu$.

Now we consider the Cauchy problem (1), (2) for the integrodifferential system with $Vz = (V_{(1,\ldots,1)}z, V_{(0,1,\ldots,1)}z, V_{(1,0,1,\ldots,1)}z, \ldots, V_{(1,\ldots,1,0)}z, V_{(0,0,1,\ldots,1)}z, \ldots \ldots, V_{(1,\ldots,1,0,0)}z, \ldots, V_{(1,0,\ldots,0)}z)$.

We introduce the following assumptions:

1° $\varphi^{(\mu)}(\cdot, y)$, $\psi^{(\mu)}(\cdot, y): I_{a_0} \to R$, $\mu \in A_m$, are measurable, $-\tau \leqq \varphi_0^{(\mu)}(x, y) \leqq x$, $-\tau \leqq \psi_0^{(\mu)}(x, y) \leqq x$, a.e. in D_a;

2° there are constants $\bar{r}_j^{(\mu)}, \tilde{r}_j^{(\mu)} > 0$, such that for all $y, \bar{y} \in R^m$ and a.e. in I_a we have

$$|\varphi_j^{(\mu)}(x, y) - \varphi_j^{(\mu)}(x, \bar{y})| \leqq \bar{r}^{(\mu)} \|y - \bar{y}\|_m^{1/|\mu|},$$

$$|\psi_j^{(\mu)}(x, y) - \psi_j^{(\mu)}(x, \bar{y})| \leqq \tilde{r}^{(\mu)} \|y - \bar{y}\|_m^{1/|\mu|}, \quad j = 1, \ldots, m, \quad \mu \in A_m;$$

3° there is a constant $d^{(\mu)}$, $0 < d^{(\mu)} < 1$, such that for every $(x, y) \in D_a$ we have

$$\prod_{j \in N_\mu} |\psi_j^{(\mu)}(x, y) - \varphi_j^{(\mu)}(x, y)| \leqq d^{(\mu)} .$$

The Assumption H_1 is satisfied for the operator V_μ defined by (8) with $q_\mu = d^{(\mu)}$, $e_\mu = \Omega[(\bar{r}^{(\mu)})^{|\mu|} + (\tilde{r}^{(\mu)})^{|\mu|}]$ and $M_\mu = d^{(\mu)}$ (here $l = 2^{m+1} - 1$).

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