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On Some Discoveries in the Field of Scientific Methods for Management within the Concept of Analytic Hierarchy Process

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Abstract

The Analytic Hierarchy Process (AHP), the multicriteria decisions making support methodology widely recognized and accepted as the prioritization and choice theory may deliver different answers for a decisional problem, providing various rankings of its alternative solutions. The variety in priorities order exists because there are many methods that can be successfully implemented within the AHP, although its inventor, Thomas Saaty strives to convince researchers that there is only one i.e. principal right eigenvector method (REV) created alongside. Facts tell us however, that REV has some drawbacks and few flaws. The research described in this paper reveals some important discoveries within this field due to application of novel scenario for computer simulations concerning the entire AHP framework (contrary to single matrix simulation results). Statistically significant findings convince that REV concedes other methods being under study what authorize to claim that from the perspective of this research, as long as reciprocal pairwise comparison matrices are considered (condition embedded in the AHP), it should not be perceived as the dominant method.

Keywords: multicriteria decisions making support methodology, analytic hierarchy process, prioritization, theory of choice, pairwise comparison and ranking

1. Introduction

1.1 Problem-Solving Activity of Human Life

Since the conception of human race we are struggling with problems demanding decision making. It is probably the fundamental reason why we continuously deal with explanation and modeling of decisional problems in the way we can comprehend them. Plenty of methods, tools and procedures have been devised in order to make decision making process easier or sometimes even possible. It can be noticed that problem-solving activity pervades all aspects and levels of human life. There are economic, political, social, and technological problems, all of which can be found at individual, group, organizational, or societal levels. Moreover, because of the interdependent nature of most problems, they can affect or be affected by other problems at any level of human activity. Then, we may call ourselves decision makers for everything we do either consciously or unconsciously is the result of some decision (Saaty, 2000). It seems that all information we gather, we need for better understanding of occurrences around us, and for some kind of judgmental analysis in order to make decisions facing these occurrences (Glimcher et al., 2009). One may notice, that the immense scope of hierarchical classification is here very clear. It occurs, this is the most powerful method of classification used by the human brain in ordering experience, observations, entities and information. Some scientists even claim that the use of hierarchical ordering must be as old as human thought, both conscious and unconscious (Whyte, 1969).

However, psychologists have proven that the human brain is limited in both its short term memory capacity and its discrimination ability. It was scientifically verified that a human being will give inaccurate answers when forced to choose from a range of twenty alternatives, because the range exceeds man's bandwidth of perception channel (Martin, 1973). It has been demonstrated that humans are not capable of dealing accurately with more than about seven things at a time. It is crucial to notice that results of numerous psychological experiments, including the well known Miller's study (Miller, 1956) verified this notion. It also seems reasonable to accept the fact that a human being learns about anything in two ways. The first way involves examining and studying a given object, feeling or an idea in itself to the extent that it has various properties, then synthesizing the findings and drawing conclusions from such observations about it. The second way, entails studying that entity relative to other similar entities and relating it to them by making comparisons (Saaty, 2006). It seems that making pairwise comparisons is for human being as natural as binary counting is for computers. This occurs because humans have limited, so called bandwidth of their channel for perception, the phenomena standing behind quests on the path to improve our cognitive efficiency. Fortunately, there are some clues which emerged from this search. The first is to enable a man making rather relative than absolute judgments. The second is to organize tasks into groups in order to make several judgments in succession. Absolute judgment involves the relation between a single stimulus and some information held in short-term memory, whereas comparative judgment entails the identification of some relation between two stimuli both present to the observer (Blumenthal, 1977).

1.2 Decision Making and Its Support Methodology

We can perceive decision making nowadays as more mathematical science than it was centuries ago (Figuera et al., 2005). The process of decision making is now more formalized in the way we can track and analyze each step we undertake in order to make better decisions. So, the activity itself is more transparent today, in all its aspects. However, there are still different kinds of decisions that may or may not need formalization, i.e. intuitive and analytical. Intuitive decisions are not supported by data and documentation and may appear arbitrary. Unfortunately, a significant number of corporate decisions could still be classified as intuitive ones. Analytic decision making on the other hand scientifically endeavor to deal with complex reality challenges.

However, a last and often crucial disadvantage of many traditional analytic decision making methods is that they demand specialized expertise to design the appropriate structure and then to bring the decision making process in it. It seems that, ideally most wanted method overwhelming disadvantages of others would be: simple in construct, natural to human intuition and general thinking process, promoting consensus and compromise, adaptable to both individuals and groups, and not requiring any sophisticated skills to master. As it occurs, these criteria are met by the decision making process called Analytic Hierarchy Process (AHP) which can be considered to be both a descriptive and prescriptive model of decision making. Its methodology compares criteria, or alternatives with respect to a criterion, in a natural, pairwise mode. The comparison process proceeds with the application of fundamental scale of absolute numbers that has been proven in practice. In fact, it has been validated in hundreds of experiments that the method does indeed generate results conforming to classic ratio scale measurement in physics, economics, and other fields where standard measures already exist (Saaty, 1988, p. 39; Saaty, 2006, pp. 192–195; Saaty, 1980, pp. 38–42; Saaty, 2008). It is also, the most widely used decision making approach in the world today, as well most validated methodology, see e.g. Kazibudzki (2012) and references in there.

Generally, in order to make a decision one needs various kinds of knowledge, information and technical data. These concern: details about the problem to be decided, the people or actors involved, their objectives and policies, the influences affecting the outcomes, and the time horizons, scenarios and constraints (Saaty, 2001). That is why the AHP is grounded on the well-defined mathematical structure of consistent matrices and their associated principal right eigenvector's ability to generate true or approximate weights (Merkin, 1979; Saaty, 1990), what also constitutes the principal source of its criticism, see e.g. (Grzybowski, 2012; Basak, 1998; Budescu, Zwick & Rapoport, 1986; Hovanov, Kolari & Sokolov, 2008; Lipovetsky & Tishler, 1997; Zahedi 1986; Bana e Costa & Vansnick, 2008; Belton & Gear, 1983; Lipovetsky & Conklin, 2002).

1.3 Basic Notations of the AHP

When we suppose that a decision maker has only judgments (estimates) of the relative weights of a set of activities, then it is possible to express them in a pairwise comparison matrix (PCM) denoted as ***A(a)*** with elements $a_{ij}=a_i/a_j$ Obviously, true relative weights themselves can be expressed analogically. Let us denote then ***A(w)*** as the symbol of a matrix with elements $w_{ij}=w_i/w_j$

If the elements of a matrix ***A(a)*** satisfy the condition $a_{ij}=1/a_{ji}$ for all $i,j=1,\dots,n$ then the matrix ***A(a)*** is said to be reciprocal (RPCM). If its elements satisfy the condition $a_{ik}a_{kj}=a_{ij}$ for all $i,j,k=1,\dots,n$ and the matrix is reciprocal, then it is called consistent. Finally, the matrix ***A(a)*** is said to be transitive (TPCM) if the following conditions hold: (i) if for any $l=1,\dots,n$, an element a_{lj} is not less than an element a_{lk} then $a_{ij} \geq a_{ik}$ for $i=1,\dots,n$, and (ii) if for any $l=1,\dots,n$, an element a_{jl} is not less than an element a_{kl} then $a_{ji} \geq a_{ki}$ for $i=1,\dots,n$. Obviously, in the case of the reciprocal PCM the two conditions (i) and (ii) are equivalent.

Thus, if we would like to recover the vector of weights $\boldsymbol{w}=[w_1, w_2, w_3,\dots, w_n]^T$ which true relative weights of a set of activities can be created from, as in the case of matrix ***A(w)***, we can apply either eigenvector method (REV) or any optimization method that seeks a vector $\boldsymbol{w}$ as a solution of the minimization problem given by the formula:

$$\text{Min } \boldsymbol{D(A(a),\ A(w))} \tag{1}$$

subject to some assigned constraints such as for instance positive coefficients and normalization condition.

Because the distance function ***D*** measures an interval between matrices ***A(a)*** and ***A(w)***, different ways of its definition lead to various prioritization concepts and prioritization outcome. It seems that the biggest competitor in comparison to REV, and also most popular method among optimization ones constitutes the logarithmic least squares method (LLSM), known also as geometric mean method (Crawford, 1987).

1.4 The Essence of the Matter

It seems prerequisite for a credible decision making support methodology to provide unambiguous answers for the alternatives of a decision. Obviously it also concerns the AHP, especially that a variety of operational procedures were invented in order to support this methodology. When decision makers preferences are more or less consistent, the priority vectors derived from their intuitive judgments rather coincide. Nevertheless, especially in managerial decisions, preferences are constantly inconsistent what leads directly to the situation that we have various rankings for different procedures. It was also demonstrated that especially in multicriteria decisional problems even when different procedures deliver priority vectors that are close to each other, both on criteria and alternatives, after standard AHP aggregation based on weighting and adding (Saaty, 1980) the alternatives priorities can change (Saaty & Hu, 1998). It seems that such ambiguity within the single decision making support methodology cannot be accepted. Certainly, we are being implicated that exclusively principal right eigenvector method (REV) counts in this matter but let us remember that REV, despite of its advantages, has also few flaws that should not be neglected. That is the basic reason we initiated novel simulations study, i.e. not such that deals exclusively with one single PCM, but such that concern hypothetic decisional problem within a given AHP framework which consists of different number of criteria and different number of alternatives.

2. Research Methodology

2.1 Examined Procedures Definitions

Thus, we have chosen three procedures that in the literature were considered as the best ones within the AHP methodology (Lin, 2007; Choo & Wedley, 2004):

– (GM) i.e. geometric mean procedure (Crawford & Williams, 1985) given by the formula:

$$w_i = \left(\prod_{j=1}^{n} a_{ij}\right)^{1/n} \Bigg/ \sum_{i=1}^{n}\left(\prod_{j=1}^{n} a_{ij}\right)^{1/n} \tag{2}$$

– (REV) i.e. principal right eigenvector method (Saaty, 1980) given by the formula:

$$w = \lim_{k\to\infty}\left(\frac{A^k \times e^T}{e \times A^k \times e^T}\right) \tag{3}$$

where $\boldsymbol{e}=[1,1,\ldots,1]$.

– (SNCS) i.e. simple normalized column sum procedure (Choo & Wedley, 2004) given by the formula:

$$w_i = \frac{1}{n}\sum_{j=1}^{n}\left(a_{ij} \Bigg/ \sum_{k=1}^{n} a_{kj}\right) \tag{4}$$

2.2 Assumptions

We assume, the hierarchy consist of three levels: goal, criteria and alternatives. This framework in our opinion reflects the hypothetic case of true managerial decision problem. Thus, we simulate distinct situations (different scales with various number of criteria and alternatives) within this framework in relation to different sources of PCMs inconsistency. Basically, the inconsistency constitutes the outcome of errors caused by the nature of human judgments and errors determined by the technical realization of the comparison procedure. The latter ones, appear mainly due to the rounding errors and errors resulting from the forced reciprocity requirement. The rounding errors are related to the numerical ratio scale whose values should be used by prospective decision makers in order to express somehow their judgments (Kazibudzki, 2012; Grzybowski, 2012; Dong, Xu, Li & Dai, 2008; Lipovetsky & Tishler, 1997; Lipovetsky & Conklin, 2002). Certainly, in conventional AHP applications the most popular is Saaty's numerical scale that comprises the integers from 1 to 9 and their reciprocals. But there are known also other scales (Dong, Xu, Li & Dai, 2008), e.g.:

- geometric scale for which the linguistic variables of Saaty's scale hold different numerical values, i.e. most commonly and as such also in our research: $2^{n/2}$ where n comprises the integers from minus 8 to 8;
- arbitrary numerical scale which comprises the integers from 1 to n and their reciprocals.

Fortunately, although more troublesome for simulation's processes, errors caused by the nature of human judgments are manageable too. They are represented as the realization of some random process in accordance with the formula (5) given below:

$$a_{ij} = e_{ij} \cdot w_i/w_j \tag{5}$$

where e_{ij} is a perturbation factor oscillating near 1, e.g. (Saaty, 2003; Sun & Greenberg, 2006; Ishizaka & Labib, 2011). In a statistical approach and many simulation studies the perturbation factor is interpreted as a realization of a random variable, e.g. (Grzybowski, 2012; Zahedi, 1986) and its probability distributions mainly involve uniform and gamma, as well truncated normal or log-normal (Basak, 1998; Choo & Wedley, 2004; Lin, 2007; Zahedi, 1986).

2.3 Simulation Scenario Description

In our simulation study we proceed in accordance with the scenario described in Kazibudzki (2012), thus we generate uniformly random and normalized 'original' priority vector for uniformly randomly chosen number of criteria. Next, we generate uniformly random and normalized 'original' priority vectors of alternatives for the given set of criteria with uniformly randomly chosen number of alternatives. Then, we calculate the 'original' total priority vector (OTPV) of weights, according to an earlier described AHP algorithm. Next, on the bases of 'original' individual priority vectors generated for the given set of criteria and the given sets of alternatives, we create correspondent pairwise comparison matrices (PCMs). Then, we make them inconsistent, first through perturbation of their elements in accordance with relation (5) and secondly through rounding their elements to a chosen particular scale. Next, on the bases of such created inconsistent PCMs we compute their respective priority vectors with the application of chosen methods: GM, REV and SNCS. Then, for each method we calculate the total priority vector (TPV) applying standard AHP aggregation algorithm. Finally, we compare values of such obtained TPV with the values of OTPV. The simulation study assumes that just described scenario is repeated given number of times. For the performance evaluation purpose we compute known from literature: the Pearson correlation coefficient (PCC) between the OTPV and its estimate TPV, Spearman rank correlation coefficient (SRCC) (Grzybowski, 2012; Moy, Lam & Choo, 1997; Budescu, Zwick & Rapoport, 1986), and mean absolute deviation (Kazibudzki, 2012; Choo & Wedley, 2004; Lin, 2007; Dong, Xu, Li & Dai, 2008).

In our simulations, we always consider approximation with forced reciprocity condition applied as it is most common and embedded assumption in the AHP methodology. As for the simulation procedure, the forced reciprocity condition operates in this way that the perturbed PCM inputs are taken only from above its diagonal elements, and the remaining ones are entered as the inverses of the corresponding symmetric units in relation to its diagonal elements.

2.4 Introductory Exemplification within Simplified Framework

The following example describes the simplified simulation procedure in the situation when only one PCM is considered as opposite to the whole AHP framework. Thus, we begin with the assumption that we know the 'original' PV. Let the 'original' normalized PV be imposed as follows: $\boldsymbol{w}=[0.75, 0.15, 0.1]^T$. Then, the 'original' matrix $\boldsymbol{A(w)}$ derived from this vector has the following entries:

$$A(w) = \begin{bmatrix} 1 & 5 & 7.5 \\ 0.2 & 1 & 1.5 \\ 0.133333 & 0.666667 & 1 \end{bmatrix}$$

Following the simulation scenario, in our next step we perturb such obtained consistent PCM in order to get an inconsistent one. We do it firstly with an application of the equation (5) and secondly rounding its entries to the chosen particular scale (here it will be Saaty's numerical scale). For the purpose of exemplification, we will proceed in this example only with steps limited to direct rounding (without earlier perturbation with an application of the equation (5)). Thus we have:

$$A(a) = \begin{bmatrix} 1 & 5 & 7 \\ 1/5 & 1 & 1 \\ 1/8 & 1/2 & 1 \end{bmatrix}$$

As we may notice due to application of rounding errors we receive nonreciprocal PCM. Thus, our further task is to make it reciprocal. Following the algorithm described earlier in this paper we get the following PCM:

$$\widetilde{A}(a)=\begin{bmatrix}1 & 5 & 7\\ 1/5 & 1 & 1\\ 1/7 & 1 & 1\end{bmatrix}$$

Basing on this PCM that can be seen as a decision maker best attempt to estimate the 'original' ***A(w)*** we can compute its possible PVs, simply choosing the right procedure, in particular GM, SNCS and REV. Certainly, we can then compare such obtained results with the 'original' PV and calculate earlier mentioned performance measures such as MAD, SRCC, and PCC. The results of such calculations and priority vectors obtained due to application of chosen methods are presented in table 1.

Table 1. Comparison of GM, SNCS and REV performances in the example with rounding errors and forced reciprocity applied for 'original' PV: $\boldsymbol{w}=[0.75, 0.15, 0.1]^T$

Method	Estimates	Performance measures		
		MAD	SRCC	PCC
GM	$[0.747053, 0.133559, 0.119389]^T$	0.0129257	1	0.99878
SNCS	$[0.745581, 0.134301, 0.120117]^T$	0.0134114	1	0.998785
REV	$[0.747053, 0.133559, 0.119389]^T$	0.0129257	1	0.99878

As we can see in the presented example both GM and REV provide the same estimates of PV, which was proven mathematically for $n \le 3$. In this case also all methods provide the same ranks for all priorities (SRCC=1). Obviously, we want to check if this is true for the entire AHP model with different number of criteria and different number of alternatives. Fortunately, we can simulate similar scenarios with the application of various scales, various probability distributions for perturbation factor in the equation (5) and various size of errors reflected by the range of perturbation factor in the equation (5). For instance in the case of AHP framework constructed for 3 criteria and 3 alternatives from the same imposed $\boldsymbol{w}=[0.75, 0.15, 0.1]^T$ for all its levels the OTPV (overall ranking) would be exactly the same as the single imposed one. The total priority vector (overall ranking) received due to application of chosen methods to respective perturbed (in the same way as above) PCMs derived from single 'original' PVs in this particular case also is the same as in the single PCM case, i.e. $\boldsymbol{w}=[0.747053, 0.133559, 0.119389]^T$ for GM, $\boldsymbol{w}=[0.745581, 0.134301, 0.120117]^T$ for SNCS, and $\boldsymbol{w}=[0.747053, 0.133559, 0.119389]^T$ for REV. Nevertheless, the study becomes truly realistic only then when simulation framework is applied without artificial assumptions and simplifications because only in this way we obtain results that reflect a big picture i.e. real life performance evaluation.

3. Research Results

Thus, let us concentrate on situations when inconsistency exists not only due to technical realization of the AHP process (rounding and forced reciprocity) but also human judgment errors that constantly take place in real decision making problems. Saaty & Hu (1998) illustrated the case where variability in PVs ranks between two procedures did not occur for each individual PCM but occurred in the overall ranking of the final alternatives due to the multicriteria process itself. Building on this example we proceeded with similarly designed simulations in order to validate the statement presented in there. However, our research framework encompasses different number of AHP frameworks (50 or 100) and various ways of inconsistency that was implemented by:

- the perturbation factor (e_{ij}) drawn uniformly, log-normally, gamma or truncated-normally from the interval $e_{ij} \in [0.05, 1.95]$ and;
- rounding entries of perturbed PCM to a particular scale (Saaty's numerical scale, geometric scale and arbitrary numerical scale for n=50.

We decided to perturb each PCM within the single AHP framework either 50 or 100 times with the application of chosen perturbation factor distribution. Tables 2–7 present selected (statistically significant) mean performance measures of GM, SNCS and REV for 2 500 (50 x 50) and 10 000 (100 x 100) cases.

Table 2. Mean performance results of GM, SNCS and REV for 2 500 cases (*)

Method	Performance measures		
	MAD	SRCC	PCC
GM	0.0169201	0.806346	0.929493
REV	0.0195622	0.758181	0.896157
SNCS	0.0183839	0.784396	0.919387

Note: (*) AHP uniformly random framework (n_k, $n_a \in [8, 12]$) with **uniformly** distributed perturbation factor $e_{ij} \in [0.05, 1.95]$ and technical errors implementation (rounding to geometric scale and forcing PCM reciprocity)

Table 3. Mean performance results of GM, SNCS and REV for 2 500 cases (*)

Method	Performance measures		
	MAD	SRCC	PCC
GM	0.0289979	0.681213	0.763865
REV	0.0318991	0.633144	0.705974
SNCS	0.0296353	0.667491	0.753209

Note: (*) AHP uniformly random framework (n_k, $n_a \in [8, 12]$) with **gamma** distributed perturbation factor $e_{ij} \in [0.05, 1.95]$ and technical errors implementation (rounding to geometric scale and forcing PCM reciprocity)

Table 4. Mean performance results of GM, SNCS and REV for 2 500 cases (*)

Method	Performance measures		
	MAD	SRCC	PCC
GM	0.0161027	0.800270	0.932354
REV	0.0192493	0.751877	0.897493
SNCS	0.0175419	0.781048	0.923762

Npte: (*) AHP uniformly random framework (n_k, $n_a \in [8, 12]$) with **uniformly** distributed perturbation factor $e_{ij} \in [0.05, 1.95]$ and technical errors implementation (rounding to arbitrary numerical scale with n=50 and forcing PCM reciprocity)

Table 5. Mean performance results of GM, SNCS and REV for 2 500 cases (*)

Method	Performance measures		
	MAD	SRCC	PCC
GM	0.0312237	0.596775	0.720996
REV	0.0370230	0.517215	0.596455
SNCS	0.0317082	0.579539	0.704673

Note: (*) AHP uniformly random framework (n_k, $n_a \in [8, 12]$) with **gamma** distributed perturbation factor $e_{ij} \in [0.05, 1.95]$ and technical errors implementation (rounding to arbitrary numerical scale with n=50 and forcing PCM reciprocity)

Table 6. Mean performance results of GM, SNCS and REV for 10 000 cases (*)

Method	Performance measures		
	MAD	SRCC	PCC
GM	0.0187484	0.835546	0.930354
REV	0.0201425	0.808539	0.914050
SNCS	0.0200100	0.822548	0.925367

Note: (*) AHP uniformly random framework (n_k, $n_a \in [8, 12]$) with **uniformly** distributed perturbation factor $e_{ij} \in [0.05, 1.95]$ and technical errors implementation (rounding to Saaty's numerical scale and forcing PCM reciprocity)

Table 7. Mean performance results of GM, SNCS and REV for 10 000 cases (*)

Method	Performance measures		
	MAD	SRCC	PCC
GM	0.0309819	0.656190	0.765560
REV	0.0322768	0.628579	0.741102
SNCS	0.0312162	0.649620	0.766790

Note: (*) AHP uniformly random framework (n_k, $n_a \in$ [8, 12]) with **gamma** distributed perturbation factor $e_{ij} \in$ [0.05, 1.95] and technical errors implementation (rounding to Saaty's numerical scale and forcing PCM reciprocity)

Noticeably, if we take SRCC as the indicator of OTPV rank preservation and ability to recover it as precisely as possible due to certain procedure application within the AHP, we may notice that REV performance is always dominated by GM or SNCS.

4. Discussion

In our simulations we studied the performance measures for three chosen prioritization procedures within the AHP framework for four classes of probability distributions of the perturbation factors: uniform, log-normal, truncated normal and gamma. Although, all results were rather similar we selected only these that were statistically significant from the perspective of two most popular and competitive procedures i.e. GM and REV.

We validated them with the application of the same AHP framework that was presented in Saaty & Hu (1998). However, as opposite to the single case study described in Saaty & Hu (1998) we simulated many thousands various AHP frameworks and observed the performance results of chosen procedures under different PCMs perturbation scenarios. We could test in this way the performance differences among chosen procedures treating SRCC as the particular procedure performance measure.

If we denote $SRCC_{GM}$ and $SRCC_{REV}$ as Spearman rank correlation coefficient of geometric mean (GM) and Spearman rank correlation coefficient of right eigenvector method (REV), we can test their difference significance using "t" statistics given by the following formula:

$$t = R\sqrt{\frac{n-2}{1-R^2}} \tag{6}$$

where R is the difference between particular SRCCs.

This statistics has a distribution of *t–student* with n minus 2 degrees of freedom df, where n equals size of the sample. We test the hypothesis H_0: $SRCC_{GM} - SRCC_{REV} = 0$ versus H_A: $SRCC_{GM} - SRCC_{REV} > 0$. In our simulations df=2 498 or df=9 998, thus for assumed significance level α=0.01 the critical value of $t_{0.01} \approx 2.32$ in both cases. When tested value of t is bigger than its critical level for accepted significance, we reject H_0 assuming equality of evaluated measures on the favor of alternative hypothesis. In the opposite situation we do not have foundations to reject H_0. Details concerning difference significance between $SRCC_{GM}$ and $SRCC_{REV}$ for six scenarios presented in tables 2–7 provides table 8.

Table 8. Statistics for six scenarios presented in tables 2–7 concerning assumption about equality of $SRCC_{REV}$ and $SRCC_{GM}$

Scenario	*R*–value	*t*–value	Decision about H_0
Tab. No.2	0.048165	2.410	REJECT
Tab. No.3	0.048069	2.405	REJECT
Tab. No.4	0.048393	2.422	REJECT
Tab. No.5	0.079560	3.989	REJECT
Tab. No.6	0.027007	2.701	REJECT
Tab. No.7	0.027611	2.762	REJECT

Taking into account the statistical hypothesis tests concerning entire simulation framework, we have serious foundation to claim that as long as reciprocity of PCMs is imposed (the requirement embedded in the AHP), it is not grounded to consider REV as the dominant method among others as it was done in Saaty & Hu (1998). Furthermore, as long as reciprocity of PCMs is forced within the AHP framework we find REV as the rule worse in its performance in all researched scenarios. Obviously, some differences in presented procedure performance were statistically insignificant so we decided to elaborate and discuss in more detail only these results that were statistically significant. However, our research clearly prove that REV is not as effective as other chosen procedures presented in this article. By the word effective, we mean the process of unique capturing the ratio scale rank order inherent in inconsistent pairwise comparison judgments as long as reciprocity of PCMs is forced in the AHP. It is clear that results of our research stand in opposition to the theorem presented in Saaty & Hu (1998). To recapitulate, REV surely is not the only valid procedure for deriving priority vectors from a reciprocal pairwise comparison matrices, what was now validated from the perspective of order preservation concept within the entire AHP framework comprising inconsistent matrices.

This seems very important from the viewpoint of multicriteria decision making in the field of scientific management because even if variability in ranks does not occur for individual judgment for different procedures applied, it may still occur in the overall ranking of the final alternatives due to the multicriteria process itself. Because REV can give less credible rankings than other procedures available for the AHP, it is advised to consider them instead, especially under some circumstances of an important and very tight managerial decisions.

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