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ON SOME INEQUALITIES INVOLVING TRIGONOMETRIC AND HYPERBOLIC FUNCTIONS WITH EMPHASIS ON THE CUSA–HUYGENS, WILKER, AND HUYGENS INEQUALITIES

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Abstract. Recently trigonometric inequalities of N. Cusa and C. Huygens (see, e.g., [9]), J. Wilker [11], and C. Huygens [4] have been discussed extensively in mathematical literature. We shall demonstrate that Wilker's inequality, Huygens' inequality, and some other related inequalities all follow from the Cusa-Huygens inequality. A generalization of the latter result is also obtained. The hyperbolic counterparts of those inequalities are also derived.

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