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ON SPECIAL TYPES OF MINIMAL AND TOTALLY GEODESIC UNIT VECTOR FIELDS

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Abstract. We present a new equation with respect to a unit vector field on Riemannian manifold M^n such that its solution defines a totally geodesic submanifold in the unit tangent bundle with Sasakian metric and apply it to some classes of unit vector fields. We introduce a class of covariantly normal unit vector fields and prove that within this class the Hopf vector field is a unique global one with totally geodesic property. For the wider class of geodesic unit vector fields on a sphere we give a new necessary and sufficient condition to generate a totally geodesic submanifold in T_1S^n.

1. Introduction

This paper is organized as follows. In Section 2 we give definitions of harmonic and minimal unit vector fields, rough Hessian and harmonicity tensor for the unit vector field. In Section 3 we give definition of a totally geodesic unit vector field and prove a basic Lemma 2 which gives a necessary and sufficient condition for the unit vector field to be totaly geodesic. Theorem 2 contains a necessary and sufficient condition on strongly normal unit vector field to be minimal. In Section 4 we apply Lemma 2 to the case of a unit sphere (Lemma 4) and describe the geodesic unit vector fields on the sphere with totally geodesic property (Theorem 5). We also introduce a notion of covariantly normal unit vector field and prove that within this class the Hopf vector field is a unique one with a totally geodesic property (Theorem 3). This theorem is a revised and simplified version of Theorem 2.1 in [27]. Section 5 contains an observation that the Hopf vector field on a unit sphere provides an example of global imbedding of Sasakian space form into Sasakian manifold as a Sasakian space form with a specific φ-curvature (Theorem 6).