

ON STRONG DELTA-CONVEXITY AND HERMITE-HADAMARD TYPE INEQUALITIES FOR DELTA-CONVEX FUNCTIONS OF HIGHER ORDER

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Abstract. In our previous paper [15], using s -convex stochastic ordering [4], we investigate Hermite-Hadamard-Fejér type inequalities in the case of higher order convex functions. In the present paper, our aim is to extend this investigation from convex to delta-convex functions of higher order [8]. We offer some useful tools for obtaining and proving of various forms of the Hermite-Hadamard-Fejér type inequalities for delta-convex functions of higher order, that generalizes results of Dragomir et al. [5]. These results are applied to derive some inequalities between quadrature operators. We define also and study strong delta-convexity of n -th order that generalizes strong n -convexity studied in [14] and [9].

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