On Strong Equilibria in the Max Cut Game

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Abstract. This paper deals with two games defined upon well known generalizations of MAX CUT. We study the existence of a *strong equilibrium* which is a refinement of the Nash equilibrium. Bounds on the price of anarchy for Nash equilibria and strong equilibria are also given. In particular, we show that the MAX CUT game always admits a strong equilibrium and the strong price of anarchy is 2/3.

1 Introduction

Suppose that n agents communicate via radio signals but only two distinct frequencies are available. In this scenario we are given a symmetric $n \times n$ matrix which indicates, for each pair of agents, the strength of the interference that they experiment if they select the same frequency. We suppose that each agent chooses her frequency in order to minimize the sum of interferences that she experiments[1], no matter what is the situation of the others. We use *strategic game theory* as a formal framework to study the following question: What would be the worst configuration that the selfish agents can reach compared to a solution where a central entity assigns frequencies optimally? When *Nash equilibria* – a situation where no agent can unilaterally deviate and benefit – are considered, this ratio is better known as the *price of anarchy* (PoA) [10]. It captures the performance of systems where selfish players interact without central coordination. Intuitively, a PoA far from 1 indicates that the system requires regulation.

Nash equilibria are considered as stable configurations. However a Nash equilibrium is not sustainable if the agents can realize that they all benefit if they perform a simultaneous deviation whereas any unilateral move is inefficient. The *strong equilibrium* introduced by Aumann [2] is a refinement of the Nash equilibrium where for every deviation by a group of agents, at least one member of the group does not benefit. The *strong price of anarchy* (SPoA) [1] is the PoA reduced to strong equilibria.

So, what are the PoA and SPoA of the above mentionned interference game? The game was already studied in [6,4,7]. It is defined upon the well known MAX CUT problem: Given a simple weighted graph, find a bipartition of the vertex set such that the weight of the edges having an endpoint in both parts of the partition, i.e. the cut, is maximum. In the MAX CUT game, a player's utility is her contribution to cut, i.e. the weight of the edges of the cut which are incident

[1] Or equivalently, maximize the sum of interferences that she does not experiment.

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to her. The game always possesses a pure Nash equilibrium since it admits a potential function [15] (the weight of the cut). It is a kind of folklore that the PoA is 1/2. Up to our knowledge, nothing is known about the existence of a strong equilibrium and the SPoA of the MAX CUT game.

In this paper, we study two generalizations of the MAX CUT game which are similarly defined upon two generalizations of MAX CUT: NAE SAT and MAX $k-$CUT. An instance of the NAE SAT problem is a set of clauses, each of them being satisfied if its literals are not all true (or not all false) and one asks a truth assignment maximizing the weight of satisfied clauses. MAX CUT is equivalent to NAE SAT if each clause is made of two unnegated variables. In the NAE SAT game, every player tries to maximize the weight of satisfied clauses where she appears. A motivation of the NAE SAT game is given in the sequel. In the MAX $k-$CUT problem, one asks a k partition of the vertex set inducing a maximum weight cut. Its associated game is the interference game with k frequencies instead of 2.

2 Definitions and Notations

A *strategic game* is a tuple $\langle N, (A_i)_{i \in N}, (u_i)_{i \in N} \rangle$ where N is the set of players (we suppose that $|N| = n$), A_i is the set of strategies of player i and $u_i : \times_i A_i \to \mathbb{R}$ is player i's utility function. A *pure state* or *pure strategy profile* of the game is an element of $\times_i A_i$. Although players may choose a probability distribution over their strategy set, we only consider pure strategy profiles in this paper. Players are supposed to be rational, i.e. each of them plays in order to maximize her utility.

Given a state a, (a_{-i}, b_i) denotes the state where a_i is replaced by b_i in a while the strategy of the other players remains unchanged. A state a is a *Nash equilibrium* (NE) if there no player $i \in N$ and a strategy $b_i \in A_i$ such that $u_i\big((a_{-i}, b_i)\big) > u_i(a)$. Given two states a, a' and a coalition $C \subseteq N$, (a_{-C}, a') denotes the state where a_i is replaced by a'_i in a for all $i \in C$. A state a is a *strong equilibrium* (SE) if there is no non-empty coalition $C \subseteq N$ and a profile $a' \in A$ such that $u_i\big((a_{-C}, a')\big) > u_i(a)$ for all $i \in C$. A state a is an *r-strong equilibrium* (r-SE) if there is no non-empty coalition $C \subseteq N$ of size at most r and a profile $a' \in A$ such that $u_i\big((a_{-C}, a')\big) > u_i(a)$ for all $i \in C$. Therefore a SE is a NE, a NE is a 1-SE and a SE is n-SE (n is the number of players).

The *price of anarchy* (PoA) measures the performance of decentralized systems [10] via its Nash equilibria. More formally, let Γ be a family of strategic games, let γ be an instance of Γ, let A_γ be the strategy space of γ, let $\mathcal{Q} : A_\gamma \to \mathbb{R}_+$ be a social function, let $\mathcal{E}(\gamma)$ be the set of all pure Nash equilibria of γ and let o_γ be a social optimum for γ (i.e. $o_\gamma = \operatorname{argmax}_{a \in A_\gamma} \mathcal{Q}(a)$). The *pure* price of anarchy of Γ is $\min_{\gamma \in \Gamma} \min_{a \in \mathcal{E}(\gamma)} \mathcal{Q}(a)/\mathcal{Q}(o_\gamma)$. If $\mathcal{SE}(\gamma)$ denotes the set of all strong equilibria of γ then the strong price of anarchy (SPoA) [1] is $\min_{\gamma \in \Gamma} \min_{a \in \mathcal{SE}(\gamma)} \mathcal{Q}(a)/\mathcal{Q}(o_\gamma)$. The r-SPoA is similarly defined when restricting ourselves to r-strong equilibria.

3 Related Work and Contribution

The MAX CUT game is a *game of congestion* [11]. Congestion games is a particular subclass of *potential games* [15] which are known to always possess a pure strategy NE. Any NE of the MAX CUT game corresponds to a local optimum whose computation is sometimes polynomial (cubic graphs [13], unweighted case) but **PLS**-complete in general [16]. Rencently Christodoulou et al. [4] studied the rate of convergence to an approximate NE in the MAX CUT game and the social welfare of states obtained after a polynomial number of best response steps.

The MAX CUT game is close to the *party affiliation game* [6] and the *consensus game* [3]. The MAX $k-$CUT game is related to the model of migration studied by Quint and Shubik [12]. A land where several animals live is partitioned into k areas and each animal has to choose one. Two animals seeking the same resources (e.g. food or living conditions) compete if they share the same area. We assume that every kind of resource exists in each area. Then each animal migrates to the area where competition is minimum.

In a broader study on *clustering games* [7], Hoefer proved that the PoA of the *unweighted* MAX $k-$CUT game is $(k-1)/k$. However, nothing is known about existence of a SE for this game and its SPoA. Up to our knowledge, nothing is known about the PoA of the NAE SAT game, the existence of a SE and the SPoA. However every game studied in this paper is a particular case of congestion games. Congestion games possess a SE in many situations (some of them are identified in [8,9,5,14]) but its existence is not always guaranteed.

In this paper we study the existence of a SE and the (S)PoA of the MAX $k-$CUT and NAE SAT games. Section 4 is devoted to the NAE SAT game. If each clause has two literals then we prove that any optimal solution is a SE and the SPoA is 2/3. With more literals per clause, we show that no 2-SE is guaranteed while a pure NE, a 1-SE in fact, must exist. The PoA of the NAE SAT game is in general 1/2 and $q/(q+1)$ if each clause if made of exactly $q \geq 3$ literals. Section 5 is devoted to the MAX $k-$CUT game. Our positive result states that any optimal solution is a 3-SE (when $k=2$, an optimal cut is a SE by the result given for the NAE SAT game). Our negative result states that for $k \geq 3$, there is an instance with two distinct optimal cuts: one is a SE while the other is not a 4-SE. Before giving a conclusion, we show that the r-SPoA of the MAX CUT game is equal to 1/2 if r is bounded above by the square root of the number of players.

Due to space limitations, proofs are sometimes sketched or skipped.

4 The NAE SAT Game

Given a set X of boolean variables and a set $\mathcal{C}$ of clauses, each of them being composed of at least two literals defined over X and a weight function $w : \mathcal{C} \rightarrow \mathbb{R}_+$, NAE SAT is to find a truth assignment $\tau : X \rightarrow \{\texttt{true}, \texttt{false}\}$ such that the weight of NAE-satisfied clauses is maximum. A clause is NAE-satisfied if its literals are not all true or not all false (NAE= not all equal). In the following q-NAE SAT refers to the case where each clause has exactly q literals.

In the NAE SAT game, each variable is controlled by a selfish player with strategy `true` or `false`. A player's utility is the weight of NAE-satisfied clauses where she appears. The social function is the weight of NAE-satisfied clauses.

As an application, imagine a population of animals cut into two groups (or gangs) denoted by T and F. Anyone can choose to live in T or F but not in both. In addition every individual i carries a set γ_i of genes (his *genotype*) that he wants to be ideally present in both groups. If i chooses T (resp. F) then all his genes are in T (resp. F) and exactly $|\gamma_i \cap \bigcup_{j \in F} \gamma_j|$ (resp. $|\gamma_i \cap \bigcup_{j \in T} \gamma_j|$) of his genes are in F. Then, in order to maximize the presence of his genotype, i prefers T if $|\gamma_i \cap \bigcup_{j \in F \setminus \{i\}} \gamma_j| \geq |\gamma_i \cap \bigcup_{j \in T \setminus \{i\}} \gamma_j|$, otherwise i prefers F. One can model the situation as a NAE SAT game: each animal i is a variable x_i, each gene g carried by at least two animals is a clause including a positive literal x_i iff $g \in \gamma_i$. Thus, $i \in T$ (resp. $i \in F$) means x_i is `true` (resp. x_i is `false`).

The NAE SAT game always has a pure Nash equilibrium since it can be defined as a congestion game. Then it is consistent to study its *pure* PoA.

Theorem 1. *The PoA of the* NAE SAT *game is*

(*i*) $q/(q+1)$ *if each clause has size exactly* q *with* $q \geq 3$
(*ii*) $1/2$ *otherwise*

Now we turn our attention to strong equilibria. We first show that every instance of the 2-NAE SAT game possesses a SE.

Theorem 2. *Every optimum of the* 2-NAE SAT *game is a SE.*

It follows that every optimum of the MAX CUT game is a SE since MAX CUT is equivalent to 2-NAE SAT if all literals are positive. When $q \geq 3$, the following result states that some instances of the q-NAE SAT game do not have a $(q-1)$-SE (the existence of a 1-SE, i.e. a NE, is guaranteed).

Theorem 3. *For any* $q \geq 3$*, the existence of a* $(q-1)$*-SE is not guaranteed for the* q-NAE SAT *game.*

Then it is consistent to study the *pure* SPoA of the 2-NAE SAT game.

Theorem 4. *The SPoA of the* 2-NAE SAT *game is* $2/3$.

Proof. Let $I = (X, \mathcal{C})$ be an instance of 2-NAE-SAT where X is the set of variables and $\mathcal{C}$ is the set of clauses weighted by w. Let σ (resp. σ^*) a strong equilibrium (resp. an optimal truth assignment) of I. Without loss of generality, we assume that $\sigma(x) = \texttt{true}$ for all $x \in X$. Indeed if $\sigma(x) = \texttt{false}$ then one can replace every $\overline{x}$ (resp. x) by x (resp. $\overline{x}$) and set $\sigma(x) = \texttt{true}$.

Let $A = \{x \in X : \sigma(x) = \sigma^*(x)\}$ and $B = X \setminus A$. In particular, we have $\sigma^*(x) = \texttt{true}$ for every $x \in A$ and $\sigma^*(x) = \texttt{false}$ for every $x \in B$. Note that the truth assignment where every variable of A is set to `false` and every variable of B is set to `true` is also optimal. Indeed switching all variables of a clause does not change its status, i.e. it remains NAE-satisfied or NAE-unsatisfied.

Let us suppose that $|A| = r$ and $|B| = s$. We rename the variables of A and B as follows. From now on $A = \{a_1, \ldots, a_r\}$ and $B = \{b_1, \ldots, b_s\}$. If A_j denotes $\{a_j, a_{j+1}, \ldots, a_r\}$ for $j = 1, \ldots, r$ (resp., B_j denotes $\{b_j, b_{j+1}, \ldots, b_s\}$ for $j = 1, \ldots, s$) then we suppose that the player associated with a_j (resp., b_j) does not benefit when every $a \in A_j$ (resp., every $b \in B_j$) plays `false` while the others play `true`. Notice that this renaming is well defined because σ is a strong equilibrium. Actually, when players in A_j form a coalition, then at least one player does not benefit because σ is a strong equilibrium.

We define some subsets of $\mathcal{C}$ as follows:

- A clause $c \in \mathcal{C}$ belongs to ζ_j^A (resp. ζ_j^B) iff σ NAE-satisfies c, c contains a literal defined upon a_j (resp. b_j) and c is not NAE-satisfied by the truth assignment where the variables of $\{a_j, a_{j+1}, \cdots, a_r\}$ (resp. $\{b_j, b_{j+1}, \cdots, b_s\}$) are `false` while any other variable is `true`. Let $\zeta^A = \bigcup_{j=1}^r \zeta_j^A$ and let $\zeta^B = \bigcup_{j=1}^s \zeta_j^B$.
- A clause $c \in \mathcal{C}$ belongs to χ_j^A (resp. χ_j^B) iff σ NAE-satisfies c, c contains a literal defined upon a_j (resp. b_j) and $c \notin \zeta_j^A$ (resp. $c \notin \zeta_j^B$). Let $\chi^A = \bigcup_{j=1}^r \chi_j^A$ and let $\chi^B = \bigcup_{j=1}^s \chi_j^B$.
- A clause $c \in \mathcal{C}$ belongs to α_j^A (resp. α_j^B) iff σ does not NAE-satisfy c, c contains a literal defined upon a_j (resp. b_j) and c is NAE-satisfied by the truth assignment where the variables of $\{a_j, a_{j+1}, \cdots, a_r\}$ (resp. $\{b_j, b_{j+1}, \cdots, b_s\}$) are `false` while any other variable is `true`. Let $\alpha^A = \bigcup_{j=1}^r \alpha_j^A$ and let $\alpha^B = \bigcup_{j=1}^s \alpha_j^B$.
- A clause $c \in \mathcal{C}$ belongs to β_j^A (resp. β_j^B) iff σ does not NAE-satisfy c, c contains a literal defined upon a_j (resp. b_j) and $c \notin \alpha_j^A$ (resp. $c \notin \alpha_j^B$). Let $\beta^A = \bigcup_{j=1}^r \beta_j^A$ and let $\beta^B = \bigcup_{j=1}^s \beta_j^B$.

In what follows, $w(C)$ denotes the weight of a given set of clauses C. Let us give some intermediate properties.

Property 1. $\zeta_j^A \cap \zeta_{j'}^A = \emptyset$ for all j, j' such that $1 \leq j < j' \leq r$ and $\zeta_j^B \cap \zeta_{j'}^B = \emptyset$ for all j, j' such that $1 \leq j < j' \leq s$.

Property 2. σ^* does not NAE-satisfy any clause $c \in \alpha^A \Delta \alpha^B$.

Property 3. σ^* does not NAE-satisfy any clause $c \in \beta^A \cup \beta^B$.

Property 4. σ^* does not NAE-satisfy any clause $c \in \zeta^A \cap \zeta^B$.

Using Properties (2), (3), (4) and $\mathcal{C} = \alpha^A \cup \alpha^B \cup \beta^A \cup \beta^B \cup \zeta^A \cup \zeta^B \cup \chi^A \cup \chi^B$ we can give the following bound on $\mathcal{Q}(\sigma^*)$:

$$\begin{aligned}\mathcal{Q}(\sigma^*) &\leq w(\alpha^A \cup \alpha^B) + w(\beta^A \cup \beta^B) + w(\zeta^A \cup \zeta^B) + w(\chi^A \cup \chi^B) \\ &\quad - \big(w(\alpha^A \Delta \alpha^B) + w(\beta^A \cup \beta^B) + w(\zeta^A \cap \zeta^B)\big) \\ &= w(\alpha^A \cap \alpha^B) + w(\zeta^A \Delta \zeta^B) + w(\chi^A \cup \chi^B) \end{aligned} \tag{1}$$

The value of $\mathcal{Q}(\sigma)$ is as follows:

$$\mathcal{Q}(\sigma) = w(\zeta^A \cup \zeta^B \cup \chi^A \cup \chi^B) = w(\zeta^A \cup \zeta^B) + w(\chi^A \cup \chi^B) \tag{2}$$

Take any variable $a_j \in A$. The utility of the associated player in the SE σ is $w(\zeta_j^A) + w(\chi_j^A)$. This utility becomes $w(\alpha_j^A) + w(\chi_j^A)$ if each player in the coalition $\{a_j, \cdots, a_r\}$ sets his variable to `false`. By construction a_j does not benefit. Therefore $w(\zeta_j^A) + w(\chi_j^A) \geq w(\alpha_j^A) + w(\chi_j^A)$ which is equivalent to $w(\zeta_j^A) \geq w(\alpha_j^A)$. Summing up this inequality for $j = 1$ to r and using Property 1, we obtain:

$$w(\zeta^A) = \sum_{j=1}^{r} w(\zeta_j^A) \geq \sum_{j=1}^{r} w(\alpha_j^A) \geq w(\alpha^A) \geq w(\alpha^A \cap \alpha^B) \tag{3}$$

One can conduct the same analysis and obtain:

$$w(\zeta^B) = \sum_{j=1}^{s} w(\zeta_j^B) \geq \sum_{j=1}^{s} w(\alpha_j^B) \geq w(\alpha^B) \geq w(\alpha^A \cap \alpha^B) \tag{4}$$

Using inequalities (3) and (4), we get:

$$\begin{aligned}
w(\zeta^A) + w(\zeta^B) &\geq 2w(\alpha^A \cap \alpha^B) \\
w(\zeta^A) + w(\zeta^B) + 2w(\zeta^A \Delta \zeta^B) &\geq 2w(\alpha^A \cap \alpha^B) + 2w(\zeta^A \Delta \zeta^B) \\
2w(\zeta^A \cup \zeta^B) + w(\zeta^A \Delta \zeta^B) &\geq 2w(\alpha^A \cap \alpha^B) + 2w(\zeta^A \Delta \zeta^B) \\
3w(\zeta^A \cup \zeta^B) &\geq 2w(\alpha^A \cap \alpha^B) + 2w(\zeta^A \Delta \zeta^B) \\
3w(\zeta^A \cup \zeta^B) + 2w(\chi^A \cup \chi^B) &\geq 2w(\alpha^A \cap \alpha^B) + 2w(\zeta^A \Delta \zeta^B) + 2w(\chi^A \cup \chi^B) \\
3\mathcal{Q}(\sigma) &\geq 2\mathcal{Q}(\sigma^*)
\end{aligned}$$

A tight example is composed of three clauses of weight one: $x_1 \vee x_2$, $x_3 \vee x_4$ and $x_1 \vee x_3$. If $\sigma(x_1) = \sigma(x_2) =$ `true` and $\sigma(x_2) = \sigma(x_4) =$ `false` then σ is a SE NAE-satisfying the first two clauses. Indeed the utility of x_2 and x_4 is maximum in this configuration (every clause where they appear is NAE-satisfied) so they have no incentive to deviate. So when $\sigma(x_2) = \sigma(x_4) =$ `false`, it is not difficult to see that both x_1 and x_3 have the same utility as in σ, whatever they play. If $\sigma(x_1) = \sigma(x_4) =$ `true` and $\sigma(x_2) = \sigma(x_3) =$ `false` then the three clauses are NAE-satisfied. □

It follows that the SPoA of the MAX CUT game is 2/3 (the tight example is made of positive literals so it can be represented as an instance of MAX CUT). When restricting ourselves to instances of the NAE SAT game which admit a SE, the proof of Theorem 4 can be extended to prove that the SPoA is $q/(q+1)$ if each clause has size exactly q and 2/3 otherwise (it suffices to give adjusted proofs of Properties 1 to 4).

5 The MAX $k-$CUT Game

Given a graph $G = (V, E)$ and a weight function $w : E \rightarrow \mathbb{R}_+$, MAX $k-$CUT is to partition V into k sets V_1, $V_2 \dots V_k$ such that the sum of the weight of the edges having their endpoints not in the same part of the partition is maximum. The MAX $k-$CUT game is defined as follows. Each vertex is controlled by a player with strategy set $\{1, 2, \ldots, k\}$. A player's utility is the total weight of the edges incident to her and such that her neighbor has a different strategy. The social function $\mathcal{Q}$ for a state a is $\sum_{\{[i,j]\in E: a_i \neq a_j\}} w([i,j])$.

The MAX $k-$CUT game always has a pure Nash equilibrium since it can be easily defined as a congestion game but an alternative proof is to observe that an optimal k-cut is a NE (it is known that an optimum is a NE for MAX CUT). In [7] it is shown that the PoA of the *unweighted* MAX $k-$CUT game is $\frac{k-1}{k}$ and one can easily extend the result to the weighted case.

Now we investigate the existence of a SE for the MAX k-CUT game. The MAX CUT game ($k = 2$) always admits a SE since an optimal cut must be a SE. It is a corollary of Theorem 2. When $k \geq 3$, our positive result is that an optimal cut of the MAX k-CUT game is a 3-SE (proof by contradiction).

Theorem 5. *Every optimum of the* MAX k-CUT *game is a* 3*-SE.*

The following result states that we can not go beyond $r = 3$ to prove that *any* optimal cut is an r-SE.

Proposition 1. *An optimum of the* MAX 3-CUT *game is not necessarily a* 4*-SE.*

Hence an optimum of the MAX $k-$CUT game is not necessarilly a SE but the existence of a SE is not compromised because the instance we found to state Proposition 1 admits two optima, one is not a 4-SE whereas the other is a SE.

To conclude this section, one can be interested in bounding the SPoA of the MAX CUT game if only coalitions of limited size are conceivable, i.e. the q-SPoA. The following result shows that, even if q is large, the q-SPoA drops to 1/2.

Theorem 6. *For any* $\varepsilon > 0$ *and* $q = O(|V|^{1/2-\varepsilon})$ *where* $|V|$ *is the number of nodes, the* $q-$*SPoA of the* MAX CUT *game is* $1/2$.

6 Concluding Remarks

We investigated two games which generalize MAX CUT and the focus was on strong equilibria, their existence and how far they are from socially optimal configurations. Some questions are left open.

For the q-NAE SAT game where $q \geq 3$, we presented an instance without any $(q-1)$-SE but can we guarantee that there is an r-SE for some $1 < r < q-1$? For example, is there a 2-SE when $q \geq 4$? Another interesting direction would be to characterize instances which possess a SE.

For the MAX $k-$ CUT game, we showed that a 3-SE exists but can we go further? Though any optimum is not guaranteed to be a 4-SE, it is possible that only some optima are 4-SE. Actually we conjecture that the MAX $k-$ CUT game always possesses a SE. If it is true then what would the SPoA?

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