2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

On Synchronization and Collision Avoidance for Mechanical Systems

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Abstract— **The current interest in coordinating nonlinear dynamical systems with safety guarantees is driven by emerging practical applications. One of the primary objectives in coordination of multiple mechanical systems (agents) is velocity synchronization with guarantees for collision avoidance among the agents. However, previous results in this direction assume point models for the agent, allow only linear dynamics and neglect delays in communication between the agents. In this paper we demonstrate that velocity synchronization and collision avoidance are simultaneously achievable in non-point, nonlinear mechanical systems in the presence of communication delays and switching interconnection topologies. A numerical example is presented to justify the proposed results.**

I. Introduction

The problem of synchronization and control of mechanical systems is important in numerous practical applications, such as unmanned air vehicles and robot networks. Inspired by the formulation in [29], [22], recently there has been considerable research devoted to the analysis and coordination control of such systems. We refer the readers to [21], [20] for exhaustive surveys on these research efforts. Group coordination, formation stability and collision avoidance problems have been recently addressed in [1], [5], [11], [7], [16], [15], [28], [19], [24], [27] among others. The recent survey [17] presents a nice overview of the research in coordination of multi-vehicle systems.

In this paper we address the problem of collision avoidance and velocity synchronization problem for nonlinear, non-point mechanical systems. The problem of collision and obstacle avoidance has been studied extensively [13], [23], [9], [6], [26]. In the multiagent coordination literature, artificial potential based algorithms for collision avoidance have been exploited by several authors [18], [27], [19] to demonstrate emergent behavior in swarm models. Typically, the unique minima of the potential function is the desired formation and the potential function goes unbounded as the relative distance between any pair of agents approaches zero. The motion of the agents is then controlled using a gradient-descent strategy that drives them to the desired formation. However, the aforementioned results have been developed for point models with simple dynamics, and hence are not directly applicable in realistic applications where nonlinear agent dynamics, non-point models and unreliable communications lead to additional challenges that need to be addressed.

In this paper we use the collision avoidance functions developed in [26] and the output synchronization results [3], [4], [2] to address these issues. These functions are used to address the non-point nature of the agent and guarantee the existence of non-penetrable regions around every agent. Furthermore, exploiting the Lagrangian dynamics of mechanical systems and output synchronization results in [4], we demonstrate that collision avoidance and velocity synchronization can be guaranteed even if there are delays in communicating velocity information between the nonlinear agents or for switching interconnection topologies. Using the notation of [27], we differentiate between (relative position) sensing and (velocity) communication networks. In the subsequent results we require the sensing graph to be undirected and the communication graph to be balanced, in comparison to [27] where the communication graph was assumed to be undirected.

The present paper is organized as follows. In section II we provide some background on nonlinear mechanical systems, functions for collision avoidance and graph theory. This is followed by the main results in Section III where velocity synchronization and collision avoidance is demonstrated simultaneously for nonlinear mechanical systems with possibly switching interconnection topologies and time delays in the network. In Section IV, a numerical example is presented to validate the proposed results and finally the results are summarized in Section V.

II. Background

To set the background and notation for what follows, consider a control affine nonlinear system of the form

$$\Sigma \begin{cases} \dot{x} &= f(x,u) \\ y &= h(x) \end{cases} \tag{1}$$

where $x \in R^n$, $u \in R^m$, and $y \in R^m$. The functions $f(\cdot) \in R^n$, and $h(\cdot) \in R^m$ are assumed to be sufficiently smooth. The admissible inputs are taken to be piecewise continuous and locally square integrable and we note that the dimensions of the input and output are the same. We assume, for simplicity, that $f(0,0) = 0$ and $h(0) = 0$.

A. Lagrangian Systems

Following [25], (in the absence of friction and gravitational forces) the Euler-Lagrange equations of motion for N mechanical systems are given as

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i = u_i \quad i = 1,\ldots,N \tag{2}$$

This research was supported by Boeing Company via the Information Trust Institute. The first author also acknowledges the support of Mechanical Engineering Department, Institute for Systems Research and the Clark School of Engineering via the Faculty Startup Grant.

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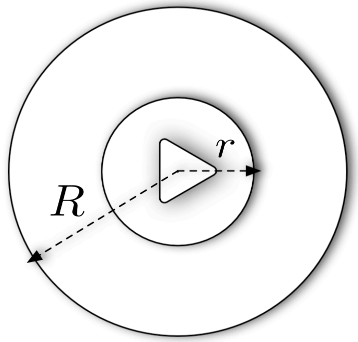

Fig. 1. The circular regions with radius r and R denote the avoidance and sensing regions respectively.

where $q \in R^n$ is the vector of generalized configuration coordinates, $u \in R^n$ is the vector of generalized forces acting on the system, $M(q) \in R^{n\times n}$ is a symmetric, positive definite inertia matrix, $C(q,\dot{q})\dot{q} \in R^n$ is the vector of centrifugal and Coriolis forces. Although the above equations of motion are coupled and nonlinear, they exhibit certain fundamental properties due to their Lagrangian dynamic structure.

- **Property 1:** The matrix $M(q)$ is symmetric positive definite and there exists a positive constant m such that $mI \leq M(q)$.
- **Property 2:** Under an appropriate definition of the matrix C, the matrix $\dot{M}$ - $2C$ is skew symmetric

For each pair of agents, define the following avoidance function [26]

$$V_{ij}(q_i,q_j) = \left(\min\left\{0, \frac{||q_i-q_j||^2 - R^2}{||q_i-q_j||^2 - r^2}\right\}\right)^2, i \neq j \quad (3)$$

where $R > r > 0$, R denotes the sensing radius of each agent in which it can detect the position of other agents and r denotes the avoidance region which is the smallest safe distance between the agents (see Figure 1).

The gradient decent based strategy to avoid collision among the agents is then given as

$$\nabla_{q_i} V_{ij} = \begin{cases} 0, & ||q_i - q_j|| \geq R \\ 4\frac{(R^2-r^2)(||q_i-q_j||^2-R^2)}{(||q_i-q_j||^2-r^2)^3}(q_i - q_j)^T & R > ||q_i - q_j|| > r \\ \text{not defined}, & ||q_i - q_j|| = r \\ 0 & ||q_i - q_j|| < r \end{cases} \quad (4)$$

Let $q = [q_1 \dots q_N]^T$, $\dot{q} = [\dot{q}_1 \dots \dot{q}_N]^T$ and $x = [q \quad \dot{q}]^T$ represent the position, velocity and state vectors respectively for the multiagent system (2). Following [26], let the avoidance sets for each pair of agents be given as

$$\Omega_{ij} = \{q : q \in R^{nN}, ||q_i - q_j|| \leq r\} \quad (5)$$

and define the sensing or detection region by defining the pairwise sensing regions as

$$\mathcal{D}_{ij} = \{q : q \in R^{nN}, ||q_i - q_j|| \leq R\} \quad (6)$$

Therefore, the overall avoidance and detection regions are given as as $\Omega = \cup_{i,j}\Omega_{ij}$ and $\mathcal{D} = \cup_{i,j}\mathcal{D}_{ij} \;\; \forall i,j$ respectively.

The goal then is to guarantee that the trajectory of the dynamical system avoids the set Ω. This condition can be formalized as follows [14].

Definition The dynamical system (2) avoids Ω if and only if for each solution $x(t,x_0), t \in \mathbf{T} = [0 \; +\infty), x_0 \notin \Omega$ implies $x(t) \notin \Omega, \forall t \in \mathbf{T}$.

Information exchange between agents can be represented as a graph. We give here some basic terminology and definitions from graph theory [8] sufficient to follow the subsequent development.

Definition By a **graph** $\mathcal{G}$ we mean a finite set $\mathcal{V}(\mathcal{G}) = \{v_i,\dots,v_N\}$, whose elements are called **nodes** or **vertices**, together with set $\mathcal{E}(\mathcal{G}) \subset \mathcal{V} \times \mathcal{V}$, whose elements are called **edges**. An edge is therefore an ordered pair of distinct vertices.

If, for all $(v_i,v_j) \in \mathcal{E}(\mathcal{G})$, the edge $(v_j,v_i) \in \mathcal{E}(\mathcal{G})$ then the graph is said to be **undirected**. Otherwise, it is called a **directed graph**.

An edge (v_i,v_j) is said to be **incoming with respect to** v_j and **outgoing with respect to** v_i and can be represented as an arrow with vertex v_i as its tail and vertex v_j as its head.

The **in-degree** of a vertex $v \in \mathcal{G}$ is the number of edges that have this vertex as a head. Similarly, the **out-degree** of a vertex $v \in \mathcal{G}$ is the number of edges that have this vertex as the tail.

If the in-degree equals the out-degree for all vertices $v \in \mathcal{V}(\mathcal{G})$, then the graph is said to be **balanced**.

A **path** of length r in a directed graph is a sequence $v_0,\dots,v_r$ of $r+1$ distinct vertices such that for every $i \in \{0,\dots,r-1\}$, (v_i,v_{i+1}) is an edge.

A **weak path** is a sequence $v_0,\dots,v_r$ of $r+1$ distinct vertices such that for each $i \in \{0,\dots,r-1\}$ either (v_i,v_{i+1}) or (v_{i+1},v_i) is an edge.

A directed graph is **strongly connected** if any two vertices can be joined by a path and is **weakly connected** if any two vertices can be joined by a weak path.

III. Velocity Synchronization with Collision Avoidance

We assume that each agent is equipped with a sensing and a communication device. The sensing device estimates the relative position between an agent and other agents in its sensing radius. On the other hand, the communication device is used to communicate velocity information between the agents. We denote the information graph underlying the sensing and the communication processes as the **sensing** and **communication graphs** respectively. It is to be noted that an agent may have different sets of neighbors in its sensing and communication graph.

The position dependent sensing graph, $\mathcal{G}_s = \{\mathcal{V},\mathcal{E}_s\}$ is assumed to be undirected. On the other hand, the directed communication graph $\mathcal{G}_v = \{\mathcal{V},\mathcal{E}_v\}$ for the velocity information exchange is assumed to be balanced. Furthermore noting (3), the sensing graph $\mathcal{G}_s$ implicitly assumes all-to-all communication in the sense that a zero value is assigned to the function (3) beyond the sensing radius R. Therefore, in the subsequent analysis, the sensing graph is a (pseudo) complete graph.

The agents are said to velocity synchronize if

$$\lim_{t\to\infty} ||\dot{q}_i(t) - \dot{q}_j(t)|| = 0 \ \ \forall i, j$$

Let the control for each agent be given as

$$u_i(x) = K \sum_{j\in\mathcal{N}_i(\mathcal{G}_v)} (\dot{q}_j - \dot{q}_i) - \sum_{j=1}^{N} \nabla_{q_i} V_{ij} \tag{7}$$

for some $K > 0$. Therefore, using (2) the closed loop system for the i^{th} agent is given as

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i = K \sum_{j\in\mathcal{N}_i(\mathcal{G}_v)} (\dot{q}_j - \dot{q}_i) - \sum_{j=1}^{N} \nabla_{q_i} V_{ij} \tag{8}$$

Our first result demonstrates that the coupling control (7) synchronizes the agents' velocities while simultaneously ensuring collision avoidance with other agents.

Theorem 3.1: Consider the dynamical system (8) together with (3) and (4). If the communication graph is balanced, time-invariant and strongly connected, the sensing graph is undirected, and $x_0 \notin \Omega$, then the set Ω is avoidable by (2) and the agents velocity synchronize.

Proof: Define a positive semi-definite function for the multi-agent system as

$$V(x) = \sum_{i=1}^{N} \frac{1}{2}\dot{q}_i^T M_i(q_i)\dot{q}_i + \sum_{i=1}^{N}\sum_{j>i} V_{ij}(q_i, q_j)$$

The derivative along trajectories of the system is given as

$$\dot{V}(x) = \sum_{i=1}^{N} (\dot{q}_i^T(-C_i(q_i, \dot{q}_i)\dot{q}_i + K\sum_{j\in\mathcal{N}_i(\mathcal{G}_v)} (\dot{q}_j - \dot{q}_i) - \sum_{j=1}^{N} \nabla_{q_i} V_{ij})$$
$$+ \frac{1}{2}\dot{q}_i^T \dot{M}_i(q_i)\dot{q}_i) + \sum_{i=1}^{N}\sum_{j>i} \{(\nabla_{q_i} V_{ij})^T \dot{q}_i + (\nabla_{q_j} V_{ij})^T \dot{q}_j\} \tag{9}$$

As the sensing graph is undirected and $\nabla_{q_i} V_{ij} = -\nabla_{q_j} V_{ij}$,

$$-\sum_{i=1}^{N}\sum_{j=1}^{N} \dot{q}_i^T \nabla_{q_i} V_{ij} + \sum_{i=1}^{N}\sum_{j>i} \{(\nabla_{q_i} V_{ij})^T \dot{q}_i + (\nabla_{q_j} V_{ij})^T \dot{q}_j\}$$
$$= 0 \tag{10}$$

Furthermore, the balanced nature of the communication graph implies that,

$$K \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i(\mathcal{G}_v)} \dot{q}_i^T \dot{q}_i \ = \ K \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i(\mathcal{G}_v)} \dot{q}_j^T \dot{q}_j \tag{11}$$

Thus, using (10), (11) and Property 3 in (9) yields

$$\dot{V}(x) = -\frac{K}{2} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i(\mathcal{G}_v)} ||\dot{q}_j - \dot{q}_i||^2 \le 0$$

Consequently, $V(x) \le V(x_0) \ \ \forall t$ and noting that

$$\lim_{||q_i - q_j||\to r^+} V_{ij}(q_i, q_j) = \infty \ \ \forall, i, j$$

we conclude that the trajectory $x(t)$ will never enter Ω and hence collisions are avoided.

As $V(x(t)) \ge 0$, $\dot{V}(x(t)) \le 0$, $\lim_{t\to\infty} V(x(t))$ exists and is finite, the agent velocities and the control input for every agent is ultimately bounded. Using this fact in the system dynamics (8), we conclude that $\ddot{q}_i(t)$ is also bounded. Thus, $\ddot{V}(x(t))$ is bounded and the function $\dot{V}(x(t))$ is uniformly continuous. Using Barbalat's Lemma [12], $\lim_{t\to\infty} \dot{V}(x(t)) = 0$ and hence the agents' velocities synchronize asymptotically to that of their neighbors. Strong connectivity of the communication graph then implies velocity synchronization for all agents. ∎

We next consider the case when the graph topology is not constant, such as in nearest neighbor scenarios. In this case the communication graph $\mathcal{G}_v(t)$ is not decided a priori, but is time varying. Consequently, we have a switched system with the continuous state $x(t)$ and the discrete state $\mathcal{G}_v(t) \in \mathcal{G}_N$ where $\mathcal{G}_N$ is the finite collection of possible directed graphs among the N agents.

The switching signal $\sigma : [0, \infty) \to \mathcal{P}$ is the right continuous switching signal and $\mathcal{P} = \{1, 2, \ldots, r\}$; $r \in \mathbb{N}$ is the finite index set associated with the elements of $\mathcal{G}_N = \{\mathcal{G}^1, \ldots, \mathcal{G}^r\}$. We assume that the switching signal is piecewise continuous and denote by $t_w, w = 1, 2, \ldots$ the consecutive discontinuities of the switching signal $\sigma(t)$.

Dwell Time Assumption We impose the restriction that there exists $d > 0$ such that for every $T_d > 0$ we can find a positive integer w for which $t_{w+1} - d \ge t_w \ge T_d$.

Let the control for each agent be given as

$$u_{\sigma_i}(x, t) = K \sum_{j\in\mathcal{N}_i(\mathcal{G}_v(t))} (\dot{q}_j - \dot{q}_i) - \sum_{j=1}^{N} \nabla_{q_i} V_{ij} \tag{12}$$

Using (2), the closed loop system for the i^{th} agent transforms to

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i = K \sum_{j\in\mathcal{N}_i(\mathcal{G}_v(t))} (\dot{q}_j - \dot{q}_i) - \sum_{j=1}^{N} \nabla_{q_i} V_{ij} \tag{13}$$

The next result demonstrates that even when the communication graph is time-varying, the agents velocities synchronize while simultaneously avoiding collision with other agents.

Theorem 3.2: Consider the dynamical system (13) together with (3), (4) and the dwell time assumption. If the communication graph is balanced and strongly connected, the sensing graph is undirected, and $x_0 \notin \Omega$, then the set Ω is avoidable by (13) and the agents velocity synchronize.

Proof: Consider a common storage function for the multi-agent system as

$$V(x) = \sum_{i=1}^{N} \frac{1}{2}\dot{q}_i^T M_i(q_i)\dot{q}_i + \sum_{i=1}^{N}\sum_{j>i} V_{ij}(q_i, q_j)$$

Even though the communication graph is switching, the function $V(x)$ is independent of the interconnection (communication and sensing) graphs and hence is continuously differentiable. Following the proof of Theorem 3.1, the derivative of $V(x)$ along trajectories of (13) is given as

$$\dot{V}(x) = -\frac{K}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(\mathcal{G}_v(t))} ||\dot{q}_j - \dot{q}_i||^2 \leq 0 \tag{14}$$

Thus, as was the case in Theorem 3.1, $V(x) \leq V(x_0) \quad \forall t$ and noting that

$$\lim_{||q_i - q_j|| \to r^+} V_{ij}(q_i, q_j) = \infty \quad \forall, i, j$$

we conclude that the trajectory $x(t)$ will never enter Ω, and hence collisions are avoided.

Due to the switching communication graph and the dwell time assumption, the function $\dot{V}(x)$ is piecewise continuous. Therefore Barbalat's Lemma cannot be directly used to establish velocity synchronization of the switched system (13). The results in [10] provide a Barbalat-like lemma for addressing switched systems and we adapt their result for demonstrating velocity synchronization (this approach was also suggested by [27] as an alternative to prove a weaker version of their results).

As $V(x) \leq V(x_0) \quad \forall t,\ \dot{q}_i(t), \ddot{q}_i(t)$ are bounded. The Barbalat-like result (see the proof of asymptotic convergence in Theorem 7 of [10]) is now used to complete the proof. Using the dwell time assumption and finiteness of the index set $\mathcal{P}$, there exists an infinite subsequence of switching times $t_{w_1}, t_{w_2}, \ldots$ such that the time intervals $t_{w_{k^*+1}} - t_{w_{k^*}} \geq d, \quad k^* = 1, 2, \ldots$ and $\sigma(t) = h$ on these time intervals.

Denote the union of these time intervals by $\mathcal{H}$ and construct the auxiliary function

$$y_{\mathcal{H}}(t) = \begin{cases} -\dot{V}(x(t)), & \text{if } t \in \mathcal{H} \\ 0 & \text{otherwise} \end{cases}$$

Using (14), $\forall t \geq 0$

$$\begin{aligned} \int_0^t y_{\mathcal{H}}(s)ds &\leq V(x(t))_{(t=t_{w_1})} - V(x(t))_{(t=t)} \\ &\leq V(x(t))_{(t=t_{w_1})} \end{aligned} \tag{15}$$

As $y_{\mathcal{H}}(t)$ is positive semi-definite, using (15) and letting $t \to \infty$, $y_{\mathcal{H}}(t) \in \mathcal{L}_1$. To show that $\lim_{t\to\infty} \dot{V}(x(t)) = 0$ we need to prove that $\lim_{t\to\infty} y_{\mathcal{H}}(t) = 0$. Let us suppose that this is not true. Then $\exists \epsilon > 0$ and an infinite sequence of times $s_k, \quad k = 1, 2, \ldots \in \mathcal{H}$ such that $y_{\mathcal{H}}(s_k) \geq \epsilon \quad \forall k$. As $\ddot{q}_1(t), \ldots, \ddot{q}_N(t)$ are bounded, $y_{\mathcal{H}}$ is uniformly continuous on $\mathcal{H}$. Consequently, $\exists \delta > 0$ such that s_k belongs to time interval of length δ on which $y_{\mathcal{H}}(t) \geq \frac{\epsilon}{2}$. This results in a contradiction as $y_{\mathcal{H}}(t) \in \mathcal{L}_1$. Thus, $\lim_{t\to\infty} y_{\mathcal{H}}(t) = 0$, and hence $\lim_{t\to\infty} \dot{V}(x(t)) = 0$. As the communication graph is strongly connected at all times, the agents output synchronize asymptotically. ■

We next address the practically important case of unknown delays in velocity communication between the agents. The delays are assumed to be constant and bounded. As there can be multiple paths between two agents, T_{ij}^k denotes the delay along the k^{th} path from the i^{th} agent to the j^{th} agent, and we henceforth denote it as the *path delay*. We only impose the restriction that delays along all paths of length one are unique, i.e. the one-hop transmission delay from one agent to the other is uniquely defined.

Definition In the presence of delays, the agents are said to delay-velocity synchronize if

$$\lim_{t\to\infty} ||\dot{q}_i(t - T_{ij}^k) - \dot{q}_j(t)|| = 0 \quad \forall i, j \quad \forall k \tag{16}$$

The delays in the velocity communication process may be due to the multi-hop wireless communication between the agents, while the sensing process, for example using infrared or sonar sensors, is assumed to be immediate and hence there are no sensing delays.

Let the control for each agent be given as

$$u_i(x) = K\sum_{j\in\mathcal{N}_i(\mathcal{G}_v)} (\dot{q}_j(t - T_{ji}) - \dot{q}_i) - \sum_{j=1}^{N} \nabla_{q_i} V_{ij} \tag{17}$$

The closed loop system for the i^{th} agent can then be written as

$$\begin{aligned} M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i &= K\sum_{j\in\mathcal{N}_i(\mathcal{G}_v)} \Big(\dot{q}_j(t - T_{ji}) - \dot{q}_i\Big) \\ &- \sum_{j=1}^{N} \nabla_{q_i} V_{ij} \end{aligned} \tag{18}$$

Our final result follows

Theorem 3.3: Consider the dynamical system (18) together with (3) and (4). If the communication graph is balanced, time-invariant and strongly connected, the sensing graph is undirected, and $x_0 \notin \Omega$, then the set Ω is avoidable by (2) and the agents delay-velocity synchronize in the sense of (16).

Proof: Define a positive semi-definite function for the multi-agent system as

$$V(x) = \sum_{i=1}^{N} \frac{1}{2}\dot{q}_i^T M_i(q_i)\dot{q}_i + \frac{K}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(\mathcal{G}_v)} \int_{t-T_{ji}}^{t} \dot{q}_j^T(s)\dot{q}_j(s)ds + \sum_{i=1}^{N}\sum_{j>i} V_{ij}(q_i, q_j)$$

The derivative along the solution of (18) is given as

$$\begin{aligned} \dot{V}(x) &= \sum_{i=1}^{N} \dot{q}_i^T(-C_i(q_i, \dot{q}_i)\dot{q}_i + K\sum_{j\in\mathcal{N}_i(\mathcal{G}_v)} (\dot{q}_j(t - T_{ji}) - \dot{q}_i) \\ &- \sum_{j=1}^{N} \nabla_{q_i} V_{ij}) + \frac{1}{2}\dot{q}_i^T \dot{M}_i(q_i)\dot{q}_i + \frac{K}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(\mathcal{G}_v)} (\dot{q}_j^T\dot{q}_j \\ &- \dot{q}_j(t - T_{ji})^T\dot{q}_j(t - T_{ji})) + \sum_{i=1}^{N}\sum_{j>i}((\nabla_{q_i} V_{ij})^T\dot{q}_i \\ &+ (\nabla_{q_j} V_{ij})^T\dot{q}_j) \end{aligned} \tag{19}$$

Again noting that the sensing graph is undirected, and $\nabla_{q_i} V_{ij} = -\nabla_{q_j} V_{ij}$, we have

$$-\sum_{i=1}^{N}\sum_{j=1}^{N} \dot{q}_i^T \nabla_{q_i} V_{ij} + \sum_{i=1}^{N}\sum_{j>i} \{(\nabla_{q_i} V_{ij})^T\dot{q}_i + (\nabla_{q_j} V_{ij})^T\dot{q}_j\} = 0$$

As the communication graph is balanced

$$K\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(\mathcal{G}_v)}\dot{q}_j^T\dot{q}_j \quad = \quad K\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(\mathcal{G}_v)}\dot{q}_i^T\dot{q}_i$$

Therefore using these facts along with Property 3 in (19) yields

$$\dot{V}(x) = -\frac{K}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}||\dot{q}_j(t-T_{ji})-\dot{q}_i)||^2 \leq 0$$

Consequently, $V(x) \leq V(x_0)\ \ \forall t$ and noting that

$$\lim_{||q_i-q_j||\to r^+} V_{ij}(q_i,q_j) = \infty \quad \forall, i, j$$

we conclude that the trajectory $x(t)$ will never enter Ω. Following the proof of Theorem 3.1 it is evident that $\lim_{t\to\infty}\dot{V}(x(t)) = 0$ and hence the agents delay-velocity synchronize in the sense of (16). ∎

IV. Numerical Example

Consider a system constituted by four agents whose equations are given as

$$m_i\ddot{q}_i \quad = \quad \tau_i \quad \forall i$$

where $q_i \in R^2$. Let the systems be coupled together using the control (7) and hence the closed loop system is given as

$$m_i\ddot{q}_i \quad = \quad K\sum_{j\in\mathcal{N}_i(\mathcal{G}_v)}(\dot{q}_j-\dot{q}_i) - \sum_{j=1}^{4}\nabla_{q_i}V_{ij} \quad \forall i$$

As the communication graph is balanced and the sensing graph undirected, $\sum_{i=1}^{4} m_i\ddot{q}_i = 0$ and therefore the momentum of the system $p_{sys} = \sum_{i=1}^{4} m_i\dot{q}_i$ is conserved.

Let the communication graph among the agents be represented by a ring topology as shown in Figure 2. A numerical

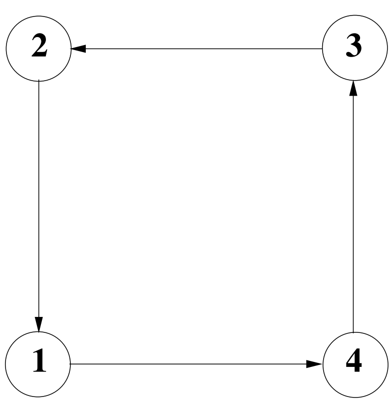

Fig. 2. The agents communicate velocities in a ring topology

simulation was done assuming $m_i = 1, K = 1\ \forall i$. The radii of the avoidance and sensing regions were chosen to be $r = 20$ and $R = 40$ units respectively. Thus, the closed system is represented as

$$\ddot{q}_1 = (\dot{q}_2-\dot{q}_1) - \sum_{j=1}^{4}\nabla_{q_1}V_{1j} \quad ; \quad \ddot{q}_2 = (\dot{q}_3-\dot{q}_2) - \sum_{j=1}^{4}\nabla_{q_2}V_{2j}$$

$$\ddot{q}_3 = (\dot{q}_4-\dot{q}_3) - \sum_{j=1}^{4}\nabla_{q_3}V_{3j} \quad ; \quad \ddot{q}_4 = (\dot{q}_1-\dot{q}_4) - \sum_{j=1}^{4}\nabla_{q_4}V_{4j}$$

As seen in Figure 3, the agents velocity synchronize.

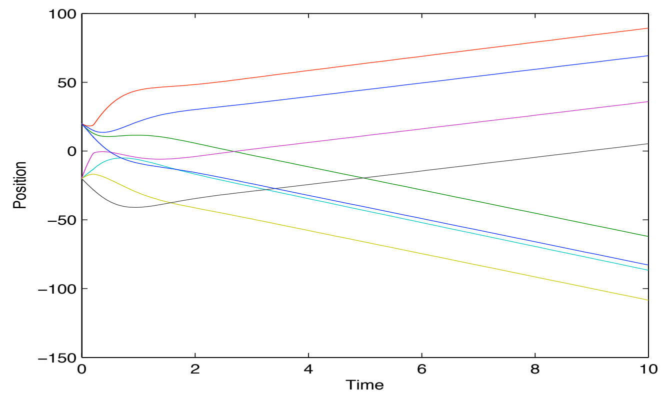

Fig. 3. The planar position components become parallel asymptotically and hence the velocities synchronize.

The figures 3, 4, 5, and 6 demonstrate that the interagent distances are a function of both the collision avoidance and the synchronizing control. Furthermore, the interagent distances do not enter the avoidance region (5) as confirmed in Figures 4, 5, and 6 and hence collisions are avoided between the agents.

V. Conclusions

In this paper control laws were developed to achieve velocity synchronization and collision avoidance simultaneously for nonlinear mechanical systems with a non-point model. Under the assumption of balanced communication graphs

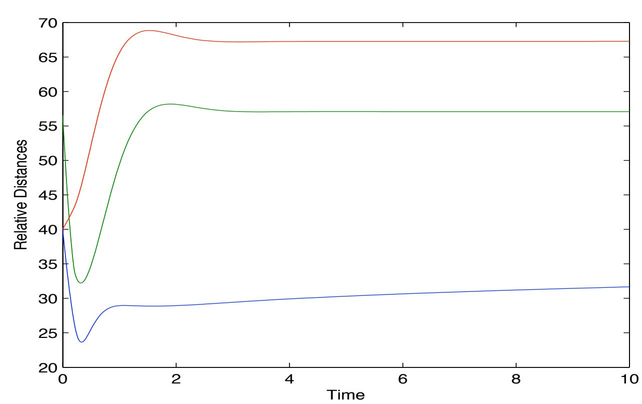

Fig. 4. Relative distances between agent 1 and the other agents do not enter the avoidance region.

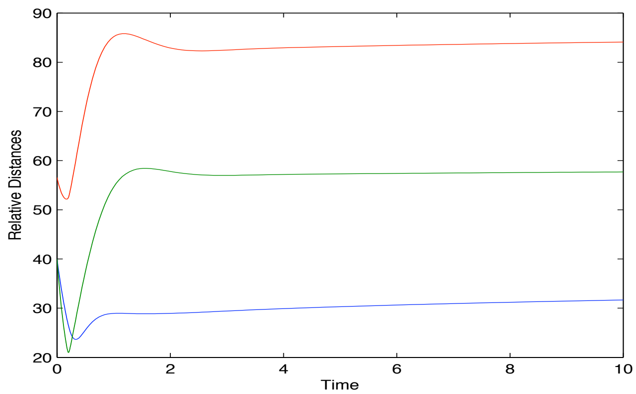

Fig. 5. Relative distances between agent 2 and the other agents do not enter the avoidance region.

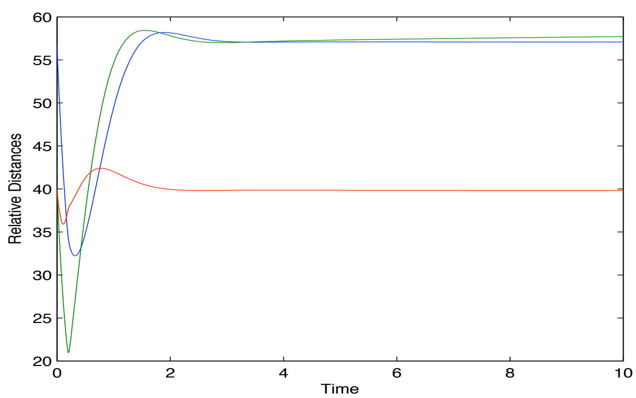

Fig. 6. Relative distances between agent 3 and the other agents do not enter the avoidance region.

and undirected sensing graphs, using avoidance functions developed in [26] and the output synchronization results in [4], velocity synchronization and collision avoidance was demonstrated under time delays and switching interconnection topologies. Future work entails extension to nonholonomic mechanical systems and experimental verification of the proposed results.

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