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ON THE ACCURACY, REPEATABILITY, AND DEGREE OF INFLUENCE OF KINEMATICS PARAMETERS FOR INDUSTRIAL ROBOTS

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Abstract

In this article, we discuss industrial robot characteristics of accuracy and repeatability. The factors that affect these characteristics are identified, and an error tree is developed. Subsequently, the accuracy and repeatability are investigated utilizing the Denavit-Hartenberg kinematics parameters, the homogeneous transformation matrix, and the differential transformation matrix theory, and corresponding measures are developed. The formulation indicates that the influence matrices associated with joint variables are constant. A new measure called degree of influence is established that qualitatively assesses the relative contribution of each kinematic parameter variation to the accuracy and repeatability of rigid manipulators. The developed formulation provides for easy evaluation of the degree of influence measures for rigid manipulators in either numerical or symbolic form. A numerical example is included in which the degree of influence of the kinematics parameters for an articulated manipulator, PUMA 560, are evaluated and analysed.

Key Words

Robotic error, accuracy, repeatability, degree of influence

1. Introduction

Robotic applications have been expanding since the introduction of robots. These applications still include the traditional manufacturing, but in recent years applications in other fields, such as the medical community, have been increasing. These new applications, such as medical surgery, require robots that are both accurate and repeatable. Also, manufacturing applications requirements are changing in an effort to address quality control issues, thus pushing the envelope of robot capabilities. Therefore, better robots are required. But what makes a better robot? This is a very ambiguous question that depends on the application. At the same time, can we utilize new technology that will allow one to improve the performance characteristics of existing robots as it relates to the positioning accuracy and repeatability?

A technological barrier in the robotics industry has been the reduction or "elimination" of the error between the tool frame and the goal frame, as shown in Fig. 1. The sources of this error were readily identified as being due to manipulator modelling differences and to hardware fixturing. The major contribution to the error between the robot base and the tool frame is attributed to modelling differences between the controller and the robot. Inaccurate fixturing and manufacturing processes account for the differences between the base frame and the goal frame.

The definition of the tool and goal frames is depicted in Fig. 1 [1]. Solutions such as building a better robot, building more rigid and repeatable fixtures, and improving manufacturing processes that could help in improving this problem are often not feasible due to the required or unavailable resources. Compensation for this error through an in-process feedback mechanism is a much more attractive alternative. As in any control system, the process parameters will define the level of required sophistication. If the requirement is to improve upon the resolution or absolute accuracy of the robot, then a precise metrology system is needed to perform these measurements.

The identified parameters relating to robot positional performance are accuracy, repeatability, and resolution.

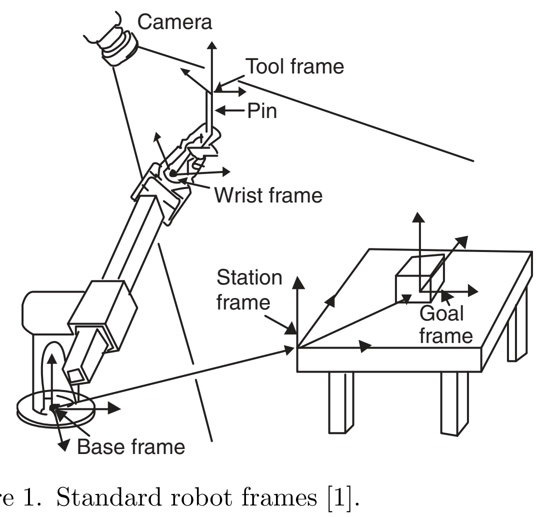

Figure 1. Standard robot frames [1].

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Each of these depends on the various components used in constructing the robot (links, motors, encoders, etc.), the construction procedure, and the capabilities of the driving actuators and the controller. The resolution is defined through the control system used to power the manipulator, but is also affected by the construction procedure, manipulator stiffness, (structural flexibility), encoders, and so on.

Resolution is defined as the smallest incremental move that the robot can physically produce. Repeatability is a measure of the ability of the robot to move back to the same position and orientation over and over again. Accuracy is defined as the ability of the robot to precisely move to a desired position in 3-D space. These concepts are shown graphically in Fig. 2.

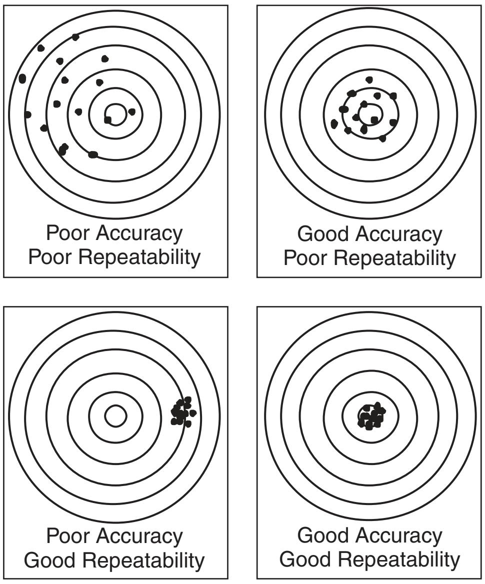

Figure 2. Accuracy versus repeatability.

Absolute accuracy and repeatability. Absolute accuracy and repeatability describe the ability of a robot to move to a desired location without any deviation. Dynamic accuracy and repeatability describe the ability of a robot to follow a desired trajectory with little or no variance. Additionally, in all robotic applications zero overshoot is a necessity to avoid disastrous collisions with other parts in the work-cell. Therefore, controller design introduces another dimension to the problem, as we would like to maximize the stiffness and bandwidth while minimizing the response time. Ideally, both the absolute and dynamic accuracy and repeatability could be minimized to the attainable resolution.

The largest effect on the robot accuracy is attributed to the robot links. Manufacturing or machining of these links inevitably introduces some variation in their dimensions from one robot to the next, as well as some variation in the orientation of the joints. The manufacturing variations are attributed to the defined machining tolerances. The differences between the physical joint zero position reported by the robot controller and the actual physical joint zero position usually has the second biggest effect on the accuracy of the robot. For a standard PUMA-type articulated six-dof robot, errors in the zero position of the waist (1), shoulder (2), and elbow (3) joints will have a larger effect on the robot positional error than that of the wrist joints (pitch 4, roll 5, and yaw 6). Although joints 1, 2, and 3 contribute primarily to the position of the tool centre point frame, the main contribution for joints 4, 5, and 6 is to the orientation of this frame. The joint zero position error is often responsible for the robot positional error on the order of 90% as identified in the analysis to be presented in this study.

A mathematical model within each robot controller assumes that the links on one robot are the same length as the links on another robot of the same model and type. Additionally, the same model also assumes that the relative orientations of the joints on one robot are the same as on another robot of the same type. Unfortunately, this assumption is not true due to manufacturing and assembly variations. Therefore, the controller will incorrectly estimate the robot endpoint given a set of joint angles. The next most significant factor in the robot positional error is joint compliance. This may be thought of as a factor representing the elasticity of each joint caused by the effects of gravity, payload, and inertia.

Each of the following robot characteristics—accuracy, repeatability, and resolution—depends upon many factors that include, but are not limited to, friction, temperature, loading, and manufacturing tolerances. Of the three robot characteristics, high accuracy is the most difficult to accomplish.

Differences between the modelled, as-designed components and the actual, as-built components will affect the accuracy. The controller software can be designed to account for discrepancies between the mathematical controller model and the actual built part. However, this approach is not cost-effective for mass-produced robots because considerable effort must be expended to identify the characteristics of the various robot components within the desired accuracy using complex, expensive, and time-consuming techniques.

The major error contributions can easily be subdivided into structural, kinematic, and dynamic. A theoretical error tree illustrating factors that contribute to the positional accuracy and repeatability of a robot is presented in Fig. 3.

This work examines the concepts of accuracy and repeatability. A literature survey on accuracy and repeatability is presented. The accuracy and repeatability principles are framed within the context of the homogeneous and differential transformation. Subsequently, an error analysis and the degree of influence of the DH-parameters on the accuracy and repeatability are evaluated and discussed within the framework of the developed methodology. A numerical example using published values for the PUMA 560 robot focused on the aforementioned concepts is presented and discussed.

2. Literature Survey

There are many available solutions on the market that could be implemented to improve the ability of a robot to

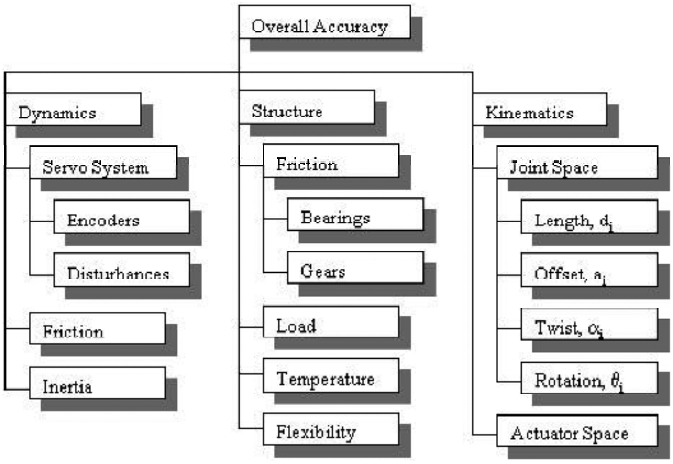

Figure 3. Positional accuracy and repeatability error tree.

accurately and repeatedly position itself. These solutions range from calibration techniques that require robot remastering to software compensation techniques, as well as many combinations of them, or to other process-based custom methods. For the majority of industrial processes, the elimination of all error is not a realistic goal [2–10]. However, improving the accuracy and repeatability of existing hardware to meet desired process parameters is always the primary focus.

An overview of robot calibration techniques and the identification of three calibration levels are introduced and discussed in [11]. Level 1 is the joint level calibration, level 2 is the entire robot kinematic model calibration, and level 3 is the nonkinematic (nongeometric) calibration. In this work, the limits on improving robot accuracy are attributed to the limits of robot repeatability and accuracy and to the accuracy of the measuring system. The extensive use of off-line-programming (OLP) systems for robot programming and path planning and the importance of calibration are described in [12]. In this work, a technique for improving the accuracy and identifying the kinematics parameters of the PUMA 560 robot was presented and experimental results were evaluated. The development and use of a tumbling technique for direct-drive robots and the identification of geometric and encoder errors are discussed in [13]. The authors observe that after the calibration the improved accuracy of the three-dof robot is very close to its repeatability. A method focused on the use of wire potentiometers attached to the robot tool is introduced in [14], where the authors use a PUMA 560 manipulator for experimental verification of the proposed method and identification of the kinematic parameters of the robot through an automated identification algorithm. The authors compared two different calibration techniques, the wire potentiometer and a CMM, and drew conclusions. A neural network-based technique for online identification of the relative position and orientation of robotic manipulators is presented in [15]. In this work, a vision system and a 3D force/torque sensor were used along with modifications in the control system using neural networks. The experimental results indicate that this method does not need the mathematical model of the robot and is simpler than other calibration procedures for identifying the robot parameters. Robot geometric errors and their identification using various techniques have been extensively discussed in the literature. The effect of nongeometric errors such as compliance and thermal is discussed in [16]. In this work, an approach for identifying geometric and nongeometric errors is presented and experimentally verified using a 6-dof robot. The robot accuracy improved by an order of magnitude after calibration.

In the surveyed literature, the focus is on the development of techniques for the identification of the geometric or nongeometric robot parameters. Once the parameters are identified, techniques to incorporate them for use in the robot model to improve accuracy were developed. However, in this literature the degree of influence of these errors on the overall robot repeatability and accuracy was not addressed. This is the premise of this work: the identification of the degree of influence of the kinematic geometric error parameters.

3. Accuracy and Repeatability Analysis

The first step in attempting to improve the accuracy and repeatability of industrial robots is to examine the current state of the robotics technology and other related technologies, such as metrology systems. It is important to understand that robot manufacturers whose precision claims cannot stand up to the world's most sophisticated measurement systems have no place in the industry.

3.1 Robotics Technology

Robot manufacturers, as an industry standard, publish the repeatability specifications of each robot. These specifications are determined by performing stringent experiments in accordance with ISO 9283 [17]. As a general rule of thumb, larger robots have larger errors in repeatability. The published repeatability values for the RX-series robots produced by Stäubli Corporation are tabulated in Table 1 [18]. The kinematically smallest robot, RX 60, has less than half the repeatability error of the largest, RX 130, indicating that the kinematic parameters of similar model robots directly influence the robot repeatability.

In most industrial applications the selected robot repeatability values are smaller than the process requirements. In those few instances when this is not true, other solutions must be found. A common approach has been

Table 1
Repeatability for Stäubli RX-Series Manipulators

Manipulator Model	Repeatability Value	
	mm	inch
RX 130	0.035	0.00138
RX 90	0.025	0.00098
RX 60	0.015	0.00059

to start from the base of the manipulator and determine if improvements can be made in each homogeneous transformation between links in order to develop a more accurate kinematics model. The homogeneous transformations between the links, however, depend not only on the resolution of the actuators for each joint, but also on the kinematic parameters of the actual machined link. Therefore, the assembly and machining accuracy directly affect the accuracy of the homogeneous transformations. There has been a wealth of research and information in the literature discussing techniques and approaches for calibration and identification of the kinematic parameters of robotic manipulators.

3.2 Kinematics Analysis

The kinematic structure of robots is often represented mathematically using a compact representation of the position and orientation of each joint relative to the previous joint. In our work, we employed the modified Denavit-Hartenberg notation as presented in [1]. The Denavit-Hartenberg notation represents a set that uniquely defines the kinematic parameters of the robot. The DH-parameters, a_{i-1}, α_{i-1}, d_i, θ_i, between two successive joints are shown in Fig. 4 [1].

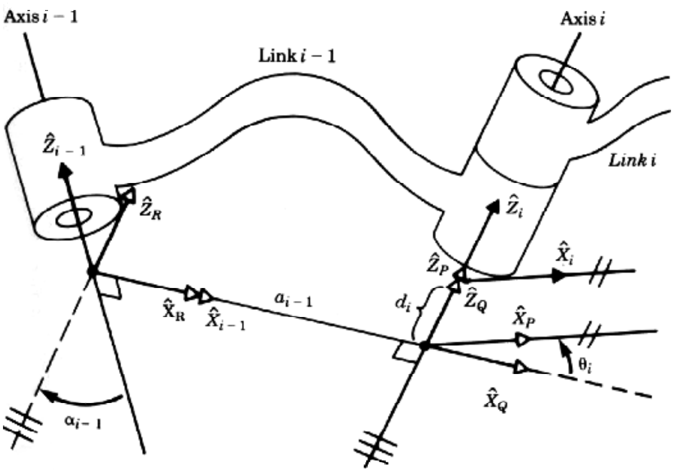

Figure 4. Graphical representation of DH-parameters [1].

The following notation applies:

a_{i-1} = link offset, distance from Z_{i-1} to Z_i measured along X_{i-1}
α_{i-1} = twist angle from Z_{i-1} to Z_i measured about X_{i-1}
d_i = distance from X_{i-1} to X_i measured along Z_i, joint variable for prismatic joints
θ_i = rotation angle from X_{i-1} to X_i measured about Z_i, joint variable for revolute joints

The DH-parameters are combined into a 4×4 matrix called the homogeneous transformation matrix, ${}^{i-1}_{\ \ i}T$, that describes the orientation and position of frame i relative to frame $i-1$. The homogeneous transformation using modified DH-parameters is given in (1) [1].

$$ {}^{i-1}_{\ \ i}T = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i\cos\alpha_{i-1} & \cos\theta_i\cos\alpha_{i-1} & -\sin\alpha_{i-1} & -d_i\sin\alpha_{i-1} \\ \sin\theta_i\sin\alpha_{i-1} & \cos\theta_i\cos\alpha_{i-1} & \cos\alpha_{i-1} & d_i\cos\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1} $$

The effect of the kinematics errors between an as-designed and as-built manipulator on its overall positioning will be examined.

The transformation that expresses the tool-centre-point frame, TCP frame, frame-n, relative to the robot base frame, frame-0, is obtained by concatenating the individual joint homogeneous transform matrices, as in (1).

$$ {}^0_nT = {}^0_1T\,{}^1_2T \cdots {}^{n-1}_{\ \ n}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & \text{tp}x \\ r_{21} & r_{22} & r_{23} & \text{tp}y \\ r_{31} & r_{32} & r_{33} & \text{tp}z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} R_{3\times3} & P_{3\times1} \\ 0_{1\times3} & 1 \end{bmatrix} \tag{2} $$

$R_{3\times3} = f(\theta_i, \alpha_{i-1}),\ i = 1, \ldots, 6$ is the orientation matrix, and $P_{3\times1} = f(\theta_i, a_i, d_i),\ i = 1, \ldots, 6$ is the position vector.

In this work, we address only the position of the TCP frame. The analysis for the orientation of the TCP frame is to be discussed in future research. The equations for the theoretical position vector, tp = {tp$x$, tp$y$, tp$z$}, provide the basis for the analysis used in this work. Using the kinematics values in Table 2, the theoretical component equations for the PUMA 560 are evaluated and presented in (3). These equations were derived using the symbolic capabilities of MATLAB.

$$ \text{tp} = \left\{ \begin{array}{l} \cos\theta_1[a_2\cos\theta_2 + a_3\cos(\theta_2+\theta_3) \\ \qquad -d_4\sin(\theta_2+\theta_3)] - d_3\sin\theta_1 \\ \sin\theta_1[a_2\cos\theta_2 + a_3\cos(\theta_2+\theta_3) \\ \qquad -d_4\sin(\theta_2+\theta_3)] + d_3\cos\theta_1 \\ -a_3\sin(\theta_2+\theta_3) - a_2\sin\theta_2 - d_4\cos(\theta_2+\theta_3) \end{array} \right\} \tag{3} $$

3.3 Error Analysis

The accuracy and repeatability of a given manipulator can be derived from (2). The theoretical zero position for a PUMA 560 manipulator is calculated by substituting the values in Table 2 for the DH-parameters and values for the joint values, θ_i. However, the actual position of the robot is a function not only of the DH-parameters, but also of their corresponding individual variations. These variations are mainly attributed to machining or manufacturing tolerances or component limitations (encoder resolution). They will be indicated as a "delta" (Δ) value for each DH-parameter. Note that in this analysis we are addressing only the effect of linear variations.

The error analysis is performed using the notion of differential transformation [19], where the differential transformation is introduced and used in a different perspective. An example cited is that of a camera observing the position of the tool frame of the manipulator and calculating the differential changes in position and orientation in order to accomplish a desired task [19].

In this analysis, the differential transformation theory is employed to obtain an estimate of the linear differential change from the theoretical position of the robot tool frame, the last frame of a manipulator. The linear differential error in the orientation and position of an arbitrary joint of the manipulator, ΔT_i, is a function of all the DH-parameters and their respective linear variations as shown in (4).

$$\Delta T_i = \frac{\partial T_i}{\partial \theta}\Delta\theta_i + \frac{\partial T_i}{\partial a}\Delta a_i + \frac{\partial T_i}{\partial \alpha}\Delta\alpha_i + \frac{\partial T_i}{\partial d}\Delta d_i \quad (4)$$

The actual orientation and position are given as the sum/difference ($\pm$) between the theoretical, T_i, and the differential change, ΔT_i as shown in (5).

$$\begin{aligned} T_{i,\text{actual}} &= T_i \pm \Delta T_i \\ &= T_i \pm \left(\frac{\partial T_i}{\partial \theta}\Delta\theta_i + \frac{\partial T_i}{\partial a}\Delta a_i + \frac{\partial T_i}{\partial \alpha}\Delta\alpha_i + \frac{\partial T_i}{\partial d}\Delta d_i \right) \end{aligned} \quad (5)$$

The variation due to each DH-parameter could be expressed as a function of the theoretical transform, T_i, multiplying an influence matrix, T_j^*, $j = (\theta, \alpha, a, d)$. This representation yields a formulation that identifies the structure of the variations as functions of the DH-parameters.

The procedure for analysing one DH-parameter, the rotational joint angle θ, and identifying the corresponding influence matrix, T_θ^*, is presented. Let $\partial T_i/\partial\theta = T_i T_\theta^*$, then, T_θ^* is evaluated as:

$$\begin{aligned} T_\theta^* &= T_i^{-1}\frac{\partial T_i}{\partial \theta} \\ &= T_i^{-1}\begin{bmatrix} -\sin\theta_i & -\cos\theta_i & 0 & 0 \\ \cos\theta_i\cos\alpha_{i-1} & -\sin\theta_i\cos\alpha_{i-1} & 0 & 0 \\ \cos\theta_i\sin\alpha_{i-1} & -\sin\theta_i\sin\alpha_{i-1} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \quad (6)$$

The influence matrix, T_i^*, for each DH-parameter is given for completeness.

$$T_\theta^* = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad T_d^* = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (7)$$

$$T_a^* = \begin{bmatrix} 0 & 0 & 0 & \cos\theta_i \\ 0 & 0 & 0 & -\sin\theta_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$T_\alpha^* = \begin{bmatrix} 0 & 0 & -\sin\theta_i & -d_i\sin\theta_i \\ 0 & 0 & -\cos\theta_i & -d_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (8)$$

The actual orientation and position of the tool frame, T_{actual}, for a six-dof manipulator is evaluated using the notion of the influence matrix and (5)–(8).

$$\begin{aligned} T_{\text{actual}} &= \prod_i^6 (T_i \pm \Delta T_i) \\ &= \prod_i^6 \{T_i \pm (T_i T_{\theta_i}^*\Delta\theta + T_i T_{a_i}^*\Delta a \\ &\qquad + T_i T_{\alpha_i}^*\Delta\alpha + T_i T_{d_i}^*\Delta d)\} \\ &= \prod_i^6 T_i\{I_4 \pm (T_{\theta_i}^*\Delta\theta + T_{a_i}^*\Delta a \\ &\qquad + T_{\alpha_i}^*\Delta\alpha + T_{d_i}^*\Delta d)\} \end{aligned} \quad (9)$$

An interesting observation is that T_θ^*, T_d^*, the influence matrices for the generalized joint variables for any manipulator, are constants, where as the other two influence matrices T_a^*, T_α^* (generally related to the geometric link properties) are functions of the generalized joint variables as shown in (7) and (8). This is an important result, especially if the differential transformation or variation information is examined on a frame-by-frame basis starting from the base and ending at the TCP frame of the manipulator. For example, for an all-revolute manipulator with equal variations on the joint variables, the actual position and orientation are evaluated using (5)–(9).

$$\begin{aligned} T_{\text{actual}} &= \prod_i^6 T_i\,\{I_4 \pm T_{\theta i}^*\Delta\theta\} \\ &= \prod_i^6 T_i \begin{bmatrix} 1 & \mp\Delta\theta & 0 & 0 \\ \pm\Delta\theta & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (10)$$

Using the developed approach, the actual orientation and position of the TCP frame can be found in symbolic and numerical form. Numerical solutions can only be evaluated if the variation of each DH-parameter is known or provided by the robot manufacturer. Even though robot manufacturers do not release this type of information, one can obtain an estimate of the errors and the level of influence or contribution of each DH-parameter using reasonable manufacturing variation (tolerances) values for machined components or resolution for encoders. A numerical example demonstrating this formulation will be presented below.

3.4 Accuracy and Repeatability Evaluation

The bounds of the actual position vector, ap = {ap$x$, ap$y$, ap$z$}, of the robot tool frame can be obtained from the theoretical position vector if one assumes that each DH-parameter (joint parameter) is associated with a variation as shown in (10). The actual position vector is the $P_{3\times 1}$ vector in T_{actual} of (10).

The position error, e_i, for each axis is given as the difference between the actual and theoretical positions as in (11). The total position error, totpos_err, is defined as the norm of the maximum individual axes position error as shown in (12), thus providing the maximum estimate for the total error.

$$e_i = \text{ap}i \pm \text{tp}i,\ i = \{x, y, z\} \tag{11}$$

$$\begin{aligned}\text{totpos\_err} &= \text{norm}(\max(e_x), \max(e_y), \max(e_z)) \\ &= [\max(\text{ap}x \pm \text{tp}x)^2 + \max(\text{ap}y \pm \text{tp}y)^2 \\ &\quad + \max(\text{ap}z \pm \text{tp}z)^2]^{1/2}\end{aligned} \tag{12}$$

An estimate of the accuracy is calculated as the positional error when variations exist in all the DH-parameters. Repeatability, on the other hand, is calculated by assuming that variations exist only for the joint variables.

This postulate is due to the fact that once a manipulator is assembled, the only dynamic (changing) components are the actuators, the joint variables. All the other kinematics parameters are dimensionally static and do not change during operation. Once a link is manufactured, its length will not change during operation, assuming no environmental changes such as temperature or deflections due to loading are present. If the dimensionally static notion is extrapolated from the link to the manipulator, variations that existed during manufacturing and assembly will not affect the repeatability as these variations will always be present in the assembled manipulator.

It is important to note that structural and joint flexibility are present in manipulators, although these effects are beyond the scope of the current analysis. This article addresses accuracy and repeatability from a pure kinematics perspective due to linear variations on the DH-parameters only.

The repeatability and accuracy position errors are defined as functions of the DH-parameters and their variations. The analysis that follows assumes a manipulator with all revolute joints. Using the differential change definitions in (9), the repeatability and accuracy are evaluated as shown in (13) and (14), respectively:

$$\begin{aligned}\text{repeatability; pos\_err} &= f(\alpha_i, a_i, d_i, \theta_i, \Delta\theta_i) \\ &= \prod_i^6 (T_i \pm \Delta T_i) \\ &= \prod_i^6 T_i(I_4 \pm T^*_{\theta_i}\Delta\theta)\end{aligned} \tag{13}$$

$$\begin{aligned}&\text{accuracy; pos\_err} \\ &= f(\alpha_i, \Delta\alpha_i, a_i, \Delta a_i, d_i, \Delta d_i, \theta_i, \Delta\theta_i) = \prod_i^6 (T_i \pm \Delta T_i) \\ &= \prod_i^6 T_i\{I_4 \pm (T^*_{\theta_i}\Delta\theta + T^*_{a_i}\Delta a + T^*_{\alpha_i}\Delta\alpha + T^*_{d_i}\Delta d)\} \\ &= \prod_i^6 [T_i\{I_4 \pm (T^*_{a_i}\Delta a + T^*_{\alpha_i}\Delta\alpha + T^*_{d_i}\Delta d)\} \\ &\qquad \pm \text{repeatability; pos\_err}]\end{aligned} \tag{14}$$

The accuracy is a function of the static (α_i, a_i, d_i) and dynamic/joint variable (θ_i) DH-parameters. The static variations $(\Delta\alpha_i, \Delta a_i, \Delta d_i)$ are attributed to machining and assembly tolerances, where the dynamic variation is attributed to the joint variable resolution $(\Delta\theta_i)$ for all revolute joints. The kinematics model of the robot residing in the robot controller and used for the inverse kinematics analysis uses the nominal values for the static parameters in estimating the joint dynamic value for a particular position. However, the variations in the static parameters are not accounted for in the inverse kinematics as they are not known. Even if the values of the variations are known, they are just variations, manufacturing or assembly tolerances, which could be positive, negative, or any number within a range from the nominal value. For example, assuming a machining tolerance of 0.0127 mm (0.005 inch) and using the values for the PUMA 560, $a_2 = 431.8 \pm 0.127$ mm. This means that any value in the range between $(431.8 - 0.127)$ 431.67 mm and $(431.8 + 0.127)$ 431.927 mm is an acceptable value for a_2. Therefore, it is impossible for the controller to account and compensate for these variations.

On the other hand, repeatability is easier to address and analyse. The repeatability of a robot indicates the robot's ability to return to the same taught position. The basis of this statement is the fact that robots return to taught positions. Therefore, any variations in the static parameters $(\Delta\alpha_i, \Delta a_i, \Delta d_i)$ do not affect the repeatability. The robot is already constructed no matter what the variations are, and nominal values are assigned to the static variables (α_i, a_i, d_i). When a position is taught to the robot, the controller will only need to remember the current state of the dynamic variables, joint variables, and is not concerned with the values of the static variables. The only information that the robot uses to return to the taught position is the joint variable values. This leads to the conclusion that the only variation that will directly affect the repeatability is the variation attributed to the joint variables ($\Delta\theta_i$).

3.5 Degree of Influence

In this work a new measure, called the degree of influence, is defined as the relative contribution of a DH-parameter variation to the base accuracy of a rigid manipulator. The degree of influence is a qualitative and not a quantitative measure. This definition allows for easy identification of the DH-parameter that contributes the most to the overall accuracy measure. For example, the accuracy due to the static parameters is evaluated due to all DH-parameter variations being nonzero. Then, only one DH-parameter is set to a nonzero variation and the accuracy measure is re-evaluated. The relative contribution of this nonzero DH-parameter to the total accuracy on a percentage basis is called the degree of influence. The same concept is also extended to include combinations of the static DH-parameters with nonzero variation values, thus providing the degree of influence for these combinations.

Identifying and understanding the degree of influence concept is very important. A robot manufacturer could possibly achieve significant improvements on the accuracy of a robot by improving the tolerance (variation) of only one of the DH-parameters. This qualitative measure will be discussed in the numerical example, where the degree-of-influence analysis is performed for two cases. For the first case a tolerance of 0.005 inch and for the second case a tolerance of 0.0025 inch (a considerable improvement over the first case) was assumed on the length parameters. The rotational positional tolerance remained the same for both cases.

4. Numerical Example

In this section a numerical example will be presented to corroborate the theory and concepts presented in this work. In this example accuracy, repeatability, and degree of influence measures for the PUMA 560 manipulator will be evaluated and examined.

The PUMA 560 robot was chosen primarily because so much has been published about it for various robotics concepts such as kinematics, dynamics, and control [1, 19]. The DH-parameters of the PUMA 560 to be used in the numerical example to demonstrate the uncertainty in accurately and repeatedly positioning the tool frame are shown in Table 2 [1, 19].

The repeatability and accuracy for the PUMA 560 are evaluated using zero for the joint variables, a rotational/angular variation of $\pm 0.005°$, a length/linear variation of ± 0.005 inch, and the DH-parameter values in Table 2. The rotational variation corresponds mainly to the encoder resolution and was chosen based on hardware in the laboratory, and the linear variation corresponds to machining or manufacturing tolerance and was chosen based on standard industry practice. All the calculations presented in this work were performed using the software package MATLAB. If the need arises, the numerical developed m-files could easily be modified to generate symbolic expressions through the symbolic toolbox of MATLAB.

Table 2
DH-Parameters for PUMA 560

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-90°$	a_3	d_4	θ_4
5	$90°$	0	0	θ_5
6	$-90°$	0	0	θ_6

$a_2 = 431.8$ mm, $a_3 = 20.32$ mm, $d_3 = 124.46$ mm, and $d_4 = 431.8$ mm. This is an all-revolute robot; therefore the vector θ_i, $i = 1, \ldots, 6$, represents the joint variables.

The repeatability and accuracy values are evaluated to be:

$$\begin{aligned} \text{Repeatability} &= \pm\, 0.1034\,\text{mm} = \pm\, 0.0041\,\text{inch} \\ \text{Accuracy} &= \text{Base} \pm \text{Repeatability} \\ &= (0.8324 \pm 0.1034)\,\text{mm} \\ &= (0.03680 \pm 0.0041)\,\text{inch} \end{aligned}$$

The degree of influence of each DH-parameter variation and their combinations for the PUMA 560 are presented in Table 3. Note that the repeatability of this manipulator will always be the same as it does not depend on the static variables, and is thus not included in this table.

In Table 3, the first column indicates the DH-parameter or combination that is set to a nonzero variation value in the analysis. The second and third columns represent the evaluated accuracy (mm) for the respective nonzero variation and the degree of influence as a percentage of the accuracy value obtained when all static variations are given a nonzero value. The same explanation applies to the third and fourth columns.

All the revolute joints (joint variables) are set to zero, and the variation of the angular variables ($\Delta\theta, \Delta\alpha$) is set to 0.005°. The analysis is performed for two different linear variation (tolerance) values in order to assess the effect of the linear variation change in both qualitative and quantitative terms. The first variation (results in columns 2 and 3) is defined to be 0.127 mm (0.005 inch), and the second variation (results in columns 4 and 5) is defined to be half of the first, 0.063 mm (0.0025 inch). The numerical results for accuracy and degree of influence are presented in Table 3.

The results presented in Table 3 provide not only the accuracy expected by a robot with the given tolerance specifications, but the degree of influence of each and combinations of the static DH-parameters on the accuracy

Table 3
Degree of Influence of DH-Parameters for PUMA 560

Nonzero DH-Parameter	Rotational/Angular Variation, $\Delta\theta$, $\Delta\alpha = 0.005°$			
	Linear Variation: Δd, $\Delta a = 0.127$ mm (0.005 inch)		Linear Variation: $\Delta d, \Delta a = 0.063$ mm (0.0025 inch)	
	Accuracy (mm)	Degree of Influence	Accuracy (mm)	Degree of Influence
Δd	0.402	0.430	0.201	0.392
Δa	0.762	0.816	0.381	0.744
$\Delta\alpha$	0.154	0.165	0.1542	0.301
Δd, Δa	0.861	0.922	0.4307	0.841
Δd, $\Delta\alpha$	0.540	0.578	0.343	0.669
Δa, $\Delta\alpha$	0.777	0.832	0.411	0.802
$\Delta d, \Delta a, \Delta\alpha$	0.934	1.000	0.512	1.000

of the manipulator being examined. The calculated entries in Table 3 depend on the values of the DH-parameters and their variations.

The repeatability of the robot remained the same for both cases as it depends only on the resolution of the joint variables. However, the calculated accuracy of the robot showed significant improvement in the second case. The accuracy improved by 0.422 inch, from 0.934 to 0.512 inch, corresponding to a 45% improvement. This drastic change is achieved just by defining stringer manufacturing and assembly tolerances.

Usually, qualitative measures like the degree of influence are better visualized in a graphical representation. The graphical representation for the degree of influence in Table 3 for the two linear variations is presented in Fig. 5.

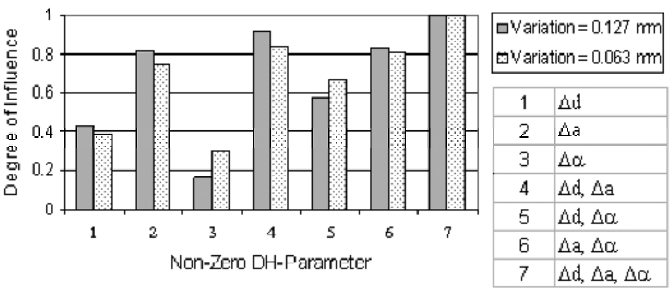

Figure 5. Qualitative representation of degree of influence versus nonzero DH-parameters.

The degree of influence of the DH-parameters and their combinations, though, remain similar between the two cases, as shown numerically in Table 3 and graphically in Fig. 5. The results presented in Table 3 indicate that for the PUMA 560 the parameter that has the least effect on the accuracy is the link twist, $\Delta\alpha$ (angular variation 0.005°), with a degree of influence of 0.165. The single DH-parameter with the highest degree of influence on the accuracy is the link offset, Δa (linear variation 0.127 mm), with a degree of influence of almost of 80%.

These calculations are performed without accounting for any other effects, such as deflections due to external loading (payload) or the mass of outer links of the manipulator. This assumes that the structural and joint flexibility are zero and rigid links. Using the presented concept for accuracy and repeatability, it has been shown that in this instance 88% of the positional error is due to errors in the initialized position of the robot. This is derived from the notion that by eliminating any error associated with the static variables, the servo system can position the robot to within a volume of 10% of the total volume seen when these errors are present. Although the analysis and numerical example presented were performed for an all-revolute articulated manipulator, they could be extrapolated and easily applied to other robot topologies.

5. Conclusion

In this article an error tree with sources that contribute to the accuracy and repeatability of manipulators is identified. Measures of accuracy and repeatability were derived and calculated indicating that high repeatability is more desirable than high accuracy in daily applications with industrial robots. In addition, using the notion of differential transformation, an error analysis technique was developed and related to accuracy and repeatability measures. This technique identified the linear variations as functions of the DH-parameters and the joint variables.

A formulation for evaluating the accuracy and repeatability of a serial link manipulator was developed employing the newly defined notion of the influence matrix. The influence matrices for joint variables are constants where for geometric variables they are not.

A new quantitative measure called the degree of influence was established as the relative contribution of a kinematic parameter variation to the accuracy when all variations are nonzero. This analysis indicated the level of accuracy improvement that could be achieved if variations on the DH-parameters are improved. In addition, the degree of influence of the DH-parameters or their combinations on the accuracy of the robot was evaluated using the developed theory.

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