The results obtained were as follow :—

Vessel.	Rate of evaporation of liquid oxygen in grms./24 hours.	Rate of cooling of hot water in cals./hour over temp. range 70°–50° C.
Vacuum-adjacent surfaces of polished copper	525	634
Vacuum-adjacent surfaces of polished silver electroplated on copper....	456	634

Silver plating and polishing the vacuum-adjacent surfaces of a 2-litre copper Dewar vessel thus increased its efficiency, when used for the storage of liquid oxygen, by nearly 15 per cent.

The Department of Scientific and Industrial Research has borne the cost of the liquid oxygen and of the copper Dewar vessels used in this work, and we wish to express our thanks for this valuable assistance.

On the Action of a Locomotive Driving Wheel.

By F. W. Carter, M.A., Sc.D., M.Inst.C.E., M.I.E.E.

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In the appendix to a paper read before the Institution of Civil Engineers,* dealing generally with the subject of the 'Electric Locomotive,' the author discussed the running qualities of locomotives from the point of view of dynamics. He based the discussion on the forces set up between wheel and rail, and these forces he referred to the creepage of the surfaces in contact due to elastic deformation of the material in the neighbourhood of the contact, defining "creepage" as the ratio of the distance gained by one surface over the other, to the distance traversed. He later introduced two quantities, f and f', which represented respectively the tractive force per unit creepage, longitudinally and transversely, to the rail. The quantities f and f', which were assumed constant in any particular problem, were not determined at the time, and the present paper is primarily an attempt to compute the first of them.

* See Minutes of 'Proc. Inst. C.E.,' vol. 201, part I, p. 248. See also the author's book 'Railway Electric Traction' (Arnold, 1922), chap. 2, p. 57, *seq.*

The area of contact between wheel and rail varies with the state of wear of the parts. For a new rail the longitudinal dimension of the contact is in general greater than the transverse dimension; but, as the rail flattens with use, the contact area approximates in shape to a uniform strip transverse to the rail. The final state is assumed herein, the wheel and rail being conceived as cylinders having their generating lines parallel. The problem proposed is accordingly a two-dimensional one. Instead of assuming the problem to be that of a cylinder rolling on a plane, however, we implicitly assume it to be that of two cylinders of like material and of equal and opposite radii, pressed together and rolling on one another, one being subject to a torque and the other to an equal counter-torque. Under this assumption, any state of stress or strain in one member, due to tangential tractive forces only, is matched by an equal reversed state in the other, and the distribution of pressure between the members is unaffected by the traction, since the radial displacements of the surfaces in contact are complementary. We may note also that any conclusion deduced for a driving wheel is true, with reversal of stresses and strains, for a wheel undergoing braking.

The radius of the wheel is large compared with the circumferential extent of the contact area; and, except in the determination of particulars of the contact, may be assumed infinite. The problem is then one of an infinite elastic medium bounded by a plane, on which is a certain local distribution of pressure and tangential traction. The stresses and strains, due to pressure, are known,* and need not be discussed further than as the means of transmitting the tractive effort.

The solution of the two-dimensional problem of an infinite elastic medium, bounded by, and on the positive side of, the plane $y = 0$, in which the portion of the boundary for which x is negative is subjected to a uniform tangential traction parallel to the x-axis, and that for which x is positive is free of externally applied stress, is given by Prof. Love.† Using the same notation as Prof. Love (viz., Δ for dilatation, ϖ for component rotation, λ, μ for elastic constants), the solution is shown to depend on the equation:

$$\frac{d(\xi + i\eta)}{d(x + iy)} = (\lambda + 2\mu)\,\Delta + i2\mu\varpi = \mathrm{C}\log(x + iy) \qquad (1)$$

in which $\mathrm{C}\pi(\lambda + \mu)/(\lambda + 2\mu)$ is the tangential traction on the half-boundary plane, being directed towards the origin when C is positive and away from it when C is negative: ξ and η are functions defined by the above equation.

* See Love's 'Mathematical Theory of Elasticity,' second edition, chap. VIII, § 138.

† *Loc. cit.*, chap. IX, § 152 (*c*).

Take the separating line between stressed and unstressed portions of the boundary at $(x', 0)$, and superpose a distribution of tangential stress extending to $(x' + dx', 0)$, and an equal reversed stress extending to $(x', 0)$. We thus obtain the solution of a problem in which the boundary stress extends over a band of width dx' only. Integrating this, in order to obtain the solution of the problem in which the tangential stress—a function of x'—extends over any desired portion of the boundary, we get as fundamental equation:

$$\frac{d(\xi + i\eta)}{d(x + iy)} = (\lambda + 2\mu)\,\Delta + i2\mu\varpi = -\int \frac{C\,dx'}{x + iy - x'}, \tag{2}$$

the integral being taken over the boundary.

The strain in the direction of the x-axis is, using Prof. Love's notation*:

$$\begin{aligned} e_{xx} &= \frac{du}{dx} \\ &= \frac{1}{2\mu}\left[-\mathrm{Y}_y + \frac{d\xi}{dx}\right]. \end{aligned}$$

At the boundary surface, Y_y is zero, and:

$$\begin{aligned} e_{xx} &= \frac{1}{2\mu} \cdot \frac{d\xi}{dx} \\ &= \frac{\lambda + 2\mu}{2\mu}\,\Delta. \end{aligned} \tag{3}$$

Thus the value of e_{xx} at the boundary is the real part, as y approaches zero, of (see equation 2):

$$-\frac{1}{2\mu}\int \frac{C\,dx}{x + iy - x'}. \tag{4}$$

The values of C with which we shall have occasion to deal are, in form, proportional to $\left(1 - \frac{x'^2}{a^2}\right)^{\frac{1}{2}}$, the limits of x' being $-a$ and a; and:

$$\int_{-a}^{a}\left(1 - \frac{x'^2}{a^2}\right)^{\frac{1}{2}} \frac{dx'}{x + iy - x'} = \pi\left\{\frac{x + iy}{a} - \left[\left(\frac{x + iy}{a}\right)^2 - 1\right]^{\frac{1}{2}}\right\}. \tag{5}$$

We discuss the pressure and contact surface between wheel and rail,† taking

* *Loc. cit*, chap. IX, § 144.

† The matter is here discussed in terms of wheel and rail. For the case of a pair of equal wheels in contact, R should be replaced by R/2 throughout.

the origin at the centre of the contact area, and employing the notation of Prof. Love.* If R is the radius of the wheel and P the total pressure:

$$A = \frac{1}{2R}$$

$$= \tfrac{3}{4}\frac{\lambda + 2\mu}{2\pi\mu(\lambda + \mu)} \cdot P\int_0^\infty \frac{d\psi}{(a^2 + \psi)^{\frac{3}{2}}(b^2 + \psi)^{\frac{1}{2}}\psi^{\frac{1}{2}}}$$

$$= \tfrac{3}{4}\frac{\lambda + 2\mu}{2\pi\mu(\lambda + \mu)} \cdot \frac{P}{b} \cdot \int_0^\infty \frac{d\psi}{(a^2 + \psi)^{\frac{3}{2}}\psi^{\frac{1}{2}}} \text{ if } b \text{ is large}$$

$$= \tfrac{3}{4}\frac{\lambda + 2\mu}{\pi\mu(\lambda + \mu)} \cdot \frac{P}{a^2 b}. \tag{6}$$

The pressure per unit area of contact near the origin is:

$$P' = \frac{3P}{2\pi ab}\left(1 - \frac{x'^2}{a^2}\right)^{\frac{1}{2}}.$$

Integrating this over the width of the contact, the pressure per unit length of contact is:

$$\int_{-a}^{a} P'\,dx' = \tfrac{3}{4}\frac{P}{b}.$$

The equivalent length of the contact is thus $4b/3$; this we call l. Accordingly:

$$\frac{1}{2R} = \frac{\lambda + 2\mu}{\pi\mu(\lambda + \mu)} \cdot \frac{P}{la^2},$$

or:

$$a = \left[\frac{\lambda + 2\mu}{\pi\mu(\lambda + \mu)} \cdot \frac{2RP}{l}\right]^{\frac{1}{2}}, \tag{7}$$

also:

$$P' = \frac{2P}{\pi la}\left(1 - \frac{x'^2}{a^2}\right)^{\frac{1}{2}}. \tag{8}$$

Assume first that the tangential traction is everywhere proportional to the pressure—an assumption which is only justifiable when the wheel is on the point of skidding. Write its value T_1P'/P, so that T_1 is the maximum available tractive effort. Then:

$$C = \frac{\lambda + 2\mu}{\pi(\lambda + \mu)} \cdot \frac{2T_1}{\pi la}\left(1 - \frac{x'^2}{a^2}\right)^{\frac{1}{2}}. \tag{9}$$

Hence, at the boundary (see equations 4 and 5), when x is in the contact area ($x^2 < a^2$):

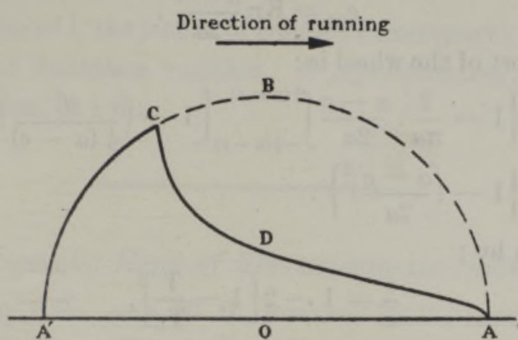
$$e_{xx} = -\frac{\lambda + 2\mu}{\pi\mu(\lambda + \mu)} \cdot \frac{T_1}{la} \cdot \frac{x}{a} \tag{10}$$

* *Loc. cit.*, chap. VIII, §§ 137, 138.

and when x is outside the contact area ($x^2 > a^2$):

$$e_{xx} = -\frac{\lambda + 2\mu}{\pi\mu(\lambda+\mu)} \cdot \frac{T_1}{la} \cdot \left[\frac{x}{a} - \left(\frac{x^2}{a^2} - 1\right)^{\frac{1}{2}}\right]. \tag{11}$$

We next consider the normal operation of the wheel. Assuming it to be running in the positive direction of the x-axis, let A′OA in the figure represent the contact surface, A being the point of first contact, and A′ the point of leaving. Let ABA′ be the curve of limiting tangential traction T_1P'/P. The actual curve of tangential traction will follow some line ADCA′, starting at A and never exceeding the limiting curve. Over the portion ADC of the curve, the surfaces



in contact are locked together, and the surface-strain is accordingly constant; for any variation of strain in one member requires an opposite variation in the other member, and this cannot be where the boundaries in contact have no relative movement. Beyond the point C, the pressure between the surfaces is insufficient to support the strain, and the surfaces accordingly slip, with limiting tangential traction. The value of the surface strain may be written:

$$e_{xx} = \text{real part of } \underset{y\to 0}{\mathrm{L}}\, \mathrm{K}\left\{\int_{-a}^{a}\left(1 - \frac{x'^2}{a^2}\right)^{\frac{1}{2}} \frac{dx'}{x+iy-x'} - \int_{c}^{a} \frac{\phi(x')\,dx'}{x+iy-x'}\right\}, \tag{12}$$

in which K is put for the coefficient of $\pi x/a$ in equation 10, and c is the abscissa of the point C in the figure. The function $\phi(x')$ is zero at the limits c and a, and positive between them: it is such that, between c and a, e_{xx} is independent of x. The first integral in equation 12 has, however (see equations 5 and 10), been shown to be proportional to x for points within the contact area; the second, accordingly, when $a > x > c$ should be a linear function of x, cancelling the first and leaving a constant.

Consider:

$$\phi(x') = \frac{a-c}{2a}\left[1 - \left(\frac{x' - \frac{1}{2}(a+c)}{\frac{1}{2}(a-c)}\right)^2\right]^{\frac{1}{2}}.$$

Changing the variable to $y' = x' - \frac{1}{2}(a + c)$, the second integral in equation 12 becomes:

$$\frac{a-c}{2a}\int_{-\frac{1}{2}(a-c)}^{\frac{1}{2}(a-c)}\left[1-\left(\frac{y'}{\frac{1}{2}(a-c)}\right)^2\right]\frac{dy'}{x-\frac{1}{2}(a+c)+iy-y'}.$$

This has the same form as the integral in equation 5, and, with $a > x > c$, and $y = 0$, its value is:

$$\frac{a-c}{2a}\cdot\pi\frac{x-\frac{1}{2}(a+c)}{\frac{1}{2}(a-c)} = \pi\left[\frac{x}{a}-\frac{a+c}{2a}\right]. \tag{13}$$

Hence, with $a > x > c$, equation 12 gives the constant value:

$$e_{xx} = \mathrm{K}\pi\frac{a+c}{2a}. \tag{14}$$

The tractive effort of the wheel is:

$$\mathrm{T} = \mathrm{T}_1\left\{1-\frac{2}{\pi a}\cdot\frac{a-c}{2a}\int_{-\frac{1}{2}(a-c)}^{\frac{1}{2}(a-c)}\left[1-\left(\frac{y'}{\frac{1}{2}(a-c)}\right)^2\right]^{\frac{1}{2}}dy'\right\}$$

$$= \mathrm{T}_1\left\{1-\left(\frac{a-c}{2a}\right)^2\right\}. \tag{15}$$

Hence c is given by:

$$\frac{c}{a} = 1-2\left[1-\frac{\mathrm{T}}{\mathrm{T}_1}\right]^{\frac{1}{2}}. \tag{16}$$

The quantity f is now readily determinable for the case considered. On entering the contact area, and for a certain distance within that area, the surface strain e_{xx} is given by equation 14. Consider a pair of points on the driving and driven wheel-rims respectively, situated an infinitesimal distance δx ahead of A (see figure), and therefore about to enter into contact with one another. The unstrained length of rim represented by δx is $(1-e_{xx})\,\delta x$ for the driving wheel, and $(1+e_{xx})\,\delta x$ for the driven wheel. The ratio of angular rotation of driving and driven wheel is therefore as $1-e_{xx} : 1+e_{xx}$ or as $1-2e_{xx} : 1$. The ratio of rolling rotation is unity, and the quantity $-2e_{xx}$ accordingly represents the creepage as defined above. Writing q for T/T_1, f is then given by:

$$f = \frac{q\mathrm{T}_1}{-2e_{xx}}$$

$$= -\frac{q\mathrm{T}_1}{\pi\mathrm{K}}\cdot\frac{a}{a+c} \qquad \text{(eqn. 14)}$$

$$= -\frac{\mathrm{T}_1}{2\pi\mathrm{K}}\frac{q}{1-(1-q)^{\frac{1}{2}}} \qquad \text{(eqn. 16)}$$

$$= \frac{\pi\mu(\lambda+\mu)}{2(\lambda+2\mu)}la\frac{q}{1-(1-q)^{\frac{1}{2}}} \qquad \text{(eqn. 10)}$$

$$= \left[\frac{\pi\mu(\lambda+\mu)}{2(\lambda+2\mu)}\mathrm{R}l\mathrm{P}\right]^{\frac{1}{2}}\frac{q}{1-(1-q)^{\frac{1}{2}}}. \qquad \text{(eqn. 7)} \tag{17}$$

Thus f depends on the tractive effort, increasing in the ratio 1 : 2 as T falls from T_1 to zero.

For the value $T = T_1$, or $q = 1$, and with forces and lengths expressed in ordinary engineering units of the subject, the approximate value of f for steel wheels and rails is as follows :

(1) With forces in kilogrammes and lengths in millimetres :

$$f = 93\ [RlP]^{\frac{1}{3}}.$$

(2) With forces in lbs. and lengths in inches :

$$f = 3500\ [RlP]^{\frac{1}{3}}.$$

The effective value of l, the length of contact transverse to the rail is matter for conjecture, and doubtless variable. A representative value is perhaps of the order of 25 mm. or 1 in.

On the Specific Heat of Ferromagnetic Substances.

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According to the Weiss theory of ferromagnetism, there is an intimate connection between the specific heat of a body and its magnetisation. Weiss* has shown that the magnetic energy per cubic centimetre of a ferromagnetic substance is :—

$$W = -\tfrac{1}{2}HI \tag{1}$$

where I is the intensity of magnetisation and H is the molecular field. Further, it is assumed that

$$H = NI \tag{2}$$

where N is a constant depending on the material itself. Thus

$$W = -\tfrac{1}{2}NI^2$$

and

$$dW/dT = -\tfrac{1}{2}N d/dT\ (I^2)$$

where T is the temperature. dW/dT will contribute to the specific heat of the substance which will become equal to

$$S = s + \frac{1}{\rho J}\frac{dW}{dT},$$

* Weiss and Beck, 'Journ. de Phys.,' vol. 7, p. 249 (1908).