On the altitude of the ELF/VLF source region generated during "beat-wave" HF heating experiments

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Received 20 July 2012; accepted 6 August 2012; published 18 September 2012.

[1] Modulated high frequency (HF, 3-10 MHz) heating of the ionosphere in the presence of the auroral electrojet currents is an effective method for generating extremely low frequency (ELF, 3-3000 Hz) and very low frequency (VLF, 3–30 kHz) radio waves. The amplitudes of ELF/VLF waves generated in this manner depend sensitively on the auroral electrojet current strength, which varies with time. In an effort to improve the reliability of ELF/VLF wave generation by ionospheric heating, recent experiments at the Highfrequency Active Auroral Research Program (HAARP) facility in Gakona, Alaska, have focused on methods that are independent of the strength of the auroral electrojet currents. One such potential method is so-called "beat-wave" ELF/ VLF generation. Recent experimental observations have been presented to suggest that in the absence of a significant D-region ionosphere ($\sim 60-100$ km altitude), an ELF/VLF source region can be created within the *F*-region ionosphere (\sim 150–250 km altitude). In this paper, we use a time-ofarrival analysis technique to provide direct experimental evidence that the beat-wave source region is located in the D-region ionosphere, and possibly the lower E-region ionosphere ($\sim 100-120$ km altitude), even when ionospheric diagnostics indicate a very weak *D*-layer. These results have a tremendous impact on the interpretation of recent experimental observations. Citation: Moore, R. C., S. Fujimaru, M. Cohen, M. Gołkowski, and M. J. McCarrick (2012), On the altitude of the ELF/VLF source region generated during "beat-wave" HF heating experiments, Geophys. Res. Lett., 39, L18101, doi:10.1029/2012GL053210.

1. Introduction

[2] Extremely low frequency (ELF, 3–3000 Hz) and very low frequency (VLF, 3–30 kHz) radio waves can be generated by modulated high frequency (HF, 3–30 MHz) heating of the *D*-region ionosphere (\sim 60–100 km) in the presence of naturally-forming current systems, such as the auroral electrojet [e.g., *Getmantsev et al.*, 1974; *Stubbe et al.*, 1982; *Papadopoulos et al.*, 2003; *Moore et al.*,

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2007; *Cohen et al.*, 2010]. The amplitude of ELF/VLF waves generated in this manner depends sensitively on the strength of the auroral electrojet currents that flow in the lower ionosphere [e.g., *Stubbe et al.*, 1981; *Rietveld et al.*, 1983, 1987; *Papadopoulos et al.*, 2003; *Payne*, 2007; *Jin et al.*, 2011]. In a directed effort to improve the reliability of ELF/VLF wave generation, recent experiments at the High-frequency Active Auroral Research Program (HAARP) HF transmitter in Gakona, Alaska, have focused on methods to generate ELF/VLF waves in a manner that is independent of the auroral electrojet currents [e.g., *Papadopoulos et al.*, 2011a, 2011b; *Kuo et al.*, 2011, 2012]. This paper focuses on one of these proposed methods: beat-wave ELF/VLF generation.

[3] During beat-wave (BW) ELF/VLF generation experiments, a high-power HF transmitter broadcasts two continuous waveform (CW) HF signals with a frequency difference in the ELF/VLF band toward the ionosphere [Kotik et al., 1986; Villaseñor et al., 1996; Barr and Stubbe, 1997]. The time-averaged Poynting flux of the transmission oscillates at the beat frequency (i.e., the difference frequency) of the two waves. Such a transmission modulates the electron temperature within the highly collisional D-region ionosphere, thereby modulating the conductivity of the region. Together with the auroral electrojet electric fields, the modulated conductivity radiates electromagnetic waves at the beat-wave frequency. Compared to the ELF/VLF source region generated using amplitude modulation (AM), the phase of the beatwave ELF/VLF source region rapidly varies with space due to the spatial separation of the two HF sources [Barr and Stubbe, 1997], which is on the order of the HF wavelength (10's of meters). Due primarily to this effect, the ELF/VLF radiation pattern for BW experiments is very different than that for AM experiments, and the ELF/VLF amplitude detected at a ground-based receiver depends strongly on the spatial separation of the two HF sources as well as on the location of the receiver [Barr and Stubbe, 1997].

[4] Recently, *Kuo et al.* [2011] suggested an electrojetindependent method for ELF/VLF wave generation using the BW technique. *Kuo et al.* [2012] directly compared the amplitude of ELF/VLF waves generated using BW to the amplitude of ELF/VLF waves generated using AM modulation. According to the theory presented by *Kuo et al.* [2011], the BW source was expected to be in the *F*-region ionosphere. *Kuo et al.* [2011, 2012] identified specific ionospheric conditions under which the ELF/VLF amplitude generated using BW was larger than that generated using AM modulation, and they concluded that it is possible to develop an ELF/VLF source region within the *F*-region ionosphere when ionospheric diagnostics indicate the "absence" of absorption in the collisional *D*-region ionosphere. At *F*-region altitudes, *Kuo et al.* [2011] suggest, the ELF/VLF

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source region would be created by the modulation of the ponderomotive force by the beat-wave transmission.

[5] In this paper, we critically evaluate experimental observations of ELF/VLF waves generated during BW and AM heating experiments at HAARP. We present observations performed under different ambient ionospheric conditions, including the same date and time (and thereby same ionospheric conditions) identified by Kuo et al. [2012] as being advantageous for the existence of an F-region ELF/VLF source. A time-of-arrival analysis technique is employed to approximate the altitude of the dominant ELF/VLF source region. For all cases, we determine that the BW- and AMgenerated ELF/VLF source regions are located at nearly the same altitude within the D-region ionosphere, and possibly within the lower edge of the E-region ionosphere. We conclude that the beat-wave ELF/VLF source region, similar to the AM-generated ELF/VLF source region, is located at whichever altitude range is dominated by collisional processes, typically identified as the *D*-region ionosphere ($\sim 60-100$ km altitude), even when ionospheric diagnostics indicate a minimal D-layer.

2. Description of the Experiment

[6] On 27 July 2011 and on 22 February 2012, the 3.6 MW HAARP HF transmitter located near Gakona, Alaska (62.39°N, 145.2°W) performed experiments generating ELF/VLF waves using both BW and AM modulation techniques. The HAARP transmitter is a 12×15 array of crossed dipole antennas. To implement BW modulation, approximately half of the array broadcast at the center frequency plus $\Delta f/2$ while the other half broadcast at the center frequency minus $\Delta f/2$, with Δf being the desired ELF/VLF frequency to be generated. On 27 July 2011, the array was split in the North–South direction, so that each frequency was broadcast using a 6×15 element array. The center frequency was 2.85 MHz, and the polarization of the HF transmission was X-mode. On 22 February 2012, the array was split in the East-West direction, so that each frequency was broadcast using a 12×7 element array, with one row of antennas at the center of the array remaining idle. In this case, the center frequency was 3.25 MHz, and the polarization of the HF transmission was X-mode. On both days, Δf was selected to vary as a linear frequencytime ramp. Two frequency-time ramps were implemented, with the first varying from 1 to 6 kHz over 5 seconds (1000 Hz/sec slope) and the second varying from 15 to 20 kHz over 5 seconds (1000 Hz/sec slope).

[7] On each day, AM modulation was performed at the same HF frequency and polarization as for the BW transmissions. Amplitude modulation was implemented using the complete 12×15 element array. The modulation waveform was selected to be sqrt-sine (or square-root-sinusoidal) AM because the power envelope of the sqrt-sine modulation is similar to that expected for beat-wave modulation. The same frequency-time ramp modulation was implemented for AM as was implemented for BW.

[8] Within the context provided above, we highlight the fact that the primary difference between the BW and AM formats is an additional phase term (for BW) that depends on the distance between the two HF sources and the location of the source in the ionosphere [*Barr and Stubbe*, 1997].

[9] In order to experimentally verify the modulation frequencies that were transmitted, HF observations were performed at Oasis (OA, 62.35°N, 145.1°W, 3 km from HAARP). The HF receiving system consists of two orthogonal 90-foot folded dipoles located approximately 12 feet above the ground. The receiver is sensitive to electric fields with frequencies between 1.0 and 10.0 MHz, and data acquisition was performed continuously at 25 MHz with 14-bit resolution. Accurate timing is provided by GPS. Each frequency-time ramp component of the received HF waveform is mixed-down to baseband and low-pass filtered to isolate the ground-wave component. A least-squares analysis is then applied to the received phase to approximate the start frequency and slope of each HF transmission, assuming speed of light propagation from the transmitter array to the receiver.

[10] ELF/VLF observations were performed at receivers located at Oasis (same location as the HF receiver) and at Chistochina (CH, 62.61°N, 144.62°W, 37 km from HAARP). Each ELF/VLF receiver system consists of two orthogonal magnetic loop antennas oriented to detect the radial and azimuthal components of the magnetic field on the ground, a preamplifier, a line receiver, and a digitizing computer. Accurate timing is provided by GPS. The receivers are sensitive to magnetic fields with frequencies between ~300 Hz and ~47 kHz. The signals were sampled at 100 kHz with 16-bit resolution.

[11] In post-processing, time-of-arrival (TOA) analysis is applied to the ELF/VLF observations at each of the receiver sites. TOA analysis is performed as described by *Fujimaru and Moore* [2011], using the HF receiver measurements at Oasis to provide the HAARP transmission parameters. The technique provides what is essentially an impulse response that describes the combined effects of HF propagation to the ionosphere and ELF/VLF propagation from the ionospheric source to the receiver. As discussed by *Fujimaru and Moore* [2011], TOA analysis requires a continuously varying waveform over a broad frequency range, like the frequency-time ramps employed herein.

[12] On 27 July 2011, the HAARP magnetometer registered only minimal activity, with the magnitude varying by less than 25 nT. 30-MHz absorption, as measured by the HAARP riometer, was less than 0.1 dB at the time of transmission. On 22 February 2012, the HAARP magnetometer registered moderate activity, with the magnitude varying by 25–50 nT. 30-MHz absorption was \sim 0.2 dB at the time of transmission.

3. Experimental Observations

[13] In order to properly describe the experimental results, we first present ELF/VLF observations performed on 22 February 2012. On this day, ELF/VLF waves were detected with high (>20 dB) signal-to-noise ratio (SNR). The four panels of Figure 1 show spectrograms of the ELF/VLF frequency-time ramps generated using BW and AM modulation techniques for different frequency ranges (1-6 kHz and 15-20 kHz) and observed at Oasis. For both modulation techniques and for both frequency ranges, the ELF/VLF signals are clearly detected in the spectrograms. The 1-6 kHz sqrt-sine AM ramp is also accompanied by a second harmonic, whereas the BW ramp is not. The spectrograms also exhibit natural and man-made ELF/VLF activity: radio atmospherics (or sferics) emitted by lightning are seen as short-duration pulses with energy content over a wide frequency band, and the NWC VLF transmitter signal (sourced in Australia) is clearly visible at 19.8 kHz. Figure 2 shows the

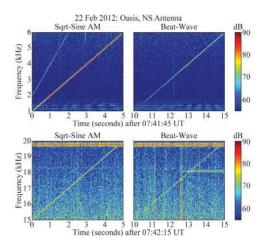


Figure 1. Spectrograms of ELF/VLF observations performed at Oasis (OA) on 22 February 2012: (top) 1–5 kHz and (bottom) 15–20 kHz and (left) BW and (right) AM.

results of TOA analysis applied to the 22 February 2012 data sets. The TOA analysis intrinsically involves integration over the 5-kHz bandwidth of the received ELF/VLF frequency-time ramp, further increasing the SNR (equivalent to a 5-second integration). The noise levels shown in the figure panels are approximated by applying the TOA analysis technique to the data set offset by 2.5 seconds (one half of a frequency-time ramp). Under good SNR conditions, clear primary peaks are observed at Oasis between 0.53 and 0.59 milliseconds for all cases, corresponding to the line-ofsight propagation path from the ionospheric source to the receiver. Assuming the ELF/VLF source is located directly above the HAARP transmitter, these time delays correspond to virtual altitudes (assuming speed-of-light propagation) of 79–88 km, which are clearly within the D-region ionosphere. The difference in virtual altitude for AM and BW sources is less than 5 km in both cases. Under good SNR conditions, each of the TOA plots exhibit ionospheric reflections (e.g., local maxima near 1.1 and 1.6 msec) and sinc function sidelobes (e.g., near 0.3 and 0.8 msec) that result from the TOA signal processing technique, as discussed in greater detail by Fujimaru and Moore [2011]. We attribute the moderate difference in the time of arrival estimates for BW and AM generation techniques to the rapid spatial variation of the ELF/VLF source region phase for the BW case. The spatial distribution of phase can reduce the amplitude at earlier time delays while increasing the amplitude at later time delays, and vice versa, depending on the receiver location. Additionally, the polarization of the received ELF/VLF wave, which varies with transmission format, can affect the perceived time delay by as much as 0.03 milliseconds, as discussed by Fujimaru and Moore [2011]. It is evident, however, that these effects are not sufficient to elevate either the BW- or the AM-generated ELF/VLF source regions out of the *D*-region ionosphere.

[14] For the particular cases shown, sqrt-sine AM modulation generates ELF/VLF waves with higher amplitudes than BW modulation, as observed at Oasis. For the 1–6 kHz ramp, the AM-generated ELF/VLF wave is \sim 13 dB higher. For the 15–20 kHz ramp, the AM-generated wave is only \sim 3 dB higher. Although the specific amplitudes of the AM and BWgenerated ELF/VLF signals are not particularly important for the purposes of this paper, we note that BW generation becomes increasingly efficient at higher modulation frequencies when compared to AM.

[15] Having described the results of the TOA analysis under good SNR conditions, we proceed to discuss the case of 27 July 2011. On 27 July 2011, ELF/VLF observations were performed at both Oasis and Chistochina. While the ELF/VLF waves generated by HAARP are clearly detectable in spectrograms on 22 February 2012, observations performed on 27 July 2011 required longer integration times to detect the HAARP-generated ELF/VLF signals. Figure 3 show the TOA signal processing results for the data sets acquired between 10:16 and 10:22 UT on 27 July 2011. This time frame is the exact same as that analyzed by Kuo et al. [2012] and described as exhibiting the "absence of a D-layer" and the presence of an *F*-region ELF/VLF source. After performing the 5-second integrations intrinsic to the TOA technique, SNR levels of only 10–15 dB are observed. For these cases, the ionospheric reflection components and the side-lobe components fall below the noise floor, but the line-of-sight component remains a reliable measurement. In all cases, the peaks of the main lobes occur between 0.62 and 0.73 milliseconds, corresponding to virtual (speed of light) altitudes of 93–109 km. The difference in virtual altitude for AM and BW sources is at most 15 km. We note that the virtual altitudes guoted here represent an upper limit of the altitudes for the dominant ELF/VLF source region because speed of light propagation is assumed. It is well-known, for instance, that the group velocities of both the HF wave and the ELF/VLF wave decrease in the lower ionosphere. The 109 km virtual altitude is thus likely to be an overestimate. Nevertheless, this maximum virtual altitude of 109 km is significantly lower than any reasonable approximation for the F-layer and is still

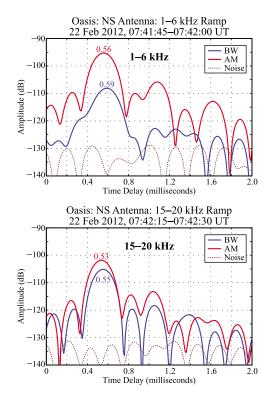


Figure 2. TOA analysis applied to observations performed at Oasis on 22 February 2012 under good SNR conditions.

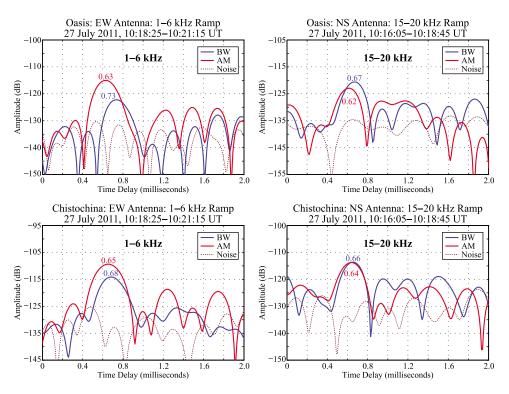


Figure 3. TOA observations performed (top) at Oasis (OA) and (bottom) at Chistochina (CH) for the (left) 1–6 kHz and the (right) 15–20 kHz ELF/VLF frequency-time ramps.

low enough in altitude to be consistent with the wellestablished collisional heating mechanism for ELF/VLF wave generation.

[16] On 27 July 2011, and for the 1–6 kHz frequency range, sqrt-sine AM modulation generates ELF/VLF waves with 5–8 dB higher amplitudes than BW modulation, as observed at both Oasis and Chistochina. For the 15–20 kHz range, however, the BW- and AM-generated ELF/VLF signals have approximately the same amplitude at Chistochina, and the BW-generated signals are \sim 2 dB larger than the AM-generated signals at Oasis. These observations are fully consistent with the experimental observations presented by *Villaseñor et al.* [1996] and the theoretical analysis presented by *Barr and Stubbe* [1997], assuming a *D*-region source for the BW generation technique.

4. Discussion and Conclusions

[17] On two different occasions for which strong and moderate ELF/VLF signal amplitudes were observed on the ground, it was found that the maximum ELF/VLF source altitude does not exceed 109 km for both BW and AM generation techniques. On both days, BW and AM techniques generated dominant ELF/VLF source regions within ~15 km (and typically 5 km) altitude of each other, shedding serious doubt on the interpretation of past experimental results that identified an F-region source for BW transmissions. TOA results show that lower D-region densities (on 27 July 2011 compared to 22 February 2012) can produce ELF/VLF source regions at higher altitudes. Even for the lower density case, however, the maximum ELF/VLF source region altitude does not exceed 109 km. In other words, even when ionospheric conditions indicate an alleged "absence" of an ionospheric D-layer, the primary physical interaction induced by HF

X-mode heating still occurs in the region of the ionosphere that is dominated by electron-neutral collisions.

[18] We further interpret the specifics of these observations in the context of the experimental observations and theoretical calculations presented by Villaseñor et al. [1996] and Barr and Stubbe [1997], who present experimental observations of ELF/VLF waves generated using both BW and AM techniques. Villaseñor et al. [1996] found that ELF/VLF waves generated using AM were generally stronger than those generated using BW, although they identified several cases where the BW ELF/VLF amplitude was stronger than the AM ELF/ VLF amplitude. Villaseñor et al. [1996] also found that, similar to AM-generated ELF/VLF waves, the amplitudes of BWgenerated ELF/VLF waves were larger for X-mode polarized HF waves than for O-mode polarized HF waves (as is also true for AM). The experimental observations presented by Barr and Stubbe [1997] were generally consistent with those presented by Villaseñor et al. [1996], with the exception that the amplitude of AM-generated ELF/VLF waves were always 5-12.5 dB stronger than those of BW ELF/VLF waves. The theoretical calculations presented by Barr and Stubbe [1997], on the other hand, indicated that BW-generated ELF/VLF waves could be stronger than AM-generated ELF/VLF waves, depending on the receiver location, the physical separation of the HF sources, and on the ELF/VLF frequency. In particular, Barr and Stubbe [1997] noted that for certain observation locations, BW generation could be more than ten times stronger than AM generation for ELF/VLF frequencies above \sim 5 kHz. Our observations are fully consistent with these results. Furthermore, the TOA analysis presented herein provides additional experimental evidence that the BW source is located in the collision-dominated D-region ionosphere, as was correctly assumed by Villaseñor et al. [1996] and Barr and Stubbe [1997].

[19] It thus not correct to assume that *D*-region absorption is negligible based on ionogram measurements that indicate little-to-no return from the *D*-region ionosphere. This conclusion further supports the analysis presented by *Jin et al.* [2011], who specifically addressed this issue. Based on the observations presented in this paper, a collision-dominated ionospheric region always exists, and a modulated HF power envelope can create an ELF/VLF source region within that altitude range.

[20] Lastly, we identify the ELF/VLF TOA analysis technique as an excellent tool for validating physical models of ELF/VLF wave generation. In this case, it provides an upper bound for the virtual altitude of the ELF/VLF source region, providing an important constraint for theoretical models of ELF/VLF wave generation. It is noted that ELF/VLF sources at higher altitudes would exhibit significant dispersion characteristics that could be detected using the TOA technique. We note that these characteristics are not exhibited in our experimental observations. Nevertheless, this work does not preclude the possibility that higher altitude ELF/VLF sources can exist. Rather, we identify the TOA analysis technique as a conclusive method for identifying the dominant ELF/VLF source height and recommend that future experiments employ this technique to validate their theoretical models.

[21] Acknowledgments. This work is supported by DARPA grant HR0011-10-1-0061, DARPA contract HR0011-09-C-0099, NSF grants AGS-0940248 and ANT-0944639, and ONR grant N000141010909 to the University of Florida. This work is further supported by ONR grant N0014-09-1-0100 and AFRL award FA9453-11-C-0011 to Stanford University. CU Denver is supported by subcontract UF-EIES-1005017-UCD and subaward 27239350-50917-B.

[22] The Editor thanks Konstantinos Papadopoulos and an anonymous reviewer for their assistance in evaluating this paper.

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