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On The Analogy between Strong Interaction and Electromagnetic Interaction

Yasunori FUJII

Department of Physics, College of Science and Engineering, Nihon University, Tokyo

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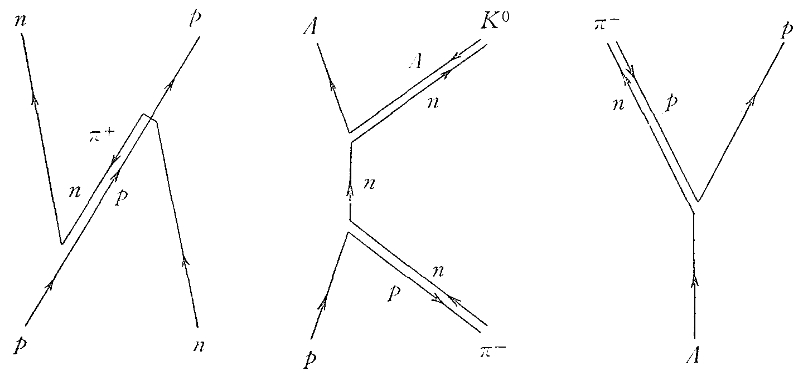
According to Sakata's composite hypothesis of heavy particles, we assume that the fundamental particles do not change their kind through strong interactions. A formulation of these interactions is given by requiring the invariance under the gauge transformation of the first kind applied to each baryon field. If we assume that the phase function depends on the space-time coordinate, it leads to the existence of a neutral vector meson analogous to the electromagnetic field the mass of which need not always vanish. Its consequences are examined and found to be quite favourable for the interpretation of the strong interaction. In particular, it leads to the conservation law of parity in strong interactions and the condition for the existence of the composite state.

§ 1. Introduction

According to the composite hypothesis of heavy particles proposed by Sakata[1-4] the following interpretation is possible. That is, the proton, neutron and Λ-particle which are assumed to be the fundamental particles do *not* change their kind through the strong interactions, while they are transformed into each other through the weak interactions. These circumstances are shown schematically in Fig. 1.



$p+n\to n+p$ $\quad$ $\pi^-+p\to\Lambda+K^0$ $\quad$ $\Lambda\to p+\pi^-$

Fig. 1.

The selection rule of this kind is very useful for the lepton processes also. As an example, consider the process

$$K^+ \to e^+ + e^- + \pi^+. \tag{1}$$

Such processes, which contain a pair of lepton and its antiparticle cannot be forbidden by the selection rules which follow from the conservation laws of so-called lepton number and neutrino charge. It has been proposed that the neutrino is the source of weak interactions. We cannot, however, take it as reasonable, because weak processes without neutrinos, such as hyperon and K-meson decays, are really observed. From the fact that the lepton-pair processes like (1) have never been observed at all,* it is necessary to assume a selection rule that a lepton-pair should not couple to any field except for the elctromagnetic field. The above selection rule leads also to the rule that a lepton should always change its kind through the interaction unless some special non-causal interaction is assumed.

On the other hand, we know the electromagnetic intraction as a prototype of interaction through which the kind of particle does not change. As for this interaction there is a remarkable feature that the interaction Lagrangian follows directly from the free Lagrangian by the requirement of invariance under the gauge transformation.

In the following it is attempted to apply this circumstance also to the strong interaction and its consequences are examined.

§ 2. Requirement of gauge invariance and its consequences

Analogous to the case of the electromagnetic interaction, consider the gauge transformation of the first kind which depends on the space-time coordinate

$$\begin{aligned} \psi_\alpha(x) &\to e^{ig_\alpha \Lambda(x)} \psi_\alpha(x), \\ \bar{\psi}_\alpha(x) &\to e^{-ig_\alpha \Lambda(x)} \bar{\psi}_\alpha(x). \end{aligned} \tag{2}$$

$$(\alpha = p,\ n,\ \Lambda)$$

If the invariance under these transformations is required, then by the replacement in the free Lagrangian,

$$\partial_\mu \psi_\alpha(x) \to (\partial_\mu - ig_\alpha A_\mu(x))\psi_\alpha(x),$$

the interaction Lagrangian must be introduced in the form

$$j_\mu(x) A_\mu(x), \tag{3}$$

where

$$j_\mu(x) = \sum_\alpha ig_\alpha \bar{\psi}_\alpha(x) \gamma_\mu \psi_\alpha(x). \tag{4}$$

$A_\mu(x)$ is a neutral vector field. Since immediate contradictions to the recent observations will occur as to the vanishing mass, we consider the mass to be non-vani-

* It is, of course, expected that the electromagnetic correction produces the processes like (1) at the ratio $\alpha^2 \sim 10^{-4}$ to the ordinary weak processes.

shing by the use of the Stückelberg formalism for the neutral vector field.[5)]

That is, by introducing a scalar field $B(x)$ in addition to $A_\mu(x)$, the theory is invariant under (2) together with the gauge transformation of the second kind,

$$A_\mu(x)\to A_\mu(x)+\partial_\mu\Lambda(x),$$
$$B(x)\to B(x)-m\Lambda(x), \tag{5}$$

where m denotes the mass of the meson, and $\Lambda(x)$ satisfies the equation

$$(\square^2-m^2)\Lambda(x)=0.$$

This formalism is shown to be equivalent to the ordinary one for the neutral vector field (See Appendix).

We assume that the interaction Lagrangian given by (3) is the only elementary interaction among the strong interactions. This is reminiscent of the electromagnetic interaction and following considerations will be possible.

(i) *Charge independence*

We know that the phenomena of strong interaction is successfully accounted for by the hypothesis of charge independence. Therefore the interaction Lagrangian (3) must be charge independent.

Since the meson here introduced is of single component, the vanishing iso-spin should be assigned. Then, the part of (3) and (4) which contains the proton and neutron, must be the iso-scalar of the form

$$\bar{\psi}_N(x)\gamma_\mu\psi_N(x)A_\mu(x),$$

where $\psi_N(x)$ denotes the usual iso-doublet

$$\psi_N(x)=\begin{pmatrix}\psi_p(x)\\ \psi_n(x)\end{pmatrix}.$$

This implies

$$g_p=g_n \tag{6}$$

in (4).

(ii) *Existence of composite particles*

The force resulting from the exchange of mesons here introduced is of opposite sign between baryon and antibaryon as compared with that between baryon and baryon. This is the immediate consequence of the vector coupling. Furthermore, between the particles of the same kind (e.g. p-$\bar{p}$, p-p), the former is attractive and the latter repulsive as long as the adiabatic approximation is valid. This is an exact analogue of the Coulomb interaction (See Appendix, (A. 7)).

As for p-$\bar{n}$, n-$\bar{p}$; p-n, the same is the case owing to (6). This allows the existence of pions as the bound state of nucleon and antinucleon.

Between nucleon and Λ-particle, it is concluded that

$$g_p = g_n \text{ and } g_\Lambda \text{ are of the same sign,} \tag{7}$$

as long as the K-meson exists as the bound state of nucleon and anti-Λ-particle, and no bound state exists between nucleon and Λ-particle.

(iii) *Invariance under space reflection and charge conjugation.*

Since the interaction Lagrangian (3), which is assumed to be the only elementary interaction derived from gauge invariance, does not allow the coexistence of γ_μ and $\gamma_\mu\gamma_5$, it is possible to assign the definite parity to $A_\mu(x)$, irrespective of the relative parity between baryons. Thus, the parity should be conserved in all strong interactions. The same consideration leads also to the invariance under the charge conjugation in strong interactions.

(iv) *Conservation law of baryon number.*

In addition to (6) and (7), we further assume

$$g_p = g_n = g_\Lambda = g, \tag{8}$$

and call it "baryonic charge". This charge has the opposite sign for antibaryon and the conservation law of this charge agrees exactly with the conservation law of so-called baryon number.*

Usually, in order to formulate the conservation law of baryon number in the frame of field theory, it is necessary to require the invariance under the phase transformation which is constant and does not depend on the space-time coordinate. It would, however, be more reasonable for the phase function to depend on the space-time coordinate, from the local character of the field quantities. Although some authors attempted to pursue the analogy between the conservation law of baryon number and that of electric charge and to relate the baryon number to the strength of the strong interaction, they did not always succeed because the associated Bose field was considered to be identified to pion field.[6]

It is remarked that in the above theory the conservation law of strangeness, or of Λ-particle is not the most general consequence of the requirement of gauge invariance. It leads merely to the conservation law of baryon number, or the sum of the number of neutron and Λ-particle. The required conservation law is guaranteed by the more special requirement that the interaction (3) should be the only elementary interaction between fundamental particles. Otherwise, it is necessary to introduce another neutral vector field associated with "strangeness charge", which might cause the mass difference between nucleon and Λ-particle.

These above considerations suggest the possible existence of a close analogy between the electromagnetic interaction and the strong interaction from the point of view of Sakata's hypothesis.

* For the leptons, $g=0$ should be assigned, the small electromagnetic correction being neglected.

§ 3. Comparison with other model

(i) Tanaka,[2] Maki,[3] and Matsumoto[4] introduced the following forms as the elementary interactions:

$$\sum_{\lambda=1}^{5} f_1^{\lambda}(\bar{N}\Omega^{\lambda}N)(\bar{N}\Omega^{\lambda}N), \quad \sum_{\lambda=1}^{5} f_1'^{\lambda}(\bar{N}\Omega^{\lambda}\tau N)(\bar{N}\Omega^{\lambda}\tau N),$$
$$\sum_{\lambda=1}^{5} f_2^{\lambda}(\bar{N}\Omega^{\lambda}N)(\bar{\Lambda}\Omega^{\lambda}\Lambda),$$
$$\sum_{\lambda=1}^{5} f_3^{\lambda}(\bar{\Lambda}\Omega^{\lambda}\Lambda)(\bar{\Lambda}\Omega^{\lambda}\Lambda),$$
$$N=\begin{pmatrix} p \\ n \end{pmatrix}. \tag{9}$$

Fermi and Yang,[7] and Tanaka[2] showed that the vector coupling, especially the product of iso-scalars, is the most favourable for the existence of the bound state only between baryon and antibaryon. In the present model, if we construct the effective Fermi couplings like (9), the only possibility is evidently the vector coupling. The necessary sign is also supplied (See § 2. (ii)). While in the models proposed by the above authors there is no principle for determining the relation between f_1^{λ}, $f_1'^{\lambda}$, f_2^{λ} and f_3^{λ}, we can determine as follows:

$$f_1^{V}=f_2^{V}=f_3^{V},^{*}$$
$$\text{others}=0, \tag{10}$$

on the basis of its analogy with the electromagnetic interaction.

(ii) Our model is not a theory of Fermion monism which is possible in Sakata's hypothesis. It is noted, however, that the neutral vector fields including the electromagnetic field are the special ones derived from the requirement of gauge invariance, which should not be considered on the same footing as pions and K-mesons.

(iii) According to Fermi and Yang,[7] and Tanaka,[2] who replaced the interaction Hamiltonian (9) by the square-well potential approximately, the potential depth necessary for the existence of pion state amounts to $\sim 26\,M$, where M is the nucleon mass and the nucleon Compton wave length is chosen as the potential range. From this a very crude estimate of the value of g^2 is obtained as

$$g^2/4\pi \sim 26, \tag{11}$$

where the meson mass is assumed to be the order of nucleon mass, though the value may depend significantly on the choice of the range, that is, the mass of the meson. If the relativistic two body problem were solved, the values of g^2 together with the mass could be determined in principle so as to give the correct values of pion mass and the effective pion-baryon coupling constant.

(iv) In the present model, the strong interaction is renormalizable in contrast with

* Mass difference between nucleon and Λ-particle is ignored.

the model starting from (9).

§ 4. Concluding remarks

We have presented a model for the strong interactions, making the best use of the assumption that the fundamental particles do not change their kind through strong interactions. Thus, analogous to the electromagnetic interactions, we have expected that the interaction Lagrangian under which the kind of particles does not change should have a close connection with the free Lagrangian, and have considered that the requirement of gauge invariance is an expression of such a connection. In this way we have been led to the introduction of a neutral vector meson.

The electromagnetic interaction has so far been distinguished from others in that it is invariant under the gauge transformation. As the result of this invariance the form of the interaction Lagrangian is restricted and the electric charge of each particle is equal. According to our formulation, these points are applied also to the strong interactions, at least to the basic ones. In this sense the electromagnetic interaction becomes less special and we could expect to get the unified view-point of the strong interactions including the electromagnetic interaction.

On the other hand, the difference between strong interactions and weak ones becomes more distinctive. Since the kind of particles always changes through the weak interactions as explained in § 1, the interaction Lagrangian for weak interactions can never be obtained from the requirement such as gauge invariance. From our standpoint, therefore, strong and weak interactions must be of quite different origins. It should also be noted that we have found a reason for the parity conservation in strong interactions but no reason in weak ones.

On account of the large coupling constant roughly estimated in (11), the meson here considered will be produced copiously in high energy collision processes and will decay rapidly into other particles (e.g. a number of pions*). In general it is expected to produce some effects both on high energy reactions and virtual processes. In this respect this meson is similar to that introduced phenomenologically by Nambu[8] in order to account for the electromagnetic size of nucleon.

In conclusion, the author wishes to express his sincere thanks to Professor S. Sakata, Professor O. Hara and Dr. T. Goto for their valuable discussions.

Appendix

— On the Stückelberg formalism —

Lagrangian is given by

$$L=L_0+L_1+L_F,$$

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* In the case of the decay into pions, decays into π^0's only and those into even number of pions are forbidden from the charge conjugation invariance and charge independence.

where

$$L_0 = -(1/4) f_{\mu\nu} f_{\mu\nu} - (1/2) m^2 A_\mu^2 - m A_\mu (\partial_\mu B) - (1/2)(\partial_\mu B)^2 - (1/2)\chi^2,$$

$$L_1 = j_\mu A_\mu,$$

$$L_F = -\sum_{\alpha=p,n,\Lambda} \bar{\psi}_\alpha (\gamma_\mu \partial_\mu + M_\alpha) \psi_\alpha, \tag{A·1}$$

where

$$f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

$$j_\mu = \sum_\alpha i g_\alpha \bar{\psi}_\alpha \gamma_\mu \psi_\alpha,$$

$$\chi = \partial_\mu A_\mu + mB, \qquad (\Box^2 - m^2)\chi = 0.$$

In addition to (A·1) the subsidiary condition must be satisfied,

$$\chi \Psi = 0. \tag{A·2}$$

The gauge transformation is given by

$$\psi_\alpha(x) \to e^{ig_\alpha \Lambda(x)} \psi_\alpha(x),$$

$$\bar{\psi}_\alpha(x) \to e^{-ig_\alpha \Lambda(x)} \bar{\psi}_\alpha(x),$$

$$A_\mu(x) \to A_\mu(x) + \partial_\mu \Lambda(x),$$

$$B(x) \to B(x) - m\Lambda(x), \tag{A·3}$$

where

$$(\Box^2 - m^2)\Lambda(x) = 0$$

is satisfied.

Lagrangian (A·1) and the subsidiary condition (A·2) is invariant under this transformation. The form for L_0 given above is particularly convenient since the invariance is shown explicitly when put into the form

$$L_0 = -(1/4) F_{\mu\nu} F_{\mu\nu} - (1/2) m^2 U_\mu^2 - (1/2)\chi^2,$$

where

$$F_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu,$$

$$U_\mu = A_\mu + (1/m)\partial_\mu B, \tag{A·4}$$

which are evidently invariant.

From (A·1) the equations of motion are derived as follows,

$$(\Box^2 - m^2) A_\mu(x) = -j_\mu(x),$$

$$(\Box^2 - m^2) B(x) = 0,$$

$$\gamma_\mu(\partial_\mu - ig_\alpha A_\mu(x))\psi_\alpha(x) + M_\alpha \psi_\alpha(x) = 0.$$

From the invariance of (A·1) under (A·3), the conservation law of current is derived in the well-known procedure, i.e.

$$0=\delta L(x)=[-\partial_\mu j_\mu(x)+m(\Box^2-m^2)B(x)]\delta\Lambda(x)$$
$$-[(\Box^2-m^2)A_\mu(x)+j_\mu(x)]\partial_\mu\delta\Lambda(x)$$
$$-f_{\mu\nu}(x)\partial_\mu\partial_\nu\delta\Lambda(x)$$
$$=-(\partial_\mu j_\mu(x))\delta\Lambda(x),$$

thus

$$\partial_\mu j_\mu(x)=0. \tag{A·5}$$

The equations (A·4) represent the relation of this formalism to the conventional one. The interaction Lagrangian is given usually by

$$j_\mu(x)U_\mu(x),$$

which can be put into the form

$$j_\mu(x)A_\mu(x)+(1/m)j_\mu(x)\partial_\mu B(x).$$

The second term is reduced to a 4-divergence owing to (A·5) and has no effect at all, suggesting the equivalence of the two formalisms.

We go further by developing the canonical formalism. Introducing the canonical momenta

$$\pi_\mu(x)=if_{4\mu}(x)+i\delta_{4\mu}\chi(x),$$
$$\pi_B(x)=\dot{B}(x)+imA_4(x),$$
$$\pi_\alpha(x)=i\psi_\alpha{}^*(x),$$

we can obtain the Hamiltonian,

$$H=(1/2)\boldsymbol{\pi}^2+(1/2)(\mathrm{curl}\,\boldsymbol{A})^2+(1/2)m^2\boldsymbol{A}^2+(1/2)\pi_B{}^2$$
$$+(1/2)(\nabla B)^2+i\boldsymbol{\pi}\cdot(\nabla A_4)-imA_4\pi_B+m\boldsymbol{A}\cdot\nabla B$$
$$-j_\mu A_\mu+\sum_\alpha\psi_\alpha{}^*(-i\boldsymbol{\alpha}\cdot\nabla+M_\alpha\beta)\psi_\alpha.$$

We perform the unitary transformation

$$F(x)\to e^{iG}F(x)e^{-iG},$$
$$G=(1/m)\int B(x)[\mathrm{div}\,\boldsymbol{\pi}(x)-ij_4(x)]\,d\boldsymbol{x},$$

then

$$(1/m)\dot{\chi}(x)\to\pi_B(x).$$

π_B in the new representation is eliminated by using

$$\dot{\chi}\,\Psi=0,$$

then the Hamiltonian reduces to

$$\begin{aligned} H &= H_0 + H' + H_1 + H_F, \\ H_0 &= (1/2)\boldsymbol{\pi}^2 + (1/2)(\mathrm{curl}\ \boldsymbol{A})^2 + (1/2)m^2\boldsymbol{A}^2 + (1/2m^2)(\mathrm{div}\ \boldsymbol{\pi})^2, \\ H' &= (1/m^2)j_0\ \mathrm{div}\ \boldsymbol{\pi} + (1/2m^2)j_0^2, \\ H_1 &= -\boldsymbol{j}\cdot\boldsymbol{A}, \\ H_F &= \sum_\alpha \psi_\alpha^*(-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + M_\alpha\beta)\psi_\alpha, \end{aligned} \qquad (\mathrm{A}\cdot 6)$$

apart from space divergences. This is the same form as the one derived from the conventioanl formalism (for example, in Wentzel's book[9]), proving the equivalence rigorously.

We further perform the unitary transformation,

$$F(x) \to e^{iG'}F(x)e^{-iG'},$$

$$G' = \int dx \int dx'\ j_0(x)U(\boldsymbol{x}-\boldsymbol{x}')\,\mathrm{div}\ \boldsymbol{A}(x'),$$

$$U(\boldsymbol{x}) = (1/4\pi)\,(e^{-m|\boldsymbol{x}|}/|\boldsymbol{x}|),$$

then

$$H_0 + H' \to H_0 + H_C,$$

where

$$\int H_C(x)\,dx = (1/2)\int dx \int dx'\ j_0(x)U(\boldsymbol{x}-\boldsymbol{x}')j_0(x'). \qquad (\mathrm{A}\cdot 7)$$

which is the analogue of the Coulomb interaction. It is noted that there remains the longitudinal part even at this stage, in contrast with the case of the electromagnetic interaction.

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