ON THE ANTENNA PROBLEM*

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The following remarks are made apropos to Brillouin's recent discussion[1] of mathematical difficulties involved in the retarded potential method of solving the antenna problem. The approximations involved in the actual solution of the final integral equation may be a source of far greater errors than the approximations in the equation itself (such as 7A and 7B of Brillouin's paper). Fig. 1 shows the first maximum

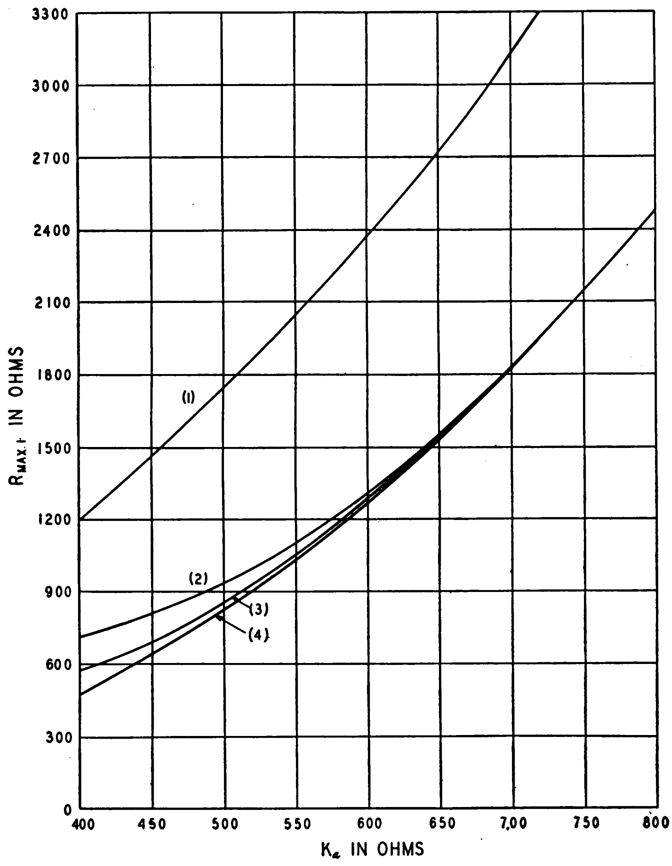

FIG. 1. The first maximum input resistance of center fed cylindrical antennas in free space as a function of the average characteristic impedance: (1) is Hallén's approximation to the solution of the integral equation for infinitely thin cylindrical shells, (2) is Schelkunoff's approximation to the solution of Maxwell's equations for solid cylinders, (3) is Schelkunoff's approximation to the solution of Maxwell's equations for infinitely thin cylindrical shells, (4) is Gray's recent approximation to the solution of the integral equation for infinitely thin cylindrical shells. $K_a = 120\ (\log 2l/a - 1) = 60\Omega - 120$ ohms, where l is the length of each half of the antenna, Ω is the parameter in Brillouin's paper. The range of K_a in the figure corresponds to $38 < l/a < 1080$ and $8.66 < \Omega < 15.33$.

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[1] Leon Brillouin, *The Antenna Problem*, Quarterly of Applied Mathematics, **1**, 201 (1943).

resistance of center fed cylindrical antennas in free space. Curves (1), (3) and (4) refer to infinitely thin cylindrical shells (for which the antenna current vanishes at the ends); (1) is Hallén's approximate solution of the integral equation, (3) is Schelkunoff's approximate solution of Maxwell's equations,[2] and (4) is Gray's recent approximation to the solution of the integral equation.[3] The difference between (1) and (4) is due solely to the difference in the methods of successive approximations. L. V. King's approximation to the integral equation gives $R_{\max,1}=4000$ for $K_a=420$; this is considerably higher than even Hallén's approximation. Curve (2) was calculated by the same method as (3) but for solid cylinders. The "end" or "cap" capacitance was estimated as explained elsewhere;[4] this estimated capacitance is probably higher than the actual capacitance and in that region where (2) and (3) diverge the true curve is likely to be somewhat lower. In the case of hemispherical ends, curve (2) is raised still further. In all these curves it is assumed that there is no excessive localized capacitance in the vicinity of the input terminals; the effect of such capacitance is to lower these curves. Incidentally it means that one should never assume a "point generator" except when the input terminals are tapered to mere points.

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[2] S. A. Schelkunoff, *Electromagnetic Waves*, D. Van Nostrand Co., New York, 1943, Chapter 11.

[3] Marion C. Gray, *A modification of Hallén's solution of the antenna problem*, Journal of Applied Physics, **15**, No. 1, Jan. 1944.

[4] Reference 2, page 465.