

Transactions Letters

On the Capacity Formula for Multiple Input–Multiple Output Wireless Channels: A Geometric Interpretation

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Abstract—The capacity of multiple input, multiple output (MIMO) wireless channels is computed for Ricean channels. The novelty is a geometrical (ray-tracing) interpretation of the MIMO channel capacity formula to find array geometries which greatly enhance channel capacity compared to single input–single output (SISO) systems.

Index Terms—Array signal processing, channel capacity, land mobile radio cellular systems.

I. INTRODUCTION

A MULTIPLE input–multiple output (MIMO) wireless communications channel with a matrix transfer function of independent complex Gaussian random variables has an information-theoretic capacity which grows linearly with the number of antenna array elements n , for fixed power and bandwidth [1].

For line-of-sight (LOS) channels, we use ray tracing to construct a matrix transfer function (channel response) explicitly for some example environments and find array geometries which result in channel matrices with close to n nonnegligible eigenvalues, with corresponding high capacity. The LOS matrix channel response will change as the receiver is moved, so that a capacity distribution is obtained from the ensemble of sample matrix elements at different receiver locations. Also, a Rayleigh matrix may be added to the LOS channel matrix to form a matrix of Ricean scalars, from which a capacity distribution is obtained. In either case, we define an outage threshold x (say 0.01), and define C_x to be that capacity for which $\text{Prob}\{C > C_x\} = 1 - x$.

In what follows, the MIMO channel capacity formula is used to compute the capacity for three example geometries, plus one Ricean example, with a discussion of the results.

II. CAPACITY CALCULATIONS AND ARRAY GEOMETRY

The capacity in b/s/Hz of a MIMO wireless system with n_T transmit antennas and n_R receive antennas with an average received SNR ρ (independent of n_T) at each receive antenna

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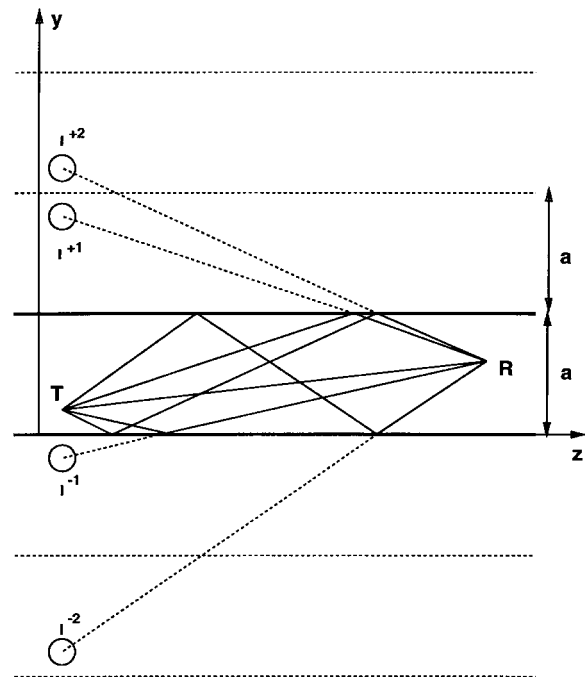


Fig. 1. Images in street canyon—top view $a = 25$ m, $\lambda = 1/3$ m, $T(x, y, z) = (1, 15, 0)$ m, $R(x, y, z) = (1, 5, 30)$ to $(1, 6, 40)$.

was obtained in [1] as $C = \log_2(\det[I_{n_R} + (\rho/n_T)HH^*])$, where the normalized channel matrix H contains complex scalars with unity average power loss, and H^* is the complex conjugate transpose of H . The capacity is expressed in b/s/Hz in the narrow-band limit with no frequency dependence. Normalization is achieved by dividing out the free space power loss and setting the parameter ρ to the desired SNR.¹ This result assumes that H is unknown to the receiver but n_R and ρ are known [1], [2]. The transmitted data has been demultiplexed into substreams which are separately independently coded and modulated on each antenna [6], so that the transmission from each antenna is different. The practicality of such MIMO wireless systems, using space-time coding with no bandwidth penalty, is illustrated in [3]–[5] and [7].

A. Line-of-Sight Channels

We first consider an environment with only free space nonfading LOS propagation and T and R arrays of $n_T =$

¹This avoids the need to compute absolute propagation loss and then select the transmitted power to obtain the desired SNR.

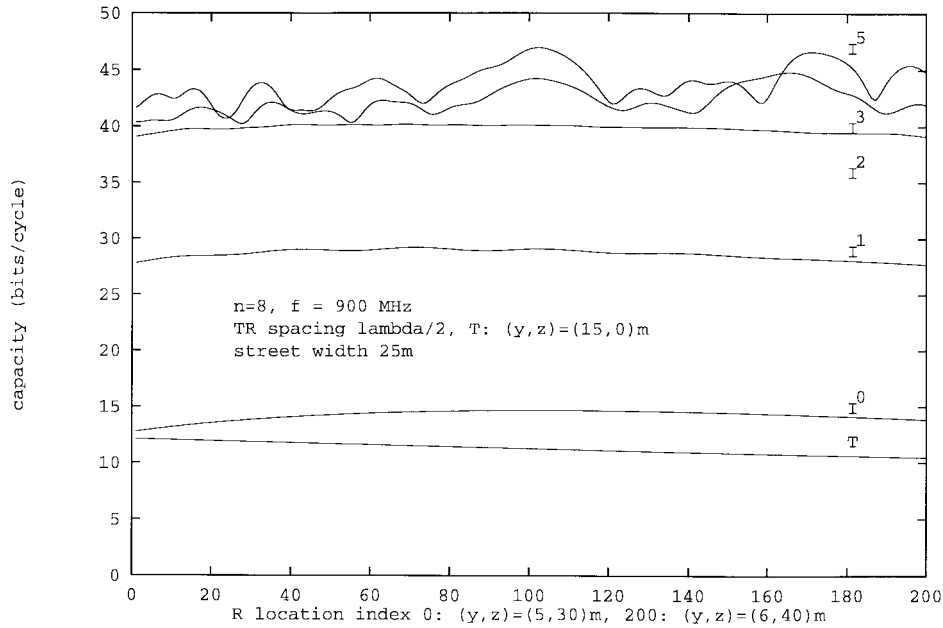


Fig. 2. Capacity versus R location in street canyon T and R height $x = 1$ m, $\rho = 20$ dB, $\lambda = 1/3$ m.

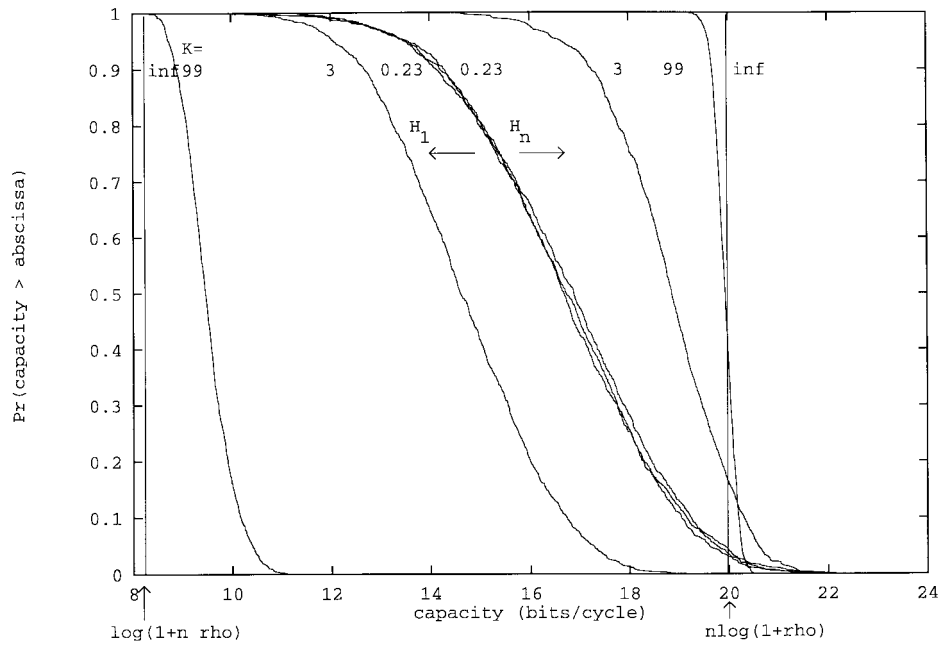


Fig. 3. Capacity on Ricean channels, $n = 3$.

$n_R = n$ antennas. The base and subscriber ends of the link are designated as T and R , respectively, but reciprocity applies. For LOS propagation, and a narrow-band channel at fixed carrier frequency $f_c = c/\lambda$, ray-tracing from T to R yields the channel transfer function matrix $H = H_{\text{LOS}}$ with complex scalar entries

$$H_{ik} = |T_1 - R_1| \frac{\exp(-j2\pi|T_i - R_k|/\lambda)}{|T_i - R_k|} \quad (1)$$

where T_i, R_i are coordinate vectors for the i th element of

T, R . H_{ik} is normalized by the distance between the reference locations T_1, R_1 , so that $H_{1,1} = 1$ and the absolute attenuation need not be calculated.

If the antennas are spaced less than $\lambda/2$ apart at both T and R , $H_{ik} = e^{j\theta_{ik}} \simeq e^{j\theta}$ for fixed θ for all i, k , $(HH^*)_{ik} \simeq n$, and $C = \log_2(1 + n\rho) = C_{\text{log}}$, so that C_{log} increases logarithmically with n . For this case, $H = H_{\text{LOS}} = H_1$ is of rank 1, and the capacity gain is essentially due to the n -fold array gain in ρ .

For arrays of n more widely spaced antennas at T and/or R , the complex scalars H_{ik} all have magnitude near one but different phases θ_{ik} . For θ_{ik} so that $HH^* = nI_n$, $H =$

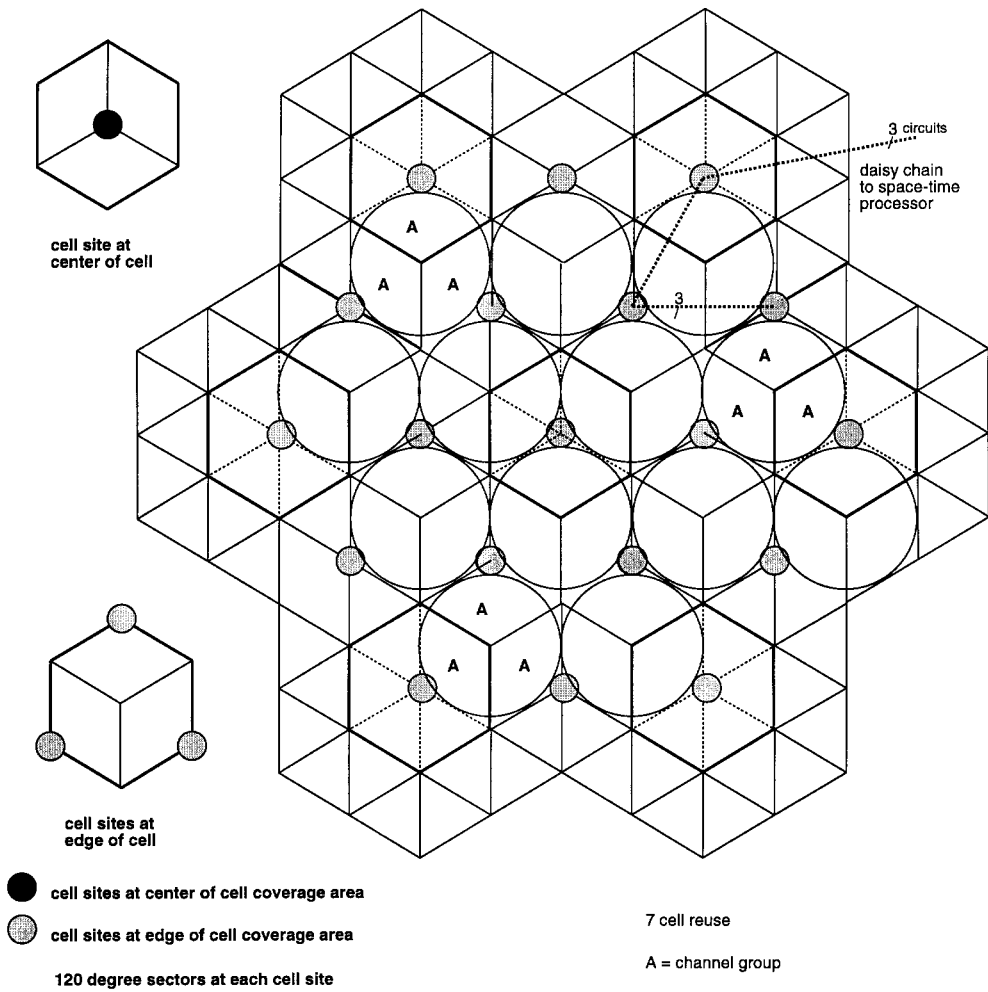


Fig. 4. Cell sites at center and edge of coverage area.

$H_{LOS} = H_n$ is of rank n , and $C = n \log_2(1 + \rho) = C_{lin}$, so that C_{lin} increases linearly with n . An example is $\theta_{ik} = \frac{\pi}{n}[(i - i_0) - (k - k_0)]^2$ where for $n = 2$ and $i_0 = k_0 = 0$, $H_{max} = \begin{pmatrix} 1 & j \\ j & 1 \end{pmatrix}$, and the corresponding array geometry is two linear arrays broadside to each other. In what follows, we show three more examples of geometric arrangements for which $H_{LOS} \simeq H_n$.

One such arrangement is an n -element T array spread along an arc at angles ϕ_{k-1} for $k = 1, \dots, n$, and a linear R array oriented broadside to the center of the arc. From (1), for arc radius $D \gg \lambda$, and interelement spacing z_r , $H_{ik} = \exp(j \frac{2\pi z_r}{\lambda} [(i-1) - \frac{n-1}{2}] \sin \phi_{k-1})$ which corresponds to the autocorrelation $R_{xx}(i-k) = \frac{1}{2\Delta} \text{Re} \int_{-\Delta}^{\Delta} e^{j \frac{2\pi z_r}{\lambda} (i-k) \sin \beta} d\beta$ from [8, eq. (A-13)] specialized for the n discrete angles of arrival ϕ_{k-1} . $C \simeq C_{lin}$ when the angle subtended by the arc $2\Delta = \phi_{n-1} - \phi_0$ is consistent with the beamwidth $2\Delta = \lambda/z_r$ at which $R_{xx}(i-k) = 0$. The radiation pattern of the R array with all elements in phase $E(\phi) = \frac{\sin(n\gamma/2)}{n \sin(\gamma/2)}$, where $\gamma = 2\pi z_r \sin \phi / \lambda$. For $z_r = \lambda/2$, and the n array elements of T placed along the arc at ϕ_{k-1} exactly in the nulls of R , $H_{ik} = \exp(j \frac{2\pi}{n} (i - i_0)(k - k_0))$ where $i_0 = k_0 = \frac{n+1}{2}$, for which $HH^* = nI_n$ so that $H = H_n$ and $C = C_{lin}$.

The second example arrangement is an n -element T array with elements spread evenly around a circle of radius $D \gg \lambda$,

and a similar R array on a circle of radius $D_r \leq \lambda$ at the center of the T array. From (1), $H_{ik} = \exp(j \frac{2\pi D_r}{\lambda} \cos[(i-k) \frac{2\pi}{n}])$. For $D_r = \lambda/2$, the T elements are not in the nulls of the R array, but the elements of HH^* for which $i-k$ is odd are zero, and the off-diagonal even elements are approximately $0.3n$. Nonetheless, the capacity approaches C_{lin} , consistent with the observation [8] that small correlation (<0.3) has negligible effect on performance. Further calculations confirm that the capacity is robust in the presence of rotation or lateral movement of R or perturbations in the placement of the T elements.

The third example is an urban street geometry with two parallel reflectors representing the building walls separated by the street width a (Fig. 1). $I^{\pm m}$ represents an image due to m specular reflections from the walls, and I^0 is the "ground reflection" image not visible in the figure. For this street geometry, the elements of H may be written

$$\frac{H_{ik}}{|T_i - R_k|} = \frac{\exp(-j2\pi|T_i - R_k|/\lambda)}{|T_i - R_k|} + \sum_{k=-m}^m \Gamma^k \frac{\exp(-j2\pi|I_i^k - R_k|/\lambda)}{|I_i^k - R_k|} \quad (2)$$

where Γ is the amplitude reflection coefficient,² m is the maximum number of reflections considered, and we have assumed isotropic array elements.

For eight-element linear arrays with $\lambda/2$ spacing oriented perpendicular to the street, Fig. 2 shows how the capacity increases as more images are added (and the angular spread of rays increases) and approaches C_{lin} with seven images ($|m| \leq 3$) plus T . Furthermore, the received signal envelope looks increasingly Rayleigh-like as more images are added.

B. Ricean Channels

Next we consider the capacity for Ricean channels, where the deterministic component H_{LOS} is fixed as either H_1 or H_n . We follow the simulation methods of [1] using the normalized Ricean channel matrix $H = (aH_{\text{LOS}} + bH_{\text{Rayleigh}})$ with $a^2 + b^2 = 1$ and Ricean K -factor $K = a^2/b^2$. The results for $n = 3$ with $\rho = 100$ (20 dB) (Fig. 3) quantify how for closely spaced ($\leq \lambda$) array elements at both T and R , and no reflectors or scatterers such that $H_{\text{LOS}} \simeq H_1$, the capacity decreases with increasing K toward $C = C_{\text{log}}$. However, for array geometries such as the above examples, where $H_{\text{LOS}} \simeq H_n$, the capacity increases with increasing K toward $C = C_{\text{lin}}$. For $K = 0$, C corresponds to that obtained in [1].

III. DISCUSSION

Capacities approaching $C_{\text{lin}} = n \log_2(1+\rho)$ can be achieved for MIMO channels in an LOS (non-Rayleigh) environment by spreading out the elements of T either explicitly (by placing one element of T at each of n sites), or implicitly (by adding reflectors which create images of T). The results of the second example suggest that three sectors in a conventional

²In general, the Γ are different for each image, since they depend on the angles of incidence and reflection, and the surface characteristics. Here we assume Γ has the same constant value 0.6 for all reflections, except the ground reflection Γ^0 which is set to -1 . This approximation is sufficient to illustrate the capacity gain.

cellular system can be combined to form one “edge-excited” (inward-facing) cell (Fig. 4) to enhance capacity for R not close to a base station. This is reminiscent of soft handoff in CDMA systems where multiple base stations serve one mobile, except that in this case, each base station carries a different substream of the transmitted data. The results of the third example suggest that in the absence of reflectors, we may use n antennas at each of n sites, thus replicating the effect of images of the n -element T array. The ray-tracing channel model for $H = H_{\text{LOS}} + H_{\text{Rayleigh}}$ described here may be useful for the performance evaluation of MIMO wireless systems which use spatial diversity through space-time coding to exploit the available capacity with no bandwidth penalty (e.g. [2], [5]–[7]).

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