On the Capacity of Cellular CDMA and TDMA over Non-dispersive Channels[*]

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Abstract. We investigate the interuser interference in cellular code-division multiple access (CDMA) and time-division multiple access (TDMA) systems and compare the two multiple access techniques at different levels of user density per cell. The study includes the interference both on the downstream channel (from base station to users) and on the upstream channel (from users to base station). The purpose is to compare the two multiple access techniques when the total bandwidth occupancy and the maximum number of users per cell are the same. The results indicate that on a nondispersive radio channel with a path loss that is proportional to the fourth power of the distance, the interuser interference is approximately 10 dB higher in CDMA. The implication of this result, which holds for both the downstream and the upstream channels, is that for the same total bandwidth and number of users per cell, TDMA gives substantially superior bit error rate (BER) performance. This is confirmed by our numerical results evaluated for both additive white Gaussian noise (AWGN) channels and memoryless flat Rayleigh fading channels. Alternatively, CDMA needs approximately 10 times more bandwidth to achieve the same signal-to-interference (S/I) ratio, a result which may seem surprising, because the fact that CDMA employs a frequency reuse factor of 1 is often given as an evidence that this technique can support a higher number of users than TDMA in a given geographic area and frequency bandwidth.

1 INTRODUCTION

Code-division multiple access (CDMA) has become very popular in recent years, not only in mobile and personal communications, but also in other applications including fixed wireless access and satellite systems. While it is undeniable that this multiple access technique has the virtue of easing frequency planning and several other interesting features, the capacity of CDMA and its comparison to time-division multiple access (TDMA) has been a very controversial issue often dominated by commercial interests. In addition, comparisons are usually made between systems in which TDMA or CDMA is only one ingredient among many others, and the attributes of a particular system design and those of the multiple access technique involved are often mixed up. The consequence of this is that the capacity of CDMA is still not a well understood issue which needs to be revisited.

The purpose of the present paper is to shed some light on the CDMA capacity and compare it to that of a TDMA scheme with the same total bandwidth. Our analysis is very general, and does not make any reference to any particular system design. Before describing the cellular CDMA schemes considered in this study, note that there are essentially two basic CDMA techniques. The first one directly derives from direct-sequence spread-spectrum (DS-SS) systems which were originally devised for military communication systems [1], [2]. DS-SS has two attractive features for those applications: The first feature is the low intercept probability which results from the fact that a DS-SS signal is virtually buried in background noise both in the time and in the frequency domains. The second feature is the robustness of DS-SS signals to intentional or unintentional jamming. It is obvious that the spreading sequence in these applications must be pseudo-random and difficult to replicate by an unauthorized user, because if such a user knows and can use the spreading sequence, the whole process becomes useless. In the sequel, we will refer to CDMA systems which use pseudo-random

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[*] Part of this work has been presented at the IEEE Vehicular Technology Conference, Houston, 1999 and at the IEEE Vehicular Technology Conference Fall, Amsterdam, 1999.

(pseudo-noise) spreading sequences as pseudo-noise CDMA (PN-CDMA).

The second basic CDMA technique is also based on direct sequence spectral spreading, but uses a set of orthogonal sequences, e.g., Walsh-Hadamard (WH) sequences [3], instead of pseudo-noise sequences. This technique is referred to as orthogonal CDMA (OCDMA). There is no randomness in this case, and the spreading sequence repeats itself from one symbol to the next. Furthermore, since the spreading sequences are orthogonal, there is no mutual interference between different user signals. It is instructive to note that, as pointed out in [4], OCDMA can be viewed as a multidimensional modulation whose bandwidth occupancy and performance are the same as those of the component two-dimensional modulation.

The paper is organized as follows: In the next section, we briefly describe the principle of TDMA and CDMA, and recall their basic features. Next, in section 3, we present the cellular TDMA and CDMA schemes that we consider in our analysis, and we compute the interuser interference. Section 4 reports bit error rate (BER) results computed for white additive Gaussian noise (AWGN) channels and memoryless flat Rayleigh fading channels, and section 5 gives a discussion of our results and our conclusions.

2 A BRIEF REVIEW OF TDMA AND CDMA

To review the basics of TDMA and CDMA, it is appropriate to start with the single-cell case. TDMA consists of sharing the available resources between different users by assigning to them different time slots of the same data stream. For simplicity, we will focus here on a simple TDMA scheme in which the data stream is formatted into frames of N time slots and each user gets one time slot per frame. That is, the channel resources are equally shared between N different users. The total bandwidth of the transmitted signal in this scheme is N times the bandwidth of the individual user signals. Let R_S designate the symbol rate of each user, and W Hz the bandwidth required to transmit this symbol rate in single-user transmission. The TDMA scheme at hand thus transmits a symbol rate of NR_S baud using a bandwidth of NW Hz. Of course, the overhead needed for signal framing is neglected in this simple example. The modulation scheme is entirely immaterial throughout our discussion, and therefore no particular modulation scheme is assumed except in section 4 where we report BER performance results for binary phase-shift keying (BPSK) modulation.

An equivalent OCDMA scheme assigns a periodic sequence of length N to each user, where all sequences are mutually orthogonal. The period of the sequences coincides with the symbol interval, giving thus N chips per symbol. This process spreads the transmitted signal bandwidth by a factor of N. Since the number of orthogonal sequences of length N is exactly N, this OCDMA scheme can accommodate N users. In other words, exactly as in TDMA, OCDMA can accommodate N users when the available bandwidth is N times that required by one user, and this is achieved without any mutual interference. This is perfectly consistent with the observations made in [4] where OCDMA was viewed as a multidimensional modulation which has the same performance and bandwidth efficiency as the component 2-D modulation.

We now analyze PN-CDMA assuming a nondispersive channel and perfect power control which implies that all user signals are received with the same signal power. Let $p_{k\ell}$ designate the portion of the kth user's PN sequence that is used during the ℓth symbol period. Since the $p_{k\ell}$ sequences are uncorrelated PN sequences of length N, we have

$$\mathrm{E}\left(p_{k\ell}\, p_{k\ell'}\right) = N\delta_{\ell\ell'} \tag{1}$$

where δ denotes the Kronecker delta which takes the value of 1 for $\ell = \ell'$ and of 0 for $\ell \neq \ell'$. Therefore, the correlator preceding the decision circuit at the receiver gives a useful signal value of N (times the transmitted symbol), while the average interference value from any other user is 0. The mean-squared value of this interference is

$$\mathrm{E}\left\{\left(p_{k'\ell}\, p_{k\ell}\right)^2\right\} = N \tag{2}$$

The signal-to-interference power ratio (S/I) is therefore $N^2/N = N$, or equivalently the interference power normalized by the useful signal power is 1/N. Interference from different users adds up, and the normalized interference power becomes n/N when the number of users is n. If we assume that the number of active users is N, then each user will get interference from the other N-1 users, and the total normalized interference power will be (N-1)/N, which indicates that for large N, the interference power is as large as the useful signal power. Obviously, no operation is possible at such a high interference level even with the best error correction codes, and this means that PN-CDMA accommodates a much smaller number of users than TDMA and OCDMA. If the interference power is to be limited to for example 20 % of the signal power, then only N/5 users can be accommodated, and this represents a capacity decrease by a factor of 5 with respect to TDMA and OCDMA. The capacity of PN-CDMA is therefore not a fixed number; It depends on the interference level that one is prepared to accept. To summarize the single-cell case, while both TDMA and OCDMA can accommodate N users without any mutual interference on a channel whose bandwidth is N times that required by a single user, PN-CDMA leads to interuser interference as soon as there are two users on the channel, and the interference power grows linearly with the number of users. As a result, the number of users that can be accommodated is substantially lower than N if the performance is to be kept at an acceptable level.

3 FREQUENCY REUSE AND INTERUSER INTERFERENCE

In the previous section, we analyzed the capacity of CDMA in a single cell. We now consider cellular systems where the basic question is the achievable frequency reuse factor among cells, along with the number of users that can be accommodated in each cell. To perform this analysis, we will assume a hexagonal cell pattern commonly used in mobile radio systems. In TDMA, two adjacent cells cannot use the same carrier frequency, and therefore the smallest number of frequencies is 3. If the S/I ratio with this frequency reuse factor is not sufficient, a frequency reuse factor of 4 or 7 can be used instead. Such frequency reuse patterns are commonly found in state-of-the-art textbooks in digital mobile radio systems, e.g. [5]. In what follows, we will assume a frequency reuse factor of 4, following the frequency reuse pattern of figure 1. Each cell in this figure is labeled with a letter which indicates the frequency allocated to it. Focusing on an A-cell (a cell with a label A), the 6 cells surrounding it are labeled B, C, D, B, C, and D, respectively. Note that in addition to the 6 nearest neighbors, also the second nearest neighbor cells employ different frequencies. The nearest base stations which employ the same frequency are at a distance 4R, where R designates the cell radius (the radius of the largest circle that fits inside the cell).

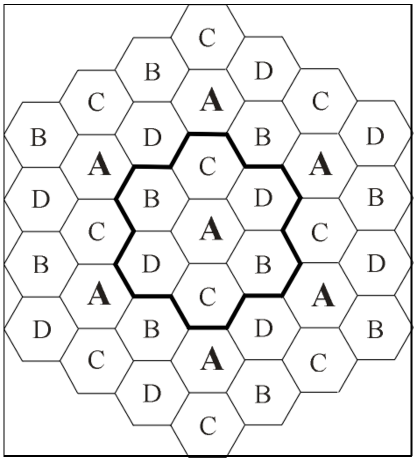

Figure 1: Hexagonal cell pattern with a frequency reuse factor of 4.

3.1 CELLULAR TDMA

Let us now examine the interference problem of TDMA in this cellular environment. Interuser interference is obviously limited to cells that employ the same carrier frequency, i.e., an A-cell gets interference only from other A-cells, a B-cell from other B-cells, and so forth. If the clocks of different cells are mutually synchronized, then the interference will be limited to users that share common time slots. In other words, there is a one-to-one correspondence between interfering users in that case. However, it is quite unlikely that clocks of different cells will be synchronized in practice. In the presence of clock frequency offset, a given user will sequentially interfere with all other users of the interfering cells, the interference resembling a periodic impulse noise when the user density is low in the interfering cells. The period is directly given by the clock frequency offset normalized by the symbol period. To proceed further, we assume that all base stations transmit the same signal level, and that the transmit power control in the upstream direction is perfect so that a base station receives the same signal level from all users in its cell.

3.1.1 Downstream channel

We first investigate the interuser interference on the downstream channel (from base station to users), assuming that the propagation path loss is proportional to the fourth power of the distance. To do this, we focus on a user located on the cell boundary in the direction of one of the nearest cells which employ the same frequency. Such a user is at a distance R from its home base station and at a distance 3R from the nearest interfering base station. Assuming that there are N active users in that cell, i.e., the cell is full, the interference power (normalized by the useful signal power S) from this particular cell for the user at hand is

$$\frac{I_{d,3R}}{S} = \frac{1}{9^2} \tag{3}$$

Considering the other 5 nearest cells with the same frequency assignment, the total interference level is easily shown to be

$$\frac{I_d}{S} = \frac{1}{9^2} + \frac{2}{13^2} + \frac{2}{21^2} + \frac{1}{25^2} \tag{4}$$

The denominators on the right-hand side of (4) are the fourth powers of the normalized distances of the interfering base stations to the user considered. Neglecting interference from cells at a larger distance, the S/I ratio for the downlink is

$$\frac{S}{I_d} = 33 \tag{5}$$

or $S/I_d = 15.2$ dB in the dB scale.

Note that the foregoing analysis holds when all the interfering cells are full. Otherwise, the downlink S/I ratio of 15.2 dB will be valid only when the time slot assigned to the user considered is also simultaneously used in all of the 6 nearest cells that employ the same frequency.

3.1.2 Upstream channel

Now we compute the S/I ratio on the upstream channel (from users to base station). Let us consider two cells that operate at the same frequency, and assume that the cells are full (N users per cell). The useful signal, received during a given symbol interval by the base station of the first cell, is disturbed by the user in the second cell that transmits during the same symbol interval. The interference caused by that particular user depends on its distance d to the base station of the first cell, and on its distance r to its home base station.

Assuming ideal power control, the transmit power of a user at a distance r from its base station is proportional to r^4, so that this base station receives a power that is independent of the user position. Normalizing by one the energy that a base station receives from its user during the considered symbol interval, the energy received during the same symbol interval from the disturbing user at distance d is equal to $(r/d)^4$, with r denoting the distance between the interfering user and its home base station. The average interfering energy per symbol is obtained by averaging $(r/d)^4$ over the position of the interfering user within its cell. Denoting by D the distance between the base stations of the considered two cells, one obtains

$$d^2 = D^2 + 2Dr\cos\theta + r^2 \qquad (6)$$

where θ is the angle indicated in figure 2. With R denoting the cell radius, the average interfering energy per symbol caused by one interfering cell, whose base station is at distance D from the considered base station, is given by

$$\frac{I_{u,cell}(D)}{S} = \frac{2}{3\sqrt{3}R^2}\iint_{cell}\frac{r^4}{\left(D^2 + 2Dr\cos\theta + r^2\right)^2}\, r\,dr\,d\theta \qquad (7)$$

Taking into account only the nearest 6 interfering cells (with D = 4R), the total interfering energy per symbol Iu is given by $I_u = 6.I_{u,cell}(4R)$. Evaluating (7) by means of a numerical integration routine, we find that the uplink S/I ratio is

$$\frac{S}{I_u} = 82.8 \qquad (8)$$

or S/I_u = 19.2 dB. Observe that the uplink S/I ratio is 4 dB larger than the downlink S/I ratio. This is not surprising, because the S/I_u ratio given by (8) is an average value which holds for all users, whereas the S/Id ratio given by (5) corresponds to a user located in one of the worst positions. Higher S/I_d ratio values are obtained with user locations closer to their home base station. Furthermore, note that the S/I ratio of 19.2 dB for the uplink will be valid only when the considered time slot is also simultaneously used in all of the 6 nearest cells that employ the same frequency. Otherwise, the S/I ratio will be higher.

It is also interesting to evaluate the worst-case S/I ratio on the uplink which corresponds to 6 interfering users located at the nearest points of the interfering cells. Each of these users is at a distance of 4R to the BS of interest, and we get $(S/I_u)_{worst\text{-}case}$ = 11.4 dB if the 6 interfering users are active simultaneously. This is 7.8 dB lower than the average S/I ratio given by (8).

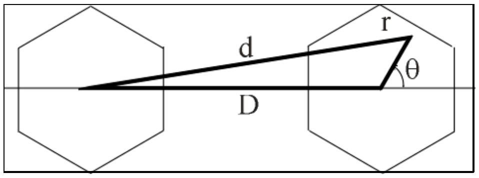

Figure 2: Illustration of interfering user location.

3.2 CELLULAR OCDMA

One possible CDMA concept consists of sharing resources in each cell using OCDMA and assigning different frequencies to adjacent cells as in TDMA. The spectral efficiency of such a scheme is identical to that of TDMA, because as indicated in the single-cell case, both multiple access techniques can accommodate the same number of users in a given bandwidth. Furthermore, the frequency reuse factor is a function of the S/I ratio that needs to be achieved, and for a given frequency reuse factor, cellular OCDMA gives exactly the same S/I ratio as TDMA when the cells are fully loaded. In fact, the same reasoning holds for any multiple access scheme which assigns orthogonal signal waveforms to different users. But this cellular CDMA concept does not exhibit the eased network planning feature of cellular CDMA with a frequency reuse factor of 1 and will not be further considered in this paper.

3.3 CELLULAR PN-CDMA

Another cellular CDMA system concept consists of assigning uncorrelated PN spreading sequences to different users whether they are located in the same cell or in different cells and use the same frequency in all cells. (We do not address the issue of generating uncorrelated PN sequences for different users, a subject that is well documented in the technical literature [6].) Contrary to the cellular OCDMA outlined in the previous subsection, this scheme does offer the feature of eased network planning, but with respect to the single-cell case, users in this scheme are also subject to intercell interference. More specifically, each user interferes with all other users in the same cell, but also with all users of all other cells that are sufficiently close to the cell considered. Since the interference level is increased with respect to the single-cell case, then also the spreading factor must be increased in order to accommodate the same number of users in each cell. For example, by expanding the spreading factor from N to 4N, the interference between two users is reduced from 1/N to 1/4N. In such a PN-CDMA scheme, intracell interference alone will lead to a normalized interference level of 0.25 as the number of users in that cell reaches N, and the effective interference level will be much higher in practice due to interference from other cells. The hybrid CDMA whose description is now to follow has exactly the same intercell interference properties without being subject to any intracell interference, and therefore the full PN-CDMA approach is of little interest and will not be considered any longer in this study.

3.4 HYBRID CDMA

A cellular CDMA concept which combines the feature of eased network planning (frequency reuse factor of 1 between cells) and orthogonal signaling in each cell is a two-layer CDMA in which the transmitted signal is first spread through multiplication by a user-specific WH sequence with N chips per bit. Then, the frequency spread user signal is multiplied by a cell-specific PN sequence either without further spreading (i.e., with the same chip rate) or with further spreading by a factor of 2 or 4. Clearly, this scheme is a particular combination of OCDMA and PN-CDMA. The WH sequences in this cellular CDMA concept are commonly known as *channelization codes*, and the PN sequences are known as *scrambling codes*. On the receiver side, the received signal is first correlated by the PN sequence assigned to the cell, and then by the WH sequence assigned to the user at hand. Within a given cell, the PN sequence is transparent and orthogonality of different user signals is achieved due the orthogonal spreading sequences used. Furthermore, the cell-specific PN scrambling sequences employed, make adjacent-cell signals look like noise and allow the use of a common frequency between adjacent cells.

This cellular CDMA concept, which is used on the downstream channel of IS-95 [5], forms the basis of the Wideband CDMA proposed for the next-generation digital mobile radio system known as universal mobile telecommunications system (UMTS) or international mobile telecommunications in the year 2000 (IMT-2000) [7]. The in-cell OCDMA avoids interference between users within the same cell, but a given user interferes with all users of all other cells. If the chip rates of the orthogonal and PN sequences are the same, the mutual interference between two users located in different cells is 1/N (attenuated by a factor which is function of the user location). If the number of PN chips per WH chip is 4 (which means 4 WH chips per PN chip), then the mutual interference is 1/4N, and in terms of bandwidth occupancy the resulting system is equivalent to the TDMA system with a frequency reuse factor of 4 considered in subsection 3.1. The basic question is whether this system can accommodate as many users as does TDMA and, if so, at what performance level. In what follows, we concentrate on this hybrid CDMA concept and evaluate the total interference level assuming first that the PN sequence has the same chip rate as the WH chip rate. As for TDMA, the analysis is carried out for both the downstream channel and the upstream channel.

3.4.1 *Downstream channel*

As previously, we consider a user located at the cell boundary and in the direction of the base station of one of the 6 neighboring cells. With n active users in each of them, it is easily verified that the total interference from these 6 cells is given by

$$\frac{I_{d,1}}{S} = \frac{n}{N}\left(1 + \frac{2}{3^2} + \frac{2}{7^2} + \frac{1}{9^2}\right) \tag{9}$$

Similarly, the interference from the 6 second-nearest cells is given by the expression

$$\frac{I_{d,2}}{S} = \frac{2n}{N}\left(\frac{1}{7^2} + \frac{1}{13^2} + \frac{1}{19^2}\right) \tag{10}$$

and, finally, the interference from the 6 third-nearest cells which also interfere in the case of TDMA with a frequency reuse factor of 4 is

$$\frac{I_{d,3}}{S} = \frac{n}{N}\left(\frac{1}{9^2} + \frac{2}{13^2} + \frac{2}{21^2} + \frac{1}{25^2}\right) \tag{11}$$

Neglecting interference from other cells, the downlink S/I ratio in hybrid CDMA is

$$\frac{S}{I_d} = \frac{1}{I_{d,1} + I_{d,2} + I_{d,3}} \tag{12}$$

which yields

$$\frac{S}{I_d} = 0.733\,\frac{N}{n} \tag{13}$$

For n = N, we have $S/I_d = -1.3$ dB, which is 16.5 dB worse than in TDMA, but in this case, we assumed that all cells use the same frequency without any further spreading due to the PN sequence. In other words, the cellular CDMA scheme at hand occupies 4 times less bandwidth than the reference TDMA scheme with a frequency reuse factor of 4 between cells. In a hybrid CDMA scheme with the same bandwidth occupancy as our reference TDMA scheme, the downlink S/I ratio becomes

$$\frac{S}{I_d} = 0.733\,\frac{4N}{n} \tag{14}$$

In the dB scale, this gives $S/I_d = 4.7$ dB for n = N. This is 10.5 dB lower than the downlink S/I ratio in the equivalent TDMA scheme.

3.4.2 Upstream channel

As in the downstream direction, we take into account only the interference coming from the 6 nearest cells (with D = 2R), the 6 second-nearest cells (with $D = \sqrt{12}R$), and the 6 third-nearest cells (with D = 4R). When the number of users per cell is n and the number of chips per symbol is N, the uplink S/I ratio is given by

$$\begin{aligned}\frac{S}{I_u} &= \frac{N}{6n\left(I_{u,\mathrm{cell}}(2R)+I_{u,\mathrm{cell}}\left(\sqrt{12}R\right)+I_{u,\mathrm{cell}}(4R)\right)} \\ &= 2.40\,\frac{N}{n}\end{aligned} \qquad (15)$$

which, for n = N, gives S/I_u = 3.8 dB. We observe that this is 15.4 dB worse than the uplink S/I ratio in TDMA.

As in the downstream channel case, increasing the number of chips per symbol to 4N so that the hybrid CDMA scheme yields the same bandwidth as the TDMA scheme with a frequency reuse factor of 4, the uplink S/I ratio becomes

$$\frac{S}{I_u} = 2.40\,\frac{4N}{n} \qquad (16)$$

This yields S/I_u = 9.8 dB for n = N, which is 9.4 dB worse than the uplink S/I ratio in the equivalent TDMA scheme. Also, note that for CDMA the uplink S/I ratio is 5.1 dB larger than the downlink S/I ratio. As mentioned for TDMA, this is due to the fact that one of the worst user positions is considered for downlink interference, while the average interference is considered for the uplink. Note also that the interference obtained through perfect averaging of interference over the cell surface is 1.6 dB larger than the worst-case interference in TDMA which occurs in the unlikely event of 6 interfering users simultaneously located in the worst position of their respective cells.

4 PERFORMANCE ANALYSIS

In this section, we compute the BER performance of TDMA and CDMA in cellular environment. The cellular CDMA is the hybrid (two-layer) CDMA presented in section 3.4 which is the best existing CDMA with a frequency reuse factor of 1. The computations are carried out for coded and uncoded systems, and for both AWGN channels and memoryless flat Rayleigh fading channels. The TDMA system has a frequency reuse factor of 4 and a capacity of N users per cell each having the same bit rate as the users in the CDMA scheme to which it is compared. The number of chips per symbol in the CDMA system considered is 4N, but only a set of N orthogonal sequences is assigned to each cell so that the maximum number of users per cell is also N. Hence, the TDMA and CDMA systems considered require the same total bandwidth and have the same maximum user capacity. The error correction code used in coded systems is the convolutional code of rate 1/2 and constraint length K = 7 [8] which has become a de facto industry standard. The BER results are given for N = 64 and several values of the number of users n.

4.1 BER COMPUTATIONS

To evaluate the BER for coded and uncoded transmissions on AWGN and Rayleigh fading channels using the same method, we found it convenient to use the Chernoff bound. In the case of CDMA, the interference is a sum of uncorrelated random processes which is closely approximated by a Gaussian probability density function (pdf), particularly with a large number of users per cell. It adds to the thermal channel noise, yielding an equivalent Gaussian noise of spectral density

$$N_{0,eq} = N_0 + \frac{E_s}{\left(\frac{S}{I}\right)_{\mathrm{CDMA}}} \qquad (17)$$

where N_0 is the spectral density of the thermal noise, ES denotes the energy per channel symbol, and $(S/I)_{\mathrm{CDMA}}$ is the S/I ratio determined in section 3.3 (see (14) and (16) for a spreading factor of 4N).

For uncoded BPSK transmission on AWGN channels, the standard expression for the BER is

$$\begin{aligned}\mathrm{BER} &= Q\left(\sqrt{\frac{2E_b}{N_{0,eq}}}\right) \\ &\le \frac{1}{2}\exp\left(\frac{-E_b}{N_{0,eq}}\right)\end{aligned} \qquad (18)$$

where Q(x) is the complement of the cumulative distribution function of a zero-mean unit-variance Gaussian random variable, and E_b denotes the energy per bit ($E_b = E_S$ for uncoded BPSK). The inequality in (18) results from the Chernoff bound $Q(x) \le 1/2\ \exp(-x^2/2)$.

For coded BPSK transmission on AWGN channels, we assume that bit errors are caused mainly by the decoder selecting the path in the trellis at the minimum Euclidean distance from the correct path. For BPSK, the minimum Euclidean distance error event is identical to the error event with minimum Hamming distance. For a rate-1/2 convolutional code, this yields [9]

$$\begin{aligned}\mathrm{BER} &\cong N_{\mathrm{info}}\,Q\left(\sqrt{\frac{N_H E_b}{N_{0,eq}}}\right) \\ &\le \frac{1}{2}N_{\mathrm{info}}\exp\left(\frac{-N_H E_b}{2N_{0,eq}}\right)\end{aligned} \qquad (19)$$

where E_b is the energy per transmitted information bit ($E_b = 2E_S$ for rate-1/2 coded BPSK), N_H is the minimum Hamming distance between code words, and N_{info} denotes the number of errored information bits corresponding to the error event with minimum Hamming distance. For the convolutional code with constraint length K = 7 used, we have $N_H = 10$ and $N_{info} = 36$.

Next, for coded BPSK on memoryless flat Rayleigh fading channels, it is again the error event at the minimum Hamming distance that dominates the BER performance. Assuming that the fading gain of the channel is known from the receiver, the BER is approximated as [10]

$$\begin{aligned} \mathrm{BER} &\cong N_{info} E\left[Q\left(\sqrt{\frac{E_b}{N_{0,eq}} \sum_{j=1}^{N_H} r_j^2} \right) \right] \\ &\leq \frac{1}{2} N_{info} E\left[\exp\left(\frac{-E_b}{2N_{0,eq}} \sum_{j=1}^{N_H} r_j^2 \right) \right] \qquad (20) \\ &= \frac{1}{2} N_{info} \left(1 + \frac{E_b}{2N_{0,eq}} \right)^{-N_H} \end{aligned}$$

In (20), the index j enumerates the errored coded bits of the error event at the minimum Hamming distance, and E[.] denotes statistical expectation with respect to the fading gains ρ_j associated with the errored coded bits. The fading gains are statistically independent random variables having a Rayleigh distribution with $E[\rho_j^2] = 1$. Statistical independence of the fading gains is achieved through interleaving.

Computation of the BER is more complicated in the case of TDMA, because not all symbols receive the same amount of interference, except when the interfering cells are entirely full. Limiting the interference to the 6 nearest cells with the same frequency allocation, the interference power during a given symbol interval is a random variable given by

$$I_{TDMA} = \sum_{i=1}^{6} u_i I_i \qquad (21)$$

where I_i is the interference power received from cell i when this cell is full, and u_i is a random variable which takes the value of 1 when a user is active in that cell during the symbol interval at hand, and the value of 0 otherwise. The distribution of the random variable u_i is clearly given by $\mathrm{Prob}[u_i = 1] = n/N$, and $\mathrm{Prob}[u_i = 0] = 1 - n/N$. Here too, we model the interference as a white Gaussian noise, although the number of interfering users is at most 6. The sum of channel noise and interference is then equivalent to a white Gaussian noise of spectral density

$$N_{0,eq} = N_0 + \frac{E_S}{\left(\frac{S}{I}\right)_{TDMA}} \qquad (22)$$

where the interference power I_{TDMA} given by (21) is used to determine $(S/I)_{TDMA}$. From (21) and (22), note that $N_{0,eq}$ is a random variable which is a function of the binary variables u_i, i = 1, 2, ..., 6. Using the Chernoff bound and taking into account only the error event which corresponds to the minimum Hamming distance as in CDMA, we obtain the following results for BPSK transmission on AWGN channels:

$$\mathrm{BER} \leq \frac{1}{2} E\left[\exp\left(\frac{-E_b}{N_{0,eq}} \right) \right] \qquad (23)$$

for uncoded transmission, and

$$\mathrm{BER} \leq \frac{1}{2} N_{info} \left(E\left[\exp\left(\frac{-E_b}{2N_{0,eq}} \right) \right] \right)^{N_H} \qquad (24)$$

for coded transmission. In these expressions, E(.) denotes ensemble averaging over the random variables u_i, i = 1, 2, ..., 6, and we have assumed that the sextuples $\{u_i$, i = 1, 2, ..., 6$\}$ corresponding to different symbols in the same error event are statically independent, which is the case with ideal interleaving. Similarly, for coded BPSK over flat Rayleigh fading channels, we have

$$\mathrm{BER} \leq \frac{1}{2} N_{info} \left(E\left[\left(1 + \frac{E_b}{2N_{0,eq}} \right)^{-1} \right] \right)^{N_H} \qquad (25)$$

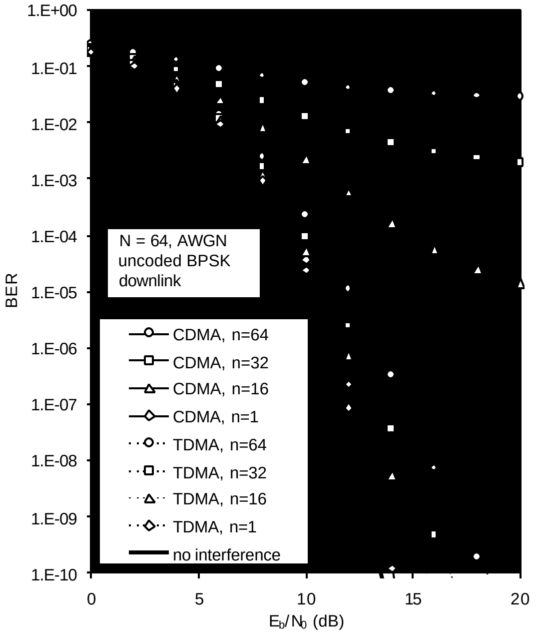

Figure 3: Downlink BER performance for uncoded transmission over an AWGN channel.

4.2 DOWNLINK PERFORMANCE

The BER results for the downstream channel are reported in figures 3 - 5. Figure 3 shows the BER performance of CDMA and TDMA using uncoded BPSK on AWGN channels. We observe that CDMA exhibits a severe BER floor even with n = 16 users which only represent one quarter of the maximum capacity, while the signal-to-noise ratio (SNR) degradation at the BER of 10^{-8} is less than 2 dB in TDMA with n = 16. Next, figure 4 shows the results in the coded case. Here, CDMA no longer exhibits a BER floor, but the degradation is still high, particularly with n = 64 users per cell. With this number of users, we can see that the SNR degradation is approximately 3.7 dB at the BER of 10^{-6}. In contrast, TDMA exhibits a degradation of no more than 0.4 dB with 64 users.

Figure 5 shows the results corresponding to coded BPSK on a memoryless flat Rayleigh fading channel. We can distinguish a similar behavior as in uncoded transmission on AWGN channels, i.e., CDMA exhibits a BER floor whereas the SNR degradation remains very modest in TDMA.

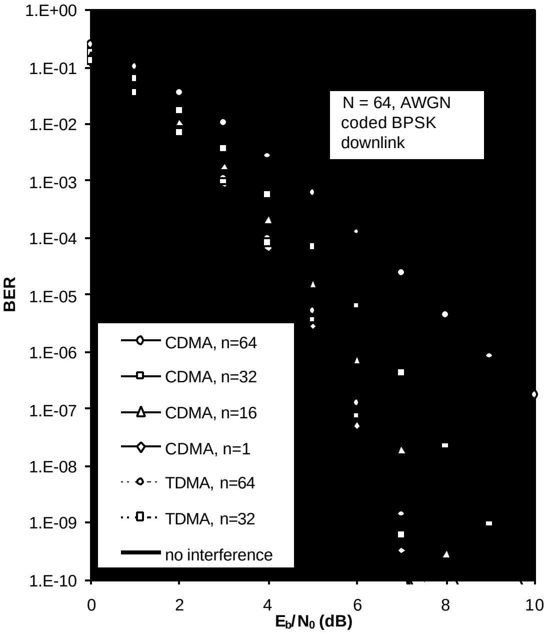

Figure 4: Downlink BER performance for coded transmission over an AWGN channel.

4.3 UPLINK PERFORMANCE

The BER results for the upstream channel are reported in figures 6 - 8. Those reported in figure 6 correspond to uncoded QPSK on AWGN channels. The first remark is that as on the downlink channel, TDMA substantially outperforms CDMA. Clearly, CDMA appears to exhibit a BER floor close to 10^{-4} with n = 64 users and between 10^{-6} and 10^{-7} with n = 32 users, while the SNR degradation of TDMA remains below 1.2 dB at the BER of 10^{-8}. The second remark is that the BER results on the uplink channel are better than those corresponding to the downlink channel, which is consistent with the S/I results reported and the explanations given in section 3. Next, the results corresponding to coded BPSK on AWGN channels are depicted in figure 7. We can see that the degradations here are much smaller, but still TDMA outperforms CDMA. Finally, the results corresponding to coded BPSK on memoryless flat Rayleigh fading channels are depicted in figure 8. Here too, the SNR degradations are larger in CDMA which exhibits a BER floor close to 10^{-8} with 64 users per cell, while the SNR degradation remains below 0.3 dB in TDMA.

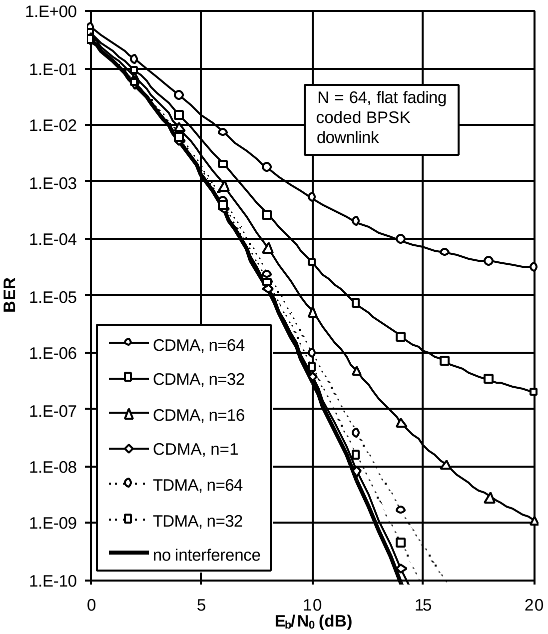

Figure 5: Downlink BER performance for coded transmission over a flat Rayleigh fading channel.

5 DISCUSSION AND CONCLUSIONS

Using nondispersive channels and ideal power control, we have compared two popular cellular system concepts with identical bandwidth occupancy and maximum number of users per cell. One of them is based on TDMA with N time slots per frame and a frequency reuse factor of 4 between cells. The other is based on hybrid CDMA with 4N chips per symbol, N orthogonal sequences per cell, and a frequency reuse factor of 1. Using a common propagation model in which the signal attenuation is proportional to the fourth-power of the distance, we have shown that TDMA has a S/I ratio advantage over CDMA of approximately 10 dB in fully-loaded cells. This result holds for both the downlink and the uplink channels. Consequently, for moderate to high user densities, the BER performance degradation due to interuser interference is substantially higher in CDMA, even when error correction coding is used. For small user densities, the interference power is much less than the thermal noise power, in which case TDMA and CDMA have a similar BER performance. These observations were confirmed by our numerical results computed for AWGN channels and flat Rayleigh fading channels.

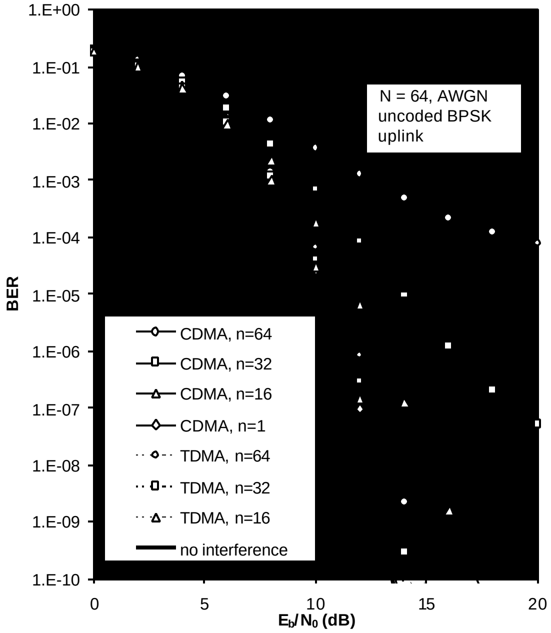

Figure 6: Uplink BER performance for uncoded transmission over an AWGN channel.

Obviously, CDMA and TDMA have a similar BER performance when they operate at the same S/I ratio. Since there is a gap of 10 dB between the two systems considered in this paper, hybrid CDMA needs to use 40N chips per symbol to achieve strictly the same S/I ratio as the TDMA with a frequency reuse factor of 4. In turn, such a CDMA system requires 10 times the bandwidth of TDMA which accommodates the same number of users per cell and achieves the same S/I ratio. This result may seem surprising, as it contradicts the usual claims that CDMA achieves a higher capacity than TDMA in cellular systems, see e.g. [11] and [12]. A reasonable answer to this discrepancy of the results lies in the assumptions made and in what is really compared. In the present paper, no other assumptions are made than hexagonal cell patterns, omnidirectional base station and user antennas, and a path loss that is proportional to the fourth power of the distance. In other comparisons, TDMA is assumed to have a frequency reuse factor of 7 rather than 4, and it is also assumed that CDMA exploits the noncontinuous voice activity of speech signals and cell sectoring at base stations. Concerning the frequency reuse factor, we have just demonstrated that even with a frequency reuse factor of 4, TDMA outperforms CDMA by approximately 10 dB in terms of S/I ratio, and therefore the assumption of a frequency reuse factor of 7 is simply inappropriate in the comparisons. Next, concerning the voice activity in speech signals, it is true that this property is naturally exploited in CDMA, but as mentioned in [12], this function is also included in most (if not in all of) state-of-the-art vocoders. Furthermore, the proportion of data transmission in telecommunications networks is rapidly increasing, and the current trend is that even voice will be carried as data packets in a nondistant future. Consequently, the voice activity factor is becoming a weaker and weaker argument. Finally, cell sectoring is quite similar to cell splitting which increases network capacity at the expense of increasing the number of base station transceivers, but this is equally applicable to TDMA.

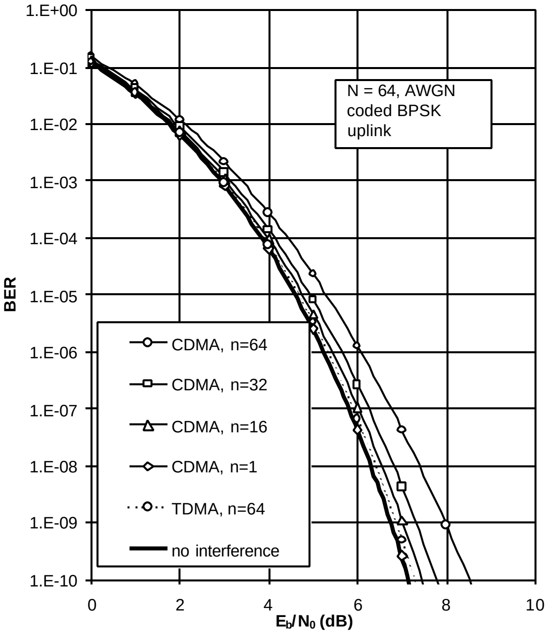

Figure 7: Uplink BER performance for coded transmission over an AWGN channel.

It may be argued that the nondispersive channel model used in this paper is too simple to draw useful conclusions on the comparison of cellular TDMA and CDMA. However, with this model, unambiguous conclusions can be drawn on the multiple access techniques themselves, and this is not always the case with more elaborate models. It is obvious that in real-world applications, dispersive channels need to be considered to assess the performance of a particular cellular system design. But the performance achieved in that environment is more closely related to the transmission technique and the receiver used than to the multiple access technique itself. In fact, multipath propagation does not affect the 10 dB S/I ratio penalty of CDMA, but this technique obviously benefits from a higher frequency diversity gain due to the larger bandwidth of the transmitted signal. We conclude that while it is undeniable that cellular CDMA has a number of attributes which include the ease of network planning, soft handoff, and lower peak-to-average power ratio on the upstream channel (when the number of spreading sequences assigned to one user in multirate transmission is a small fraction of the total number of sequences), this multiple access technique is inherently capacity-limited with respect to TDMA. In other words, reduced capacity is precisely the price paid for the higher flexibility of cellular CDMA with a frequency reuse factor of 1.

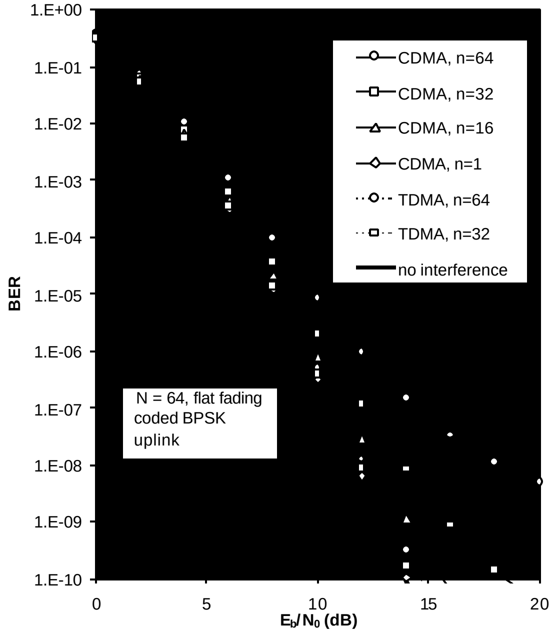

Figure 8: Uplink BER performance for coded transmission over a flat Rayleigh fading channel.

Before closing, it is quite instructive to point out another seemingly counterintuitive result which was recently reported in [13]. Similar to its perception as a multiple access technique which increases network capacity, CDMA is also commonly perceived as being superior to TDMA in terms of robustness to narrowband interference. The analysis and the results reported in [13] demonstrate that contrary to intuition and commonly spread ideas, TDMA slightly outperform CDMA in that environment.

Manuscript received on June 3, 1999

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