On the Capacity of Hybrid Wireless Networks

Benyuan Liu *, Zhen Liu +, Don Towsley *

* Department of Computer Science
University of Massachusetts
Amherst, MA 01002

+ IBM T.J. Watson Research Center
P.O. Box 704
Yorktown Heights, NY 10598

Abstract— **We study the throughput capacity of hybrid wireless networks. A hybrid network is formed by placing a sparse network of base stations in an ad hoc network. These base stations are assumed to be connected by a high-bandwidth wired network and act as relays for wireless nodes. They are not data sources nor data receivers. Hybrid networks present a tradeoff between traditional cellular networks and pure ad hoc networks in that data may be forwarded in a multi-hop fashion or through the infrastructure. It has been shown that the capacity of a random ad hoc network does not scale well with the number of nodes in the system [1]. In this work, we consider two different routing strategies and study the scaling behavior of the throughput capacity of a hybrid network. Analytical expressions of the throughput capacity are obtained. For a hybrid network of n nodes and m base stations, the results show that if m grows asymptotically slower than $\sqrt{n}$, the benefit of adding base stations on capacity is insignificant. However, if m grows faster than $\sqrt{n}$, the throughput capacity increases linearly with the number of base stations, providing an effective improvement over a pure ad hoc network. Therefore, in order to achieve non-negligible capacity gain, the investment in the wired infrastructure should be high enough.**

***Keywords*: Throughput capacity, hybrid wireless networks, ad hoc networks.**

I. INTRODUCTION

Throughput capacity is a key characteristic of wireless networks. It represents the long-term achievable data transmission rate that a network can support. The throughput capacity of a wireless network depends on many aspects of the network: network architecture, power and bandwidth constraints, routing strategy, radio interference, etc. A good understanding of the capacities of different network architectures allows a designer to choose an architecture appropriate for his or her specific purpose.

Several network models are available for wireless data networks. In a wireless cellular network or a wireless LAN, nodes communicate with each other through base stations or access points. A node first connects to the nearest base station or access point in order to communicate with other nodes. A base station (access point) serves as a communication gateway for all the nodes in its cell (basic service area).

In situations where there is no fixed infrastructure, for example, battle fields, catastrophe control, wireless ad hoc networks become valuable alternatives to wireless cellular networks or wireless LANs for nodes to communicate with each other. An ad hoc network is a communication network formed by a collection of nodes without the aid of any fixed infrastructure. In an ad hoc network, due to the lack of infrastructure and the limited transmission range of each node, data needs to be routed to the destination by the nodes in a multi-hop fashion.

In a recent study [2], the authors proposed a hybrid network model to improve network connectivity. In the model, a sparse network of base stations connected by a wired network is placed within an ad hoc network. The resulting network consists of normal nodes and some well-connected base stations. It is called a hybrid network since it presents a tradeoff between traditional cellular networks and pure ad hoc networks. In ad hoc networks, there is no infrastructure, data can only be forwarded by the nodes in a multi-hop fashion. In cellular networks, data are always forwarded through the infrastructure. While in a hybrid network, data may be forwarded in a multi-hop fashion or through the infrastructure. Communications using multi-hop forwarding and communications using the infrastructure coexist in the network. It is of great interest to understand what performance gains can be achieved by the hybrid networks.

While the capacity performance of cellular networks has been well studied [3], researchers have started to investigate the capacity of wireless ad hoc networks only recently. In [1], Gupta and Kumar studied the throughput capacity of a random wireless network, where fixed nodes are randomly placed in the network and each node sends data to a randomly chosen destination. The throughput capacity per node is shown to be $\Theta(\frac{W}{\sqrt{n\log n}})$, as n approaches infinity, where n is the number of nodes in the network (the same below) and W is the common transmission rate of each node over the wireless channel. Thus the aggregate throughput capacity of all the

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nodes in the network is $\Theta(\sqrt{\frac{n}{\log n}}W)$. In [4], Gupta and Kumar studied the capacity of a random three-dimensional wireless ad hoc network, and showed that the aggregate throughput scales as $\Theta\left(\left(\frac{n}{\log n}\right)^{\frac{2}{3}}W\right)$. In [5], Grossglauser and Tse proposed a scheme that takes advantage of the mobility of the nodes. By allowing only one-hop relaying, the scheme achieves an aggregate throughput capacity of $O(n)$ at the cost of unbounded delay and buffer requirement. Gastpar and Vetterli studied the capacity under a different traffic pattern in [6]. There is only one active source and destination pair, while all other nodes serve as relay, assisting the transmission between the source and destination nodes. The capacity is shown to scale as $O(\log n)$. In [7], Li et al. examined the effect of IEEE 802.11 on network capacity and presented specific criteria of the traffic pattern that makes the capacity scale with the network size.

The above studies all focus on the capacities of pure ad hoc network models. It is not clear how much capacity gain a network can achieve by adding a certain number of base stations to an ad hoc network and forming a hybrid network. Intuitively, on one hand, the infrastructure helps to reduce the wireless transmissions, resulting in less interference and a higher capacity. On the other hand, too much use of the infrastructure may cause hot spots around base stations and inefficient use of spatial concurrency, leading to a sub-optimal capacity. It is the purpose of this work to study the capacity of hybrid networks. In particular, we are interested in the following questions:

- How does the throughput capacity scale with the number of nodes and the number of base stations?
- How does the capacity of a hybrid network model compare to that of a pure ad hoc network?

In this paper, we consider two different routing strategies and obtain the analytical expressions of the throughput capacity of hybrid networks. Moreover, we derived the maximum throughput capacities and the conditions to achieve them. We assume a hybrid network of m base stations and n nodes, each capable of transmitting at W bits/sec over the wireless channel.

In the first routing strategy, a node sends data through the infrastructure if the destination is outside of the cell where the source is located. Otherwise, the data are forwarded in a multi-hop fashion as in an ad hoc network. Under this strategy, if m grows asymptotically slower than $\sqrt{n}$, the maximum throughput capacity is $\Theta\left(\sqrt{\frac{n}{\log \frac{n}{m^2}}}W\right)$. In this case, the benefit of adding base stations is insignificant. However, in the case where base stations can be added at a speed asymptotically faster than $\sqrt{n}$, the maximum capacity is $\Theta(mW)$, which increases linearly with the number of base stations.

Similar results are obtained for a probabilistic routing strategy. In the strategy, a node chooses whether to use infrastructure to send data according to some probability. Under this routing strategy, if m grows slower than $\sqrt{\frac{n}{\log n}}$, the maximum throughput capacity has the same asymptotic behavior as a pure ad hoc network. There is no benefit to use the infrastructure in this case. If m grows faster than $\sqrt{\frac{n}{\log n}}$, the maximum throughput capacity scales as $\Theta(mW)$, which increases linearly with the number of base stations.

For both routing strategies, if the number of base stations scales slower than some threshold, the throughput capacity is dominated by the contribution of ad hoc mode transmissions. The benefit of adding base stations is minimal. If the number of base stations scales faster than the threshold, the capacity contributed by the infrastructure dominates the overall network throughput capacity. In this case, the maximum throughput capacity scales linearly with the number of base stations, providing an effective improvement over pure ad hoc networks. Therefore, in order to achieve non-negligible capacity gain, the investment in the wired infrastructure should be high enough: the number of base stations should be at least $\sqrt{n}$ for the first routing strategy and $\sqrt{\frac{n}{\log n}}$ for the probabilistic routing strategy.

The rest of the paper is organized as follows: In Section II, we describe the hybrid wireless network model. In Section III and IV, we present the analytical results of throughput capacity of a hybrid network, under the two different routing strategies. In Section V, we draw the conclusions.

II. Hybrid Network Model

In this section, we present the hybrid wireless network model and define the throughput capacity of hybrid networks.

A. Network Components

A hybrid network consists of two components. The first component is an ad hoc network containing only normal nodes, the same as the model defined in [1]. The second component is a sparse network of base stations. The base stations are assumed to be connected together by a wired network and are placed within the ad hoc network in a regular pattern.

We scale space and assume that a population of n nodes are randomly, i.e., independently and uniformly, located within a disk of area 1 square meter in the plane. We further assume that the nodes are homogeneous, employing the same transmission range or power. Every node is a data source. The destination for each node is independently chosen as the node nearest to a randomly located point within the unit area disk.

In addition to the n nodes in the network, a sparse network of m base stations is regularly placed in the unit area disk. The base stations divide the area into a hexagon tessellation, as shown in Fig 1. As in a cellular network, each hexagon is called a cell and there is a base station in the center of each cell. Unlike normal nodes, the base stations are neither data sources nor data receivers. They are added as relay nodes to improve network performance and they only engage in routing and forwarding data for normal nodes. The base stations are assumed to be connected together by a wired network. Furthermore, we assume the link bandwidth in the wired network are all large enough so that there are no bandwidth constraints in the wired network. We also assume there are no power constraints for the base stations.

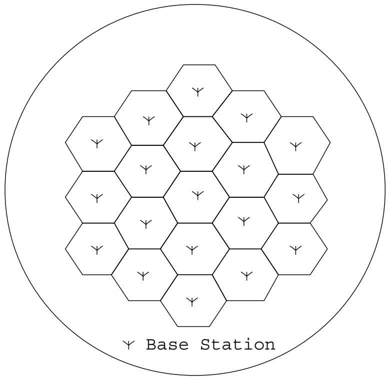

Fig. 1. Hybrid Wireless Network Model

B. Interference Model

All the nodes and the base stations share a common wireless channel. We assume a time-division multiplexing (TDMA) scheme for the data transmission over the wireless channel. Time is divided into slots of fixed durations. In each time slot, a node is scheduled to send data. A node cannot transmit and receive data simultaneously and a node can only receive data from one other node at the same time.

The wireless transmissions in the network are assumed to be homogeneous. Nodes including the base stations employ the same transmission range, denoted by r. For the interference model, we adopt the Protocol Model introduced in [1].

A transmission from node X_i is successfully received by node X_j if the following two conditions are satisfied:

1) Node X_j is within the transmission range of node X_i, i.e.,

$$|X_i - X_j| \le r$$

where $|X_i - X_j|$ represents the distance between node X_i and node X_j in the plane.

2) For every other node X_k that is simultaneously transmitting over the same channel,

$$|X_k - X_j| \ge (1+\Delta)|X_i - X_j|$$

This condition guarantees a guard zone around the receiving node to prevent a neighboring node from transmitting on the same channel at the same time. The radius of the guard zone is $(1+\Delta)$ times the distance between the sender and receiver. The parameter Δ defines the size of the guard zone and we require that $\Delta > 0$.

C. Routing Strategy

In a hybrid network, there are two transmission modes: ad hoc mode and infrastructure mode. In the ad hoc mode, data are forwarded from the source to the destination in a multi-hop fashion without using any infrastructure. In the infrastructure mode, data are forwarded through the infrastructure. It can be shown that in terms of throughput capacity, it is optimal to enter and exit the infrastructure only once. Also, it is optimal for a node to communicate with the nearest base station in order to reach the infrastructure. Denote the base station nearest to node X_i as $B(X_i)$. In this work, by infrastructure mode we mean that data are first transmitted from the source (X_s) to $B(X_s)$ over the wireless channel; the base station then transmits the data through the wired infrastructure to $B(X_d)$, which finally transmits the data to the destination X_d.

In this work, we consider two routing strategies. In the first routing strategy, if the destination is located in the same cell as the source node, data are forwarded in the ad hoc mode. Otherwise, data are forwarded in the infrastructure mode. Since the destination for a source node is randomly chosen in the unit area disk, the probability that a node commits to intra-cell communications is $1/m$; the probability that a node commits to inter-cell communications is $1-1/m$. We can generalize the routing strategy to represent a family of routing strategies by relaxing the condition that the ad hoc mode is chosen to send data. Instead of requiring the destination be located in the same cell as the source, a node uses ad hoc mode to send data as long as the destination is located within k nearest neighboring cells from the source node, where $k \ge 0$ defines the range within which ad hoc mode transmissions should be used. We call this family of routing strategies the *k-nearest-cell* routing strategies.

The second routing strategy is a probabilistic routing strategy. A transmission mode is independently chosen for each source destination pair. With probability p, the ad hoc mode is employed, and with probability $1-p$, the infrastructure mode is used. By varying the probability p, a family of probabilistic routing strategies can be obtained.

We assume each node can transmit at W bits/sec over the common wireless channel. We divide the wireless channel so that ad hoc mode transmissions and infrastructure mode transmissions go through different sub-channels. We further divide the sub-channel for infrastructure mode transmissions into uplink and downlink parts, according to the direction of the transmissions relative to the base station. Since intra-cell traffic, uplink traffic and downlink traffic use different sub-channels, there is no interference between the three types of traffics. The bandwidth assigned to intra-cell, uplink, and downlink sub-channels are W_1, W_2, and W_3, respectively. The transmission rates should sum to W, i.e., $\sum_{i=1}^{3} W_i = W$. Since there are same amount of uplink traffic and downlink traffic, we let $W_2 = W_3$.

D. Definition of Throughput Capacity

To make the formulas more concise, we present the aggregate throughput capacity of the whole network instead of the throughput capacity of each node. Note that the throughput capacity per node is simply $1/n$ of the aggregate throughput capacity of the whole network. In this paper, we adopt the asymptotic notations defined in [8]. We now define the feasible aggregate throughput and the aggregate throughput capacity of the hybrid network model.

Definition 1: Feasible Aggregate Throughput. For a hybrid network of n nodes and m base stations, an aggregate throughput of $T(n,m)$ bits/sec is feasible if by transmitting data in the ad hoc or infrastructure mode, there is a spatial and temporal scheduling scheme that yields an aggregate network throughput of $T(n,m)$ bits/sec on average. Here the aggregate throughput is the sum of the individual throughputs from each node to its chosen destination.

Definition 2: Aggregate Throughput Capacity of Hybrid Networks. The aggregate throughput capacity of a hybrid wireless network is of order $\Theta(f(n,m))$ bits/sec if there are deterministic constants $c > 0$, and $c' < +\infty$ such that

$$\lim_{n\to\infty} \text{Prob(T(n,m) = cf(n,m) is feasible)} = 1$$
$$\lim_{n\to\infty} \inf \text{Prob(T(n,m) = } c'f(n,m) \text{ is feasible)} < 1$$

III. Capacity of Wireless Hybrid Networks under k-nearest-cell routing strategies

In this section, we derive the throughput capacity of the hybrid network under the k-nearest-cell routing strategies. In particular, we analyze the case where $k = 0$, i.e., a node sends data in ad hoc mode if the destination is located in the same cell as the source. We conjecture that the results hold for the family of routing strategies when k is a constant. After this, we compare the capacity of hybrid networks to the capacity of pure ad hoc networks.

A. Throughput Capacity

Since the intra-cell, uplink and downlink traffics are transmitted in three different sub-channels, there is no interference between the three types of traffic. However, within a sub-channel, interference exists between the same type of traffic in different cells. Fortunately, the effect of this interference is minimal. There is a spatial transmission schedule that yields efficient frequency reuse. More specifically, the cells can be spatially divided into a constant number of different groups. Transmissions in the cells of the same group do not interfere with each other. If the groups are scheduled to transmit in a round robin fashion, each cell will be able to transmit once every fixed amount of time without interfering with each other. The degradation of network capacity due to the interference between the same types of traffic is thus bounded by a constant factor. We now provide the formal proof.

We adopt the notion of interfering neighbors introduced in [1], and compute the number of cells that can be affected by a transmission in one cell. Two cells are defined to be interfering neighbors if there is a point in one cell which is within a distance $(2+\Delta)r$ of some point in the other cell, where r is the transmission range of the nodes. By the definition of the Protocol Interference Model, if two cells are not interfering neighbors, transmissions in one cell do not interfere with transmissions in the other cell.

Lemma 1: Each cell has no more than c interfering neighbors, where c is a constant that only depends on Δ.

Proof. We denote the length of each side of a cell (hexagon) as l and assume that $l = c_1 r$. Therefore, each cell is contained by a disk of radius $c_1 r$ and contains a disk of radius $\frac{\sqrt{3}}{2}c_1 r$.

If a cell H' is an interfering neighbor of a cell H, one point in H' must be within a distance of $(2+\Delta)r$ of a point in H. Therefore, for the Protocol Interference Model, all the interfering cells of H must be contained by a disk D of radius $3c_1 r + (2+\Delta)r$. We know that each cell contains a disk of radius $\frac{\sqrt{3}}{2}c_1 r$. The area of each cell is then larger than the area of the contained disk.

The number of cells contained in disk D is thus bounded by:

$$\begin{aligned} c &= \pi((3c_1+2+\Delta)r)^2/\pi\left(\frac{\sqrt{3}}{2}c_1 r\right)^2 \\ &= \frac{4}{3}\left(\frac{3c_1+2+\Delta}{c_1}\right)^2 \end{aligned}$$

□

Lemma 2: In the Protocol Model, there is a spatial scheduling policy such that each cell gets one slot to transmit data in every $(1+c)$ slots.

Proof. We construct the following graph. Each cell is represented by a vertex and edges are added between interfering neighbors. It is a well known fact in graph theory that a graph of degree no more than c can have its vertices colored by using no more than $(1+c)$ colors, while no two adjacent vertices have the same color. Therefore, the cells in the network can be colored with no more than $(1+c)$ colors, while no two interfering neighbors have the same color. We allow cells of the same color to transmit in the same time slot. Transmissions from non-interfering cells do not interfere with each other. Therefore, there exist scheduling schemes where each cell receives a slot for transmission every $(1+c)$ slots. The degradation of network capacity due to the interference between the same type of traffic is bounded by a constant factor. □

Now we derive the aggregate throughput capacity of the hybrid network model. Depending on the asymptotic behavior of m as a function of n, the hybrid network exhibits different capacities.

Theorem 1: For a hybrid network of n nodes and m base stations, if $m = o(\sqrt{n})$, under the routing protocol and channel allocation scheme, the aggregate throughput capacity of the hybrid network is:

$$T(n,m) = \Theta\left(\sqrt{\frac{n}{\log\frac{n}{m^2}}}W_1 + mW_2\right) \tag{1}$$

Proof. To derive the aggregate network throughput capacity, we first compute the per cell capacity contributed by the ad hoc mode transmissions (T_a) and the per cell capacity contributed by the infrastructure mode transmissions (T_i), respectively.

Consider an arbitrary cell k, let Y_i be a random variable that represents whether node i $(1 \le i \le n)$ and its destination are both located in cell k. The random variables are defined as follows:

$$Y_i = \begin{cases} 1 & \text{both node i and its destination are in cell } k \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

In a hybrid network, there are m cells. Nodes and the corresponding destinations are randomly and independently and placed in the unit area disk. The probability that a node i is located in cell k is $1/m$; the probability that the destination of node i is located in cell k is also $1/m$. Therefore, $E[Y_i] = 1/m^2$.

We then define a random variable $N_k = \sum_{i=1}^{n} Y_i$, representing the number of source and destination pairs communicating using the ad hoc mode within cell k. Since $\{Y_i\}_1^n$ is an i.i.d. sequence of random variables with $E[Y_i] = 1/m^2$. By Strong Law of Large Numbers, with probability 1,

$$\frac{N_k}{n} = \frac{1}{n}\sum_{i=1}^{n} Y_i \to \frac{1}{m^2} \text{ as } n \to \infty \tag{3}$$

Given $m = o(\sqrt{n})$, we have $\lim_{n\to\infty} n/m^2 \to \infty$, and thus $\lim_{n\to\infty} N_k \to \infty$. According to [1], for a random ad hoc network of N_k nodes and a common transmission rate of W_1, the per node capacity is $\Theta(\frac{W_1}{\sqrt{N_k \log N_k}})$, as N_k goes to infinity. Therefore, the capacity of cell k contributed by ad hoc transmissions is $T_a(N_k) = \Theta(\sqrt{\frac{N_k}{\log N_k}} W_1)$. Denote

$$c_2 = \lim_{n\to\infty} \inf \frac{T_a(N_k)}{\sqrt{\frac{N_k}{\log N_k}} W_1}$$
$$c_3 = \lim_{n\to\infty} \sup \frac{T_a(N_k)}{\sqrt{\frac{N_k}{\log N_k}} W_1}$$

By (3), we have $\lim_{n\to\infty} \frac{\sqrt{\frac{N_k}{\log N_k}}}{\sqrt{\frac{n/m^2}{\log(n/m^2)}}} = 1$. Therefore,

$$\lim_{n\to\infty} \inf \frac{T_a(N_k)}{\sqrt{\frac{n/m^2}{\log(n/m^2)}} W_1} = \lim_{n\to\infty} \inf \frac{T_a(N_k)}{\sqrt{\frac{N_k}{\log N_k}} W_1} \frac{\sqrt{\frac{N_k}{\log N_k}}}{\sqrt{\frac{n/m^2}{\log(n/m^2)}}} = c_2$$

$$\lim_{n\to\infty} \sup \frac{T_a(N_k)}{\sqrt{\frac{n/m^2}{\log(n/m^2)}} W_1} = \lim_{n\to\infty} \sup \frac{T_a(N_k)}{\sqrt{\frac{N_k}{\log N_k}} W_1} \frac{\sqrt{\frac{N_k}{\log N_k}}}{\sqrt{\frac{n/m^2}{\log(n/m^2)}}} = c_3$$

The term $\lim_{n\to\infty} T_a(N_k)/\sqrt{\frac{n/m^2}{\log(n/m^2)}} W_1$ is upper and lower bounded by two constants. Therefore, the per cell throughput capacity contributed by the ad hoc mode communications is

$$T_a = \Theta\left(\sqrt{\frac{\frac{n}{m^2}}{\log \frac{n}{m^2}}} W_1\right) \tag{4}$$

Now we calculate the capacity contributed by the infrastructure mode communications. Since the same packet has to go through an uplink and a downlink when transmitted in the infrastructure mode, it should be only counted once in the throughput capacity. We consider uplink throughput, for example. Since all the infrastructure mode traffic has to go through the base station and the base station can only receive data at the rate of W_2 bits/sec at any time. T_i is upper bounded by W_2. For the lower bound, if each node in the infrastructure mode employs a transmission range of l (the side length of each cell), there is a schedule for each node to transmit to the base station in a round robin fashion, yielding a throughput of W_2. Therefore,

$$T_i = \Theta(W_2). \tag{5}$$

Since there is no interference between the the ad hoc mode and the infrastructure mode, the aggregate throughput capacity of a cell is just $T_a + T_i$. In Lemma 2 we proved that there is a scheduling policy such that each cell gets a slot to transmit in every constant number $(1+c)$ of time slots. Therefore, the aggregate capacity of the network is

$$\begin{aligned} T(n,m) &= \Theta(mT_a + mT_i) \\ &= \Theta\left(\sqrt{\frac{n}{\log \frac{n}{m^2}}} W_1 + mW_2\right) \end{aligned}$$

□

Note that the above capacity results correspond to the specific channel allocation scheme. To obtain the maximum capacity, we must maximize the capacity over different channel allocation schemes. More specifically, the maximum capacity should be obtained over all possible combinations of W_1 and W_2.

Corollary 1: The aggregate throughput capacity is maximized when $W_2/W \to 0$. And the corresponding capacity is:

$$T(n,m) = \Theta\left(\sqrt{\frac{n}{\log \frac{n}{m^2}}} W\right) \tag{6}$$

Proof. Since $W_1 + W_2 + W_3 = W$ and $W_2 = W_3$, we have $W_1 = W - 2W_2$. Replace W_1 in (1) with $W - 2W_2$.

$$\begin{aligned} T(n,m) &= \Theta\left(c_4\sqrt{\frac{n}{\log \frac{n}{m^2}}} W + \left(c_4 - 2c_4\sqrt{\frac{\frac{n}{m^2}}{\log \frac{n}{m^2}}}\right) mW_2\right) \\ &= \Theta\left(\sqrt{\frac{n}{\log \frac{n}{m^2}}} W + \left(c_6 - \sqrt{\frac{\frac{n}{m^2}}{\log \frac{n}{m^2}}}\right) mW_2\right) \end{aligned}$$

Since $m = o(\sqrt{n})$, n/m^2 goes to infinity as n increases. Hence, $c_6 - \sqrt{\frac{n/m^2}{\log(n/m^2)}} < 0$ when n is large enough. The throughput capacity is maximized if $W_2/W \to 0$. And the corresponding capacity is

$$T(n,m) = \Theta\left(\sqrt{\frac{n}{\log \frac{n}{m^2}}} W\right)$$

□

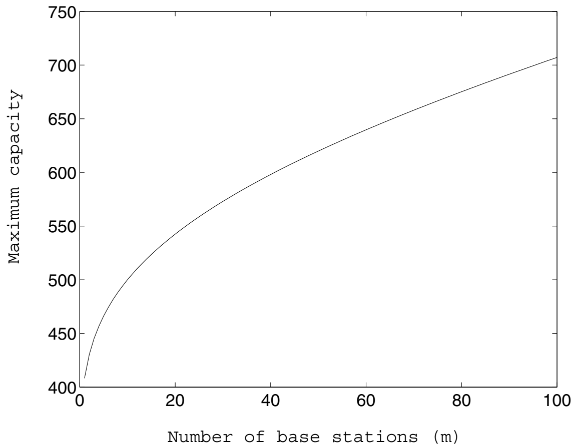

Fig. 2. Maximum aggregate throughput capacity for $m = o(\sqrt{n})$

The maximum capacity is achieved when $W_2/W \to 0$ or equivalently, $W_1/W \to 1$. W_1 is the bandwidth assigned to carry the intra-cell traffic via ad hoc mode in each cell. Therefore, the condition implies that in order to maximize the throughput capacity of the network, we should assign most of the wireless channel bandwidth to carry intra-cell traffic. And only a minimal amount of channel bandwidth should be assigned to carry inter-cell traffic. An intuitive explanation for the bandwidth assignment to achieve maximum capacity is as follows. The capacity contributed by ad hoc mode transmissions increases at a rate of m as W_2 increases, while the capacity contributed by infrastructure mode transmissions increases at a rate of roughly $\sqrt{n}$ as W_1 increases. When $m = o(\sqrt{n})$, it is more beneficial to assign bandwidth to ad hoc mode transmissions.

Fig. 2 presents a numerical example to illustrate the behavior of the maximum aggregate throughput capacity as the number of base stations increases. In this example, the number of nodes in the network is fixed at $n = 1,000,000$ and we increase the number of base station (m) from 1 to 100. Note that the y-axis in the plot is the scaling term $\sqrt{n/\log\frac{n}{m^2}}$ in (6). The real throughput capacity should be scaled by a constant factor. However, this does not affect the trend of the capacity. We can observe that as the number of base stations increases, the maximum throughput capacity increases. However, the increase of the maximum capacity is dominated by a logarithmic term of m, the number of base stations. More specifically, if we increase the number of base stations from m to km, the maximum capacity of the resulting hybrid network is

$$\begin{aligned} T(n,km) &= \Theta\left(\sqrt{\frac{n}{\log\frac{n}{k^2m^2}}}W\right) \\ &\simeq \Theta\left(\sqrt{\frac{n}{\log\frac{n}{m^2}}}W\left(1+\frac{\log k}{\log\frac{n}{m^2}}\right)\right) \\ &= T(n,m)\left(1+\frac{\log k}{\log\frac{n}{m^2}}\right) \end{aligned} \tag{7}$$

In the above derivation, we used the fact that $\log\frac{n}{m^2} >> 1$ since $n/m^2 >> 1$. Therefore, the addition of km base stations only provides a less than $\log k$-fold increase in the maximum capacity.

Note that the above capacity result is only valid when the number of nodes communicating using ad hoc mode is large. In the case where $m = \Omega(n)$, n/m^2 is upper bounded by a constant. The capacity result in [1, Main Result 3] cannot be applied to obtain T_a. In this case, the capacity can be obtained in a different way.

Theorem 2: For a hybrid network of n nodes and m base stations, if $m = \Omega(\sqrt{n})$, under the routing protocol and channel allocation scheme, the aggregate throughput capacity of the hybrid network is:

$$T(n.m) = O\left(\sqrt{n}W_1\right) + \Theta(mW_2) \tag{8}$$

Proof. The value of n/m^2 does not change the the capacity contributed by the infrastructure mode. We have $T_i = \Theta(W_2)$.

However, if $m = \Omega(\sqrt{n})$, n/m^2 is upper bounded by a constant, we cannot use [1, Main Result3] to derive T_a.

According to [1, Theorem 2.1], if n nodes are optimally placed in a disk of area A, each transmission's range is optimally chosen, and the average distance traversed by a packet is $\overline{L}$, the network transport capacity (bit-distance product per unit time) is bounded as follows:

$$\lambda n\overline{L} \le \sqrt{\frac{8}{\pi}}\frac{1}{\Delta}W\sqrt{nA} \text{ bit-meters/sec} \tag{9}$$

Consider a cell in a hybrid network of n nodes and m base stations, the area of the cell is $A = 1/m$ and the average distance traversed by a packet using ad hoc mode within the cell is $\overline{L} = \Theta\left(\sqrt{1/m}\right) = \Theta\left(\sqrt{A}\right)$. Recall that $N_k = \sum_{i=1}^{n} Y_i$ is a random variable representing the number of source and destination pairs within cell k. By definition, the aggregate throughput capacity of a random ad hoc network is necessarily smaller than that of the optimal network. Therefore, the capacity of cell k contributed by ad hoc mode transmissions is

$$T_a(N_k) \le \lambda N_k \le \sqrt{\frac{8}{\pi}}\frac{1}{\Delta}W_1\sqrt{N_k} = O\left(\sqrt{N_k}W_1\right)$$

From (3), we know that with probability 1, $N_k/n \to 1/m^2$ as $n \to \infty$. Hence, $\lim_{n\to\infty} \sqrt{N_k}/\sqrt{\frac{n}{m^2}} = 1$, and $\lim_{n\to\infty} T_a(N_k) = O\left(\sqrt{\frac{n}{m^2}} W_1\right)$. Therefore,

$$\begin{aligned} T(n,m) &= \Theta(mT_a + mT_i) \\ &= O\left(\sqrt{n}W_1\right) + \Theta(mW_2) \end{aligned}$$

□

Since $m = \Omega(\sqrt{n})$, the aggregate throughput capacity is maximized when $W_2/W \to 1/2$, resulting in a maximum capacity of $\Theta(mW)$. Therefore, for the case $m = \Omega(\sqrt{n})$, we have the following corollary.

Corollary 2: For a hybrid network of n nodes and m base stations, if $m = \Omega(\sqrt{n})$, the aggregate throughput capacity is maximized when $W_2/W \to 1/2$, and the maximum capacity is:

$$T(n,m) = \Theta(mW) \tag{10}$$

In this case, it is more effective to assign bandwidth to carry inter-cell traffic and the maximum capacity is obtained when $W_2/W \to 1/2$, or equivalently, $W_1/W \to 0$. In other words, in order to maximize the throughput capacity, almost all of the channel bandwidth should be assigned to uplink and downlink sub-channels to carry inter-cell transmissions, while the intra-cell ad hoc mode transmissions are suppressed. Note that the optimal channel assignment to achieve maximum throughput capacity is the opposite to the case where $m = o(\sqrt{n})$. Compared to the logarithmic growth when $m = o(\sqrt{n})$, the maximum throughput capacity increases linearly with the number of base stations in this case.

B. Comparison to pure ad hoc networks

Now we compare the maximum capacity of a hybrid network to the capacity of a pure ad hoc network. Since a hybrid network contains base stations which are connected by a high link bandwidth wired network, it is not fair to compare the capacity of a hybrid network to the capacity of an ad hoc network directly. The purpose of this section is to investigate the benefit of constructing a hybrid network if some number of base stations can be added to a pure ad hoc network. We define the capacity gain factor to quantify the benefit of introducing infrastructure in ad hoc networks.

Definition 3: Capacity gain factor. The capacity gain factor $g(n,m)$ of a hybrid network of n nodes and m base stations is the ratio of the maximum throughput capacity of the hybrid network to the throughput capacity of an ad hoc network of n nodes, i.e., $g(n,m) = T(n,m)/T_a(n)$, where $T_a(n)$ represents the aggregate throughput capacity of an ad hoc network of n nodes.

Gupta and Kumar have shown in [1], for an ad hoc wireless network of n nodes, if the nodes are randomly placed and the destination of each node is randomly chosen, the aggregate throughput capacity as $n \to \infty$ is

$$T_a(n) = \Theta\left(\sqrt{\frac{n}{\log n}} W\right) \tag{11}$$

where W is the common transmission rate of the nodes over the wireless channel. Note that splitting the channel into several sub-channels does not change the capacity results.

Based on the capacity result, Gupta and Kumar pointed out that if m additional homogeneous nodes are deployed as pure relays in random positions, the aggregate throughput capacity becomes $\Theta\left(\sqrt{\frac{n+m}{\log(n+m)}} W\right)$. Therefore, the addition of kn pure relay nodes merely provides a less than $\sqrt{k+1}$-fold capacity gain. Note that there is no wired links between the relay nodes in Gupta and Kumar's model. While in our model, we assume base stations are connected by high-bandwidth wired links.

In the previous section, we obtained the maximum aggregate capacity of the hybrid network. Depending on the scaling behaviors of m relative to n, the capacity results have different dependencies on n and m. In the following, we present the capacity comparisons for the two different regimes of m.

1. $m = o(\sqrt{n})$.

When the number of nodes (n) goes to infinity, the maximum capacity of a hybrid network of n nodes and m base stations is obtained when $W_2/W \to 0$ and the capacity scales as $\Theta\left(\sqrt{\frac{n}{\log \frac{n}{m^2}}} W\right)$, while the capacity of a pure ad hoc network of n nodes scales as $\Theta\left(\sqrt{\frac{n}{\log n}} W\right)$. The capacity gain factor is

$$\begin{aligned} g(n,m) &= \Theta\left(\sqrt{\frac{\log n}{\log(n/m^2)}}\right) \\ &= \Theta\left(\frac{1}{\sqrt{1 - \frac{2\log m}{\log n}}}\right) \end{aligned} \tag{12}$$

We now derive the capacity gain when the number of base stations scales as a polynomial of the number of nodes. Assume $m = n^\alpha$ where $0 < \alpha < 1/2$, from (12), we have

$$g(n,m) = \Theta\left(\frac{1}{\sqrt{1-2\alpha}}\right) \tag{13}$$

Note that the gain factor is independent of n and m, it remains a constant as m goes to infinity in the form of n^α. The gain factor is an increasing function of α, as shown in Fig. 3. If the number of base stations grows asymptotically faster, the capacity gain is larger. Therefore, if m is bounded by two polynomials of n, n^{α_1} and n^{α_2} where $\alpha_1 < \alpha_2$, the capacity gain is bounded by two constants:

$$\frac{c_7}{\sqrt{1-2\alpha_1}} \le g(n,m) \le \frac{c_7}{\sqrt{1-2\alpha_2}} \tag{14}$$

where c_7 represents the constant factor in (12).

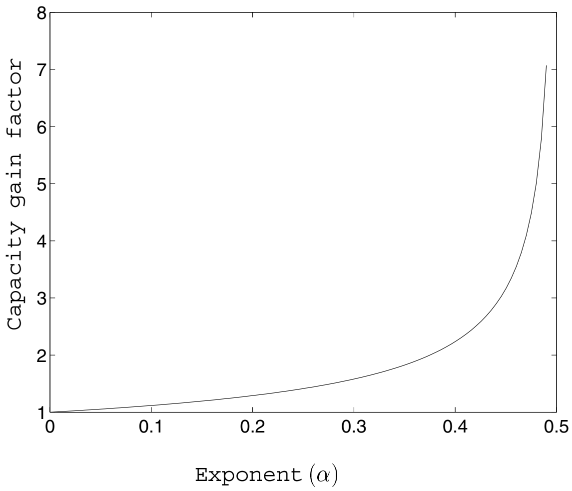

Fig. 3. Capacity gain factor as a function of α

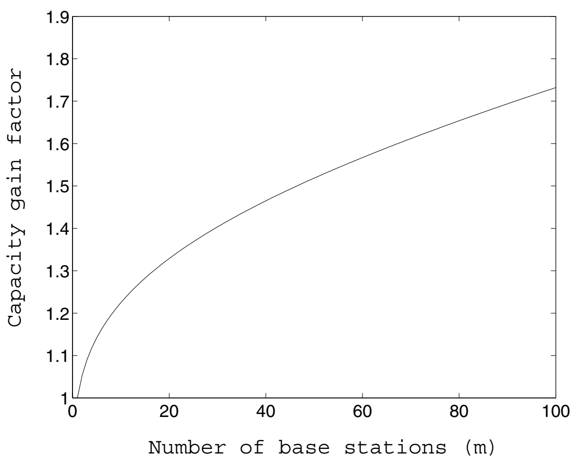

Fig. 4. Capacity gain factor

Above we show that the capacity gain is a constant if m scales as a polynomial of n. Now we show the benefit of placing more base stations on network capacity, if the number of nodes in the network is fixed. We use the same example as in the previous subsection, there are 1,000,000 nodes and the number of base stations varies from 1 to 100. According to (7), increasing the number of base stations from m to km only provides a less than $\log k$ fold increase in the maximum capacity. The throughput capacity of the pure ad hoc network does not change since n is fixed. As a result, the capacity gain factor only increases logarithmically with the number of base stations, as shown in Fig. 4.

2. $m = \Omega(\sqrt{n})$.

In this scenario, the maximum aggregate capacity of the hybrid networks is achieved when $W_1/W \to 0$, and the maximum capacity scales linearly with the number of base stations. The capacity gain factor is

$$g(n,m) = \Theta\left(m\sqrt{\frac{\log n}{n}}\right) \tag{15}$$

Again, we are interested in the scaling behavior of capacity gain when m scales as a polynomial of n. Assume $m = n^{\alpha}$ where $1/2 \le \alpha \le 1$, a simple derivation yields

$$g(n,m) = \frac{1}{\sqrt{\alpha}} m^{1-\frac{1}{2\alpha}} \sqrt{\log m} \tag{16}$$

If $\alpha = 1/2$, i.e., $m = \Theta(\sqrt{n})$, we have $g(n,m) = \Theta\left(\sqrt{\log m}\right)$. The capacity gain grows nearly logarithmically with the number of base stations. If $\alpha > 1/2$, $1 - \frac{1}{2\alpha} > 0$, the capacity gain grows polynomially with the number of base stations.

In an extreme case where $\alpha = 1$, we have $m = \Theta(n)$, and $g = \Theta\left(\sqrt{m \log m}\right)$. The per node throughput capacity is

$$\lambda(n,m) = \Theta(1)$$

This suggests that if the number of base stations grows asymptotically at the same speed as the number of nodes, each node gets a constant throughput capacity. The reason is that in this case, each base station serves a constant number of nodes. Therefore, each node can connect to the wired network using a constant share of the bandwidth, resulting in a $\Theta(1)$ per node capacity or a $\Theta(n)$ aggregate capacity.

IV. Capacity of Wireless Hybrid Networks under Probabilistic Routing Strategies

In this section we briefly present the throughput capacity of a hybrid network under the probabilistic routing strategies. We skip some of the derivations since the techniques are very similar to those used in Section III.

We first compute the throughput capacity contributed by the ad hoc mode transmissions. Let Z_i be a random variable that represents whether node i chooses to send data using ad hoc mode.

$$Z_i = \begin{cases} 1 & \text{node i chooses ad hoc mode} \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

According to the routing strategy, $\{Z_i\}_1^n$ is an i.i.d. sequence of random variables with $E[Z_i] = p$. The random variable $N_a = \sum_{i=1}^n Z_i$ represents the number of nodes that choose to send data using ad hoc mode in the network. By Strong Law of Large Numbers, with probability 1,

$$\frac{N_a}{n} = \frac{1}{n}\sum_{i=1}^{n} Z_i \to p \text{ as } n \to \infty$$

Using the same technique as we used to derive (4) in Section III, the throughput capacity contributed by ad hoc mode transmissions can be obtained as follows.

$$T_a = \Theta\left(\sqrt{\frac{np}{\log(np)}} W_1\right) \tag{18}$$

In each cell, in order to utilize the capacity provided by the infrastructure, at least one node must choose the infrastructure mode to send data. Otherwise, the base station does not receive or forward data. The uplink and downlink sub-channels are not used and the bandwidth is wasted.

Actually, if $p < 1$, the probability that at least one node chooses to send data in infrastructure mode approaches 1 as n goes to infinity. We define the random variable $\overline{Z}_i$ to be the opposite of Z_i, i.e., $\overline{Z}_i = 1$ if node i chooses the infrastructure mode and $\overline{Z}_i = 0$ otherwise. For cell k, denote the number of nodes in the cell as N_k, we have $\lim_{n\to\infty} N_k/n \to 1/m$ and thus $\lim_{n\to\infty} N_k \to \infty$. The capacity of cell k contributed by the infrastructure mode is

$$\begin{aligned} T_i(N_k) &= \Theta(W_2)E\left[\mathbf{1}\left(\sum_{i=1}^{N_k}\overline{Z}_i\right)\right] \\ &= \Theta(W_2)P\left(\sum_{i=1}^{N_k}\overline{Z}_i \geq 1\right) \\ &= \Theta(W_2)\left(1-p^{N_k}\right) \end{aligned}$$

Since $\lim_{n\to\infty} N_k \to \infty$, if $p < 1$,

$$\lim_{n\to\infty} T_i(N_k) = \Theta(W_2)$$

There are a total number of m cells in the network. For the whole network, the throughput capacity contributed by infrastructure mode transmissions is $\Theta(mW_2)$. Combine this with (18), we have the following theorem.

Theorem 3: For a hybrid network of n nodes and m base stations, under the probabilistic routing strategy, the aggregate throughput capacity of the hybrid network is:

$$T(n,m) = \Theta\left(\sqrt{\frac{np}{\log(np)}}W_1 + mW_2\right) \tag{19}$$

As can be seen from (19), for any channel allocation scheme, the throughput capacity is maximized when $p \to 1$, which implies that almost all the nodes should choose ad hoc mode in order to maximize the capacity. This is because the bandwidth of a base station is fully utilized as long as there is a node communicating using infrastructure mode. The bandwidth of the base station is shared among the nodes that use it to forward data. More nodes communicating through the base station does not increase the capacity contributed by the infrastructure. Since p is strictly less than 1, as n goes to infinity, the probability that at least one node chooses the infrastructure mode approaches 1. Therefore, the bandwidth of the base stations is guaranteed to be fully utilized when n goes to infinity, $p \to 1$ implies almost all the nodes should communicate using ad hoc mode in order to fully take advantage of the spacial concurrency.

To derive the maximum throughput capacity, we should account for all channel allocation schemes. With the same technique as we used to derive (6), we obtain the following corollaries.

Corollary 3: If $m = o\left(\sqrt{\frac{n}{\log n}}\right)$, the aggregated capacity is maximized when $W_2/W \to 0$ and $p \to 1$. The corresponding capacity is $\Theta\left(\sqrt{\frac{n}{\log n}}W\right)$.

Note that the maximum capacity in this case has the same asymptotic behavior as the capacity of a pure ad hoc network. When m grows asymptotically slower than $\sqrt{\frac{n}{\log n}}$, there is no significant benefit to use the infrastructure in terms of the throughput capacity.

Corollary 4: If $m = \omega\left(\sqrt{\frac{n}{\log n}}\right)$, the aggregate capacity is maximized when $W_1/W \to 0$ and $p \to 1$. The corresponding capacity is $\Theta(mW)$.

When the number of base station grows faster than $\sqrt{\frac{n}{\log n}}$, the maximum throughput capacity increases linearly with the number of base stations. Compared to a pure ad hoc network, the capacity gain factor is $g(n,m) = \Theta\left(m\sqrt{\frac{\log n}{n}}\right)$ And if $m = n^{\alpha}$ where $1/2 \leq \alpha \leq 1$, $g(n,m) = \frac{1}{\sqrt{\alpha}}m^{1-\frac{1}{2\alpha}}\sqrt{\log m}$, which increases polynomially with the number of base stations. Note that this is the same as the case when $m = \Omega(\sqrt{n})$ under the *k-nearest-neighbor* routing strategy.

V. Conclusions

In this paper, we studied the throughput capacity of hybrid wireless networks. A hybrid network consists of an ad hoc network and a sparse network of base stations. The base stations are connected by a wired network and placed in the ad hoc network in a regular pattern. Data may be forwarded in a multi-hop fashion as in ad hoc networks or forwarded through the infrastructure as in cellular networks. The goal of this paper is to investigate the benefit of the infrastructure to the throughput capacity and derive the asymptotic capacity of hybrid networks.

We consider a hybrid network of m base stations and n nodes, each capable of transmitting at W bits/sec over the common wireless channel. Under the k-nearest-cell routing strategies, if m grows slower than $\sqrt{n}$, the maximum aggregate throughput capacity is $\Theta\left(\sqrt{\frac{n}{\log\frac{n}{m^2}}}W\right)$. In this case, the benefit of adding base stations is insignificant. However, if base stations can be added at a speed asymptotically faster than $\sqrt{n}$, the maximum throughput capacity scales as $\Theta(mW)$, which increases linearly with the number of base stations.

In a probabilistic routing strategy, a transmission mode is independently chosen for each source destination pair with certain probability. Under this strategy, if m grows slower than $\sqrt{\frac{n}{\log n}}$, the maximum throughput capacity is $\Theta\left(\sqrt{\frac{n}{\log n}}W\right)$, which is of the same asymptotic behavior as the capacity of a pure ad hoc network. There is no benefit to use the infrastructure in this case. If m grows faster than $\sqrt{\frac{n}{\log n}}$, the maximum throughput capacity scales as $\Theta(mW)$, which increases linearly with the number of base stations.

For both routing strategies, there is a threshold for the scaling of the number of base stations (m) with respect to the number of nodes (n), where the maximum capacity changes the asymptotic behavior. When the number of base stations scales slower than the threshold, the capacity is dominated by the contribution of ad hoc mode transmissions. In this case, the effect of adding base stations on capacity is minimal. When the number of base stations scales faster than the threshold, the capacity is dominated by the contribution of the infrastructure. In this case, the maximum throughput capacity scales linearly with the number of base stations, providing an effective improvement over pure ad hoc networks.

Therefore, in order to achieve non-negligible capacity gain, the investment in the wired infrastructure should be high enough: the number of base stations should be at least $\sqrt{n}$ for the k-nearest-cell routing strategies and $\sqrt{\frac{n}{\log n}}$ for the probabilistic routing strategies.

The maximum throughput capacities are achieved when $W_1/W \to 0$ or $W_1/W \to 1$. Recall that W_1 is the channel bandwidth assigned to carry ad hoc mode transmissions. The conditions suggest that in order to maximize the throughput capacity, one of the two transmission modes will get almost all of the bandwidth while the other will get zero. In either case, some of the nodes will not get any bandwidth to send data. One way to avoid this situation is to assign some minimum amount of bandwidth to each sub-channel. In this case, the maximum capacities would be achieved when W_1 takes its minimum or maximum possible values. If the previous requirement is $W_1/W \to 0$, the new condition would be that W_1 takes the minimum value assigned to the ad hoc mode sub-channel. Note that this does not change the dominating scaling behavior of the maximum capacity. Our results about the scaling of the maximum capacity and the comparison to pure ad hoc networks still hold.

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