On the Coefficient of Heat Transfer from the Internal Surface of Tube Walls.

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1. Introductory.

The present work on the transfer of heat from a heated brass tube to water flowing through it was undertaken for the British Electrical and Allied Industries Research Association though the experimental methods pursued were left to the discretion of the experimenters. Two important innovations compared with the methods of previous experimenters were : First, the direct heating of the tube by a low tension alternating current ; and secondly, the discarding of the use of thermocouples to obtain the temperature of the water, this being calculated for any cross-section from the amount of heat put into the water up to that cross-section. The temperature so calculated is what an engineer always understands as the temperature of the water. The only other way of getting a definite water temperature is to take the temperature at points in the axis—a quite unimportant temperature in practice. Naturally the value that is obtained for the coefficient of heat transfer depends considerably on what is taken as the water temperature.

2. Historical.

Many experiments on the transfer of heat across condenser tubes have been made with the outside of the tube steam heated, the amount of heat transfer being obtained from the rise of temperature of the water flowing through the tube. None of this work has any scientific value, however, except in the few cases where the actual temperature of the tube was directly obtained by means of thermocouples. Here are included the experiments of Webster,[*] Clement and Garland[†] and McAdams and Frost.[‡] Also worthy of mention are some

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[*] " Some experiments in the Condensation of Steam," ' Inst. Engineers and Shipbuilders in Scotland,' vol. 57 (1913).

[†] " A Study in Heat Transmission," ' Bulletin No. 40 University of Illinois ' (1909).

[‡] ' J. Ind. Eng. Chem.,' vol. 5, pp. 13-18 and 1101-1105 (1922).

early experiments by Stanton* in which the temperature of the tube was obtained from its increase in length.

Webster's experiments suffer from the disadvantage, among other things, that the difference of temperature between the tube and the water was so great (the steam outside the tube being above atmospheric pressure and therefore above 100° C.) that, in view of the greatly varying properties of water over the range from 20° C. to 100° C., his results have comparatively little application to the problems of modern condensers. Clement and Garland's experiments suffer from the fact that they used an iron tube of about one-eighth of an inch (3 mm.) thick. Thermocouples gave the temperature of the outside of the tube whereas it was the temperature of the inside surface which was required; and it has been pointed out by McAdams and Frost that the uncertainty of the thermal conductivity of the iron render Clement and Garland's results uncertain by about 50 per cent. Moreover the increasing film of iron oxide which must develop with use must make reliance on their results difficult.

Probably the most accurate experiments published when the present work began were those of McAdams and Frost; but they had only obtained the coefficient of heat transfer for comparatively isolated cases, whereas, owing to its great variation, a systematic examination was required covering a wide range of water velocities, water temperatures, tube diameters and heat flow rates. Only when this has been done can any useful theoretical deductions be made from the experimental results. The experiments of Stanton also covered too limited a range; moreover Stanton's tubes were too short to give accurate coefficients of heat transfer since it is not till the water has traversed a sufficient length of the heated portion of the tube, to establish the limiting distribution of temperature over the cross section, that a definite coefficient can be obtained. The present experimental values refer entirely to these limiting coefficients.

3. *Range of Conditions covered by the present Experiments.*

The present experiments on commercial brass tubes covered the desired wide range of conditions. The water temperature was varied from about 40° F. (5° C.) to over 140° F. (60° C.); the water velocity from 1 ft. per second (30 cm. per second) to 11 ft. per second (330 cm. per second); the outside tube diameters from ½ inch to 1½ inches (inside diameters from 1·02 cm. to 3·56 cm.); while the heat flow rates varied from 4000 British thermal units

* 'Phil. Trans.,' vol. 190, pp. 67-88 (1897).

per square foot per hour to 20,000 British thermal units per square foot per hour (0·30 to 1·50 calories per square centimetre per second).

For each tube the water velocity was adjusted as nearly as possible to the desired round figure of so many feet per second, and tests were taken at various inlet water temperatures and rates of heat flow at this velocity.

In the present experiments the experimental tube was cleaned at the commencement of every morning's or afternoon's experiments, by drawing a wash-leather plug through it. Other experimenters do not seem to have sufficiently realised the necessity of this if accurate coefficients are to be obtained. The dirt removed was chiefly hydrated oxide of iron from the connecting pipes and was especially pronounced at the higher water temperatures.

4. *Symbols.*

The following symbols are used, all the quantities being in C.G.S. units :—

$d = 2a =$ diameter of tube ;

$0 =$ temperature of fluid (° C.) ;

$\Delta 0 =$ the temperature difference between the inside tube surface and the calculated mean temperature of the fluid at that cross section ;

$H =$ heat flow in calories per square centimetre per second of internal tube surface ;

k or $k_H \equiv H/\Delta 0 =$ coefficient of heat transfer ;

$k_0 =$ value of k when $H = 0$;

$\rho =$ density of fluid ;

$s =$ specific heat of fluid (at constant pressure in the case of a gas) ;

$c =$ thermal conductivity of fluid ;

$\mu =$ viscosity of fluid ;

$v =$ mean velocity of flow ;

$v_m \equiv \rho v =$ " mass velocity " ;

$f =$ " theoretical " thickness of viscous flow film at tube wall ;

$\phi f =$ actual thickness of above film ;

$\sigma \equiv \mu s/c$. This measures the departure from the ideal gaseous state for thermal conductivity, and might appropriately be called the " coefficient of liquidity " ;

$\tau \equiv \rho v d/\mu$. This is known as Reynolds' number, and may appropriately be called the " coefficient of turbulence " ;

$R =$ wall friction (in dynes per square centimetre) at tube surface causing the loss of hydrostatic head.

$Z \equiv R/\rho v^2$ is a function of τ only which has been experimentally determined by Stanton and Pannell.* Z is sometimes known as "Lees' function," he having given an empirical formula for it.

5. *Dimensional Equations.*

The consideration of the dimensions of the quantities involved, shows that if Q is the amount of heat gained or lost in unit time by a surface of any fixed shape, and of linear dimensions proportional to l, when a fluid of density ρ, specific heat s, viscosity μ, and thermal conductivity c, flows past it with a velocity v; and if the difference in temperature between the surface and any point in the fluid (in any given direction and at a distance proportional to l) is θ, then Q is given by

$$Q = \rho l^2 v^3 \cdot F \{s\theta/v^2, \quad \mu s/c, \quad \rho v l/\mu\}, \qquad (1)$$

where F is any function of its three arguments.†

If we assume that the temperature difference, θ, is infinitesimal it is obvious that Q will be infinitesimal and that Q/θ will, in the limit, be independent of θ. In this case Q can only involve θ through $s\theta/v^2$ as a factor. Hence we have

$$\text{limit } Q/\theta l^2 = \rho v s \cdot F \{\mu s/c, \quad \rho v l/\mu\}. \qquad (2)$$

As the left-hand side is proportional to the heat transfer coefficient, k_0, while, for a pipe of diameter d at points too far, both from the entrance and the point at which the heating begins to be affected by the initial irregularities due to these causes, no linear dimension is concerned save d, we get, on replacing l by d in (2) and inverting the formula for convenience,

$$\rho v s/k_0 \equiv v_m s/k_0 = F (\sigma, \tau), \qquad (3)$$

where $\sigma = \mu s/c$, $\tau = \rho v d/\mu$ and $v_m = \rho v$ is the mass flow. Experimentally, ρv is the quantity W/At, where W is the weight flowing past a cross section of area A in the time t. For water, we have termed v_m the "nominal velocity."

Equation (3) does not accurately hold for finite rates of heat flow because some or all of the quantities μ, s, c and ρ change with the temperature ; and

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* 'Phil. Trans.,' A, vol. 214, p. 199 (1914).

† See *e.g.* A. H. Gibson, "The Mechanical Properties of Fluids," p. 180. This formula, of course, entirely neglects any transfer of heat through the fluid by direct radiation, and so is not applicable to most gases where this may be a considerable fraction of the whole transfer. The external pressure to which the fluid is subjected is also assumed by this formula to have no effect on k. This must practically be so for both gases and ordinary liquids, but need not be so for liquids and vapours near their critical point where the molecular complexity changes greatly with a change of pressure.

hence to get k then by (3) would necessitate substituting the values of μ, s, c and ρ for some unknown temperature intermediate between that of the tube and the liquid.

By the principle of small variations and the theory of dimensions we get k_H, the coefficient for a small but not infinitesimal, rate of heat flow, H, given by

$$k_H - k_0 = \left\{ \frac{\lambda_1}{\mu} \frac{d\mu}{d\theta} + \frac{\lambda_2}{s} \frac{ds}{d\theta} + \frac{\lambda_3}{c} \frac{dc}{d\theta} + \frac{\lambda_4}{\rho} \frac{d\rho}{d\theta} \right\} H, \qquad (4)$$

where the λ's are mere numerical constants or functions of the dimensionless quantities σ and τ. In the case of many liquids $ds/d\theta$ and $d\rho/d\theta$ are negligible.

6. *Osborne Reynolds' Theory of Heat Transfer and its Modifications.*

Osborne Reynolds' well-known theory of heat transfer, founded on the similarity of the manner in which turbulent fluids convey both heat and momentum, gives, if we ignore the presence of a viscous flow film over the surface,

$$v_m s/k_0 = Z^{-1}. \qquad (5)$$

If a viscous film of small thickness is supposed to cover the tube wall and the whole of the turbulent core is supposed to have the velocity v and no thermal resistance, it is easily found, by calculating the thickness of the film from the viscosity, wall friction and transverse velocity gradient, that

$$v_m s/k_0 = \sigma Z^{-1}. \qquad (5\text{A})$$

G. I. Taylor* allowed for the experimentally demonstrated fact that the velocity at the film boundary is only a fraction, say ϕ, of v, so that the film thickness is only ϕ of the above amount ; in this case it can be shown that

$$v_m s/k_0 = (1 - \phi + \sigma\phi) Z^{-1} = \{1 + (\sigma - 1) \phi\} Z^{-1}, \qquad (6)$$

an equation first explicitly given by Sir T. E. Stanton† though he derived it from, and it is implied in, Taylor's paper just cited which was originally published prior to Stanton's paper. Equation (6) may conveniently be called the Taylor formula though neither Stanton nor Taylor specified that the heat flow rate was to be infinitesimal.

It is to be noted that when $\sigma = 1$—and it is not greatly different from unity for most gases and vapours—the presence or the viscous flow film is without effect on the thermal resistance as (6) then reduces to (5). But equation (6)

* 'Technical Report of Advisory Committee for Aeronautics,' vol. 9, p. 423 (1917).
† 'Technical Report of Advisory Committee for Aeronautics,' vol. 8, p. 16 (1916).

is in strong contradiction to what the present results give when extrapolated to $\sigma = 1$. For $\sigma = 1$ it will be seen that the results in Table I (p. 556) give

$$v_m s/k_0 = 1 \cdot 48\ Z^{-1} \quad \text{when} \quad \log \tau = 3 \cdot 7$$

falling to

$$v_m s/k_0 = 1 \cdot 04\ Z^{-1} \quad \text{when} \quad \log \tau = \infty,$$

showing a very large discrepancy at low values of τ which indicates that when the turbulence is feeble the thermal resistance of the fluid is much larger than is given by Reynolds' theory.

Now Stanton and Pannell's results on the loss of hydrostatic head show that if the loss of head for turbulent flow is extrapolated past the unstable and discontinuous region, it will become identical with the loss of head for viscous flow when $\log \tau \approx 3 \cdot 07$, *i.e.*, say when $\tau \approx 1175$. Since Z^{-1} is the main variable factor in (6) in the expression for $v_m s/k_0$ it may be expected with some reason that the k_0's when extrapolated, first to $\sigma = 1$ and then to low values of τ, should agree with the calculable results for pure viscous flow at about $\log \tau = 3 \cdot 07$. Also k_0 for viscous flow is calculated in Appendix A, as

$$v_m s/k_0 = 11\sigma/6Z. \tag{7}$$

At $\sigma = 1$ this is $v_m s/k_0 = 1 \cdot 833\ Z^{-1}$. The present results indicate a very satisfactory tending to this value at about $\log \tau = 3 \cdot 07$.

It is noteworthy that though the present results differ so markedly at low turbulence from the Taylor equation (6) they closely agree with a *general* linear formula in σ, say,

$$v_m s/k_0 = \{\alpha + \beta (\sigma - 1)\}\ Z^{-1}, \tag{8}$$

where α and β are functions of τ only. In fact this equation could be made to represent the *whole* of the present results covering from $\sigma = 3 \cdot 0$ to $\sigma = 10 \cdot 6$ and all values of τ to within $1\frac{1}{2}$ per cent. Nevertheless the deviation of the results from (8) was perfectly regular and systematic and in the manner which theory indicated was to be expected. Taking (8) to represent the results with sufficient accuracy, (7) shows that both α and β must become $11/6$ for viscous flow.

It is, of course, possible to calculate ϕ in (6) so as to make k_0 agree with any experimental k_0, but the ϕ's so calculated are not constants but vary with both σ and τ. Now if ϕ has the physical meaning attributed to it, it is absurd to suppose it varies with σ since it obviously has nothing to do with the thermal conductivity which is involved in σ; nor can the heat flow upset the value of ϕ for we are only dealing with *infinitesimal* rates of heat flow.

Since the thermal resistance of the viscous flow film cannot, apparently,

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be very different from ϕ times the value in (5A),* it appears that the expression in (5) cannot be considered as valid for the core but only as the value to which its thermal resistance would tend for very high rates of turbulence. Comparing equations (6) and (8) for both the film and the core we see that $\beta = \phi$, and that the thermal resistance of the core is $(\alpha - \beta)/(1 - \phi)$ times its value given by Reynolds' theory. At $\tau = 5000$ (*i.e.*, $\log \tau = 3 \cdot 7$) this ratio is (see Table I below) $(1 \cdot 483 - 0 \cdot 480) \div (1 - 0 \cdot 480) = 1 \cdot 93$; at $\tau = 1,000,000$ this ratio has fallen to $1 \cdot 055$; and it is shown in § 14 that there is evidence that it is below this, and presumably, exactly unity.

It is easy to see on theoretical grounds that the thermal resistance of the turbulent core must be considerably more than its value given by Reynolds' theory when the turbulence is feeble and sluggish. It can also be seen that in this case the thermal resistance must depend appreciably upon the thermal conductivity for complete temperature equalisation; that is the thermal resistance of the core itself must vary (at least when τ is small) with σ. It follows from this that it is impossible to apportion the thermal resistances of the core and film. Since the resistance of the core will increase with $1/c$ it must increase with σ. Hence the $\beta\sigma$ term in (8) will represent part of the thermal resistance of the core as well as the resistance of the film. Consequently ϕ, the fractional thickness of the film, is less than β.

7. *The Formula Adopted.*

As there must be, between the film and the core a region whose mean behaviour is intermediate between the behaviour of these, theory indicates that $v_m s/k_0$ instead of being a linear function of σ should be a continually increasing function of it with a gradient continuously decreasing to some asymptotic value, which asymptotic value would give the value of ϕ for the viscous film. This variation with σ is exactly what the present results indicate, except that in the variation with σ (for a constant τ) the curvature of the curves is so small for the range of σ covered ($3 \cdot 0$ to $10 \cdot 6$) that no *rate of change* in the rate of change of slope could be detected.

It follows that the present experiments can be exactly represented by

$$v_m s/k_0 = A' + B'\sigma - C\sigma^2,$$

or, as we prefer to write it, by

$$v_m s/k_0 = A + B(\sigma - 1) - C(\sigma - 1)^2, \tag{9}$$

* The effect of the curvature of the tube wall is easily calculated (since $f/d = 1/Z\tau$) and will be found to be quite small.

where A, B and C are positive functions of τ only. Inserting Z^{-1} as a factor in the right-hand side, we may write

$$v_m s/k_0 = \{\alpha + \beta (\sigma - 1) - \gamma (\sigma - 1)^2\} Z^{-1}. \qquad (10)$$

The experimental results gave ample justification for inserting Z^{-1} as a factor, in that α, β and γ were all found to be continuously decreasing functions of τ apparently tending asymptotically to definite limiting values.

Tables of A, B, C, α, β, γ and Z^{-1}—the latter from Stanton and Pannell— are given below (p. 556). It may be noted that Z^{-1} is, as is easily proved, equal to 8N, where N is the number of diameters of length in which the loss of hydrostatic head is equal to the velocity head. It should also be noted that formula (10) has only been arrived at for the range $3 \cdot 0 < \sigma < 10 \cdot 6$, and it should not be used when σ is, say, > 15. The quantity $v_m s/k_0$ is *necessarily* always an increasing function of σ whereas (10) gives a maximum value when $\sigma - 1 = \beta/2\gamma$—a value in the neighbourhood of 40 to 50.

As pointed out above, the resistance of both the core and the intermediate layer must increase with σ, so the resistance of the film is less than the $\beta\sigma$ term in (10); in fact, the β appertaining to the film resistance proper should be less than the *minimum slope* given by $\alpha + \beta (\sigma - 1) - \gamma (\sigma - 1)^2$ over the experimental range. This slope is $\beta - 20\gamma$ when $\sigma - 1 = 10$, which is roughly equal to $0 \cdot 25$.

From the present experiments none of the quantities A, B, C or α, β, γ appear to be simple analytic functions of τ so that no attempt is here made to find formulæ for them. A set of large scale graphs which can easily be constructed from the tabulated values given below is far preferable to complicated empirical formulæ.

Although the present experiments have been exclusively confined to water, it will be seen from the above analysis that the functions A, B, C and α, β, γ must be the same for all fluids, liquid and gaseous, just as Z is the same for all fluids. The limitations on these results, viz., that they only apply to an infinitesimal rate of heat flow and after the initial irregularity has died down, must not be overlooked. Also, in the case of gases, it must be remembered that the transfer of heat by direct radiation may be an appreciable fraction of the total amount.

8. *Description of Apparatus.*

A diagram of the experimental tube with its salient fittings is shown in fig. 1. The dimensions there given apply to the $\frac{3}{4}$-inch tube, though tubes of nominal outside diameters of $\frac{1}{2}$ inch, $\frac{3}{4}$ inch, 1 inch and $1\frac{1}{2}$ inches and of mean internal
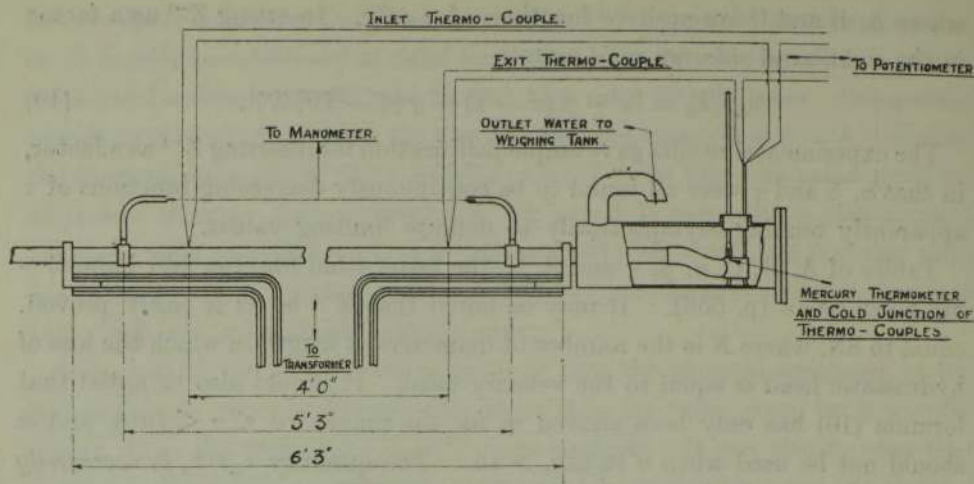
FIG. 1.—Diagram of Apparatus.

diameters of about 1·025 cm., 1·659 cm., 2·29 cm., and 3·56 cm., respectively, were all employed.

A length of 6 feet 3 inches of a ¾-inch tube, which was 15 feet in total length, was heated electrically by a low tension alternating current, fed into it by six copper straps of section 1½ inches × ⅛ inch in parallel* at the heavy brass lugs shown, which were soldered on to the tube. Two copper-constantan thermo-couples were soldered on to the tube at 13½ inches respectively from the ends of the electrically heated portion, care being taken that the two wires emerge separately, but near together, from a small blob of solder. To prevent any cooling of the thermo-junction the two wires were laid down close to the tube for about an inch and insulated from it by a thin flake of mica, and the wires and the tube at this place were then wrapped round with asbestos rope. Two grooved rings were also soldered to the tube at two points 5 feet 3 inches apart where four small holes had been drilled through the tube wall. These served to determine the loss of hydrostatic head over the included portion of the tube. Immediately beyond the heated portion the tube projected into a water box, also shown. A brass end-piece shaped somewhat like a tobacco pipe fitted into the end of the ¾-inch tube and the "bowl" served to contain the bulb of a mercury thermometer, which was used to obtain the temperature of the outlet water, together with the "cold junctions," immersed in oil in thin glass tubes, of the two thermocouples mentioned above. A third thermocouple with its hot junction in this bowl was used to obtain the temperature rise of the water

* Twelve straps in parallel were used for the 1½-inch tube.

and so to check the electrical heat supplied. With the 1-inch and the 1½-inch tubes this " tobacco pipe " was not necessary, nor was it necessary with the ¾-inch tube at the higher velocities, while it was required in all cases for the ½-inch tube.

The water supply came from a tank (not shown) 18 feet above the experimental tube. The water in this tank could be heated by direct steam heating—the steam issuing from nozzles and mixing with the water. A motor driven stirrer was also provided.

The value of the electric current was given by a bus-bar transformer and an ammeter which were calibrated together against a Kelvin current balance in the Electrical Standards Department of the Manchester College of Technology. The electrical resistance of the heated portion was obtained by previously comparing on the potentiometer the drop of potential across the brass lugs with that across a standard low resistance of about $0·002$ ohm in series with it when a direct current passed through both. This was done with water at different temperatures flowing down the tube. The electrical input was taken as given by C^2R, the error due to the unequal distribution of current over the cross section being utterly negligible. The extreme currents employed varied from 520 to 2750 amperes for the different tubes.

The third thermocouple mentioned above had its cold junction immersed in oil in a thin glass tube, inserted into the condenser tube about 2 feet before the commencement of the heated portion. It served to check the electrical heat supplied ; and the two heats were found to be in quite satisfactory agreement, being often within 1 per cent. and nearly always within 2 per cent. The heat supplied could be determined with much greater *consistent* accuracy electrically than by this thermocouple, as the arrangement at the inlet water junction was not so satisfactory for getting the temperature of the water as that at the outlet end, and the diameter of the tube being too small to permit the temperature being taken up accurately by the thermojunction, so that it was the C^2R determination of heat which was always used in reducing the results. The check served, however, to show that there was no reason for doubting the accuracy of the electrical heat to more than 1 per cent.

The two thermocouples soldered on the tube were calibrated *in situ* by running water at various temperatures and a fairly high velocity through the tube and placing the " cold-junctions " either in ice or in a thermos flask which contained oil or water heated to any desired temperature up to about 230° F. From these results a graph was constructed showing the " thermoelectric power " plotted against the mean temperature of the junctions over a range from about

40° F. to 190° F.   The E.M.F. of the thermocouples was obtained on a potentio-meter in the usual way with a Weston normal cell employed to ensure a constant potential gradient down the potentiometer.   The sensitivity was about 10 cm. of wire to 1° F.

The mean internal diameters of the tubes were determined by weighing them empty and full of water. . This readily gave the weight of water per second corresponding to a (nominal) velocity of 1 foot per second.

The mean water temperatures at the positions of the thermocouples were calculated from the heat put into the water up to those positions ; that is, it was taken to rise linearly along the tube.   The temperature drop across the tube wall was allowed for, but since the heat was generated throughout the bulk of the material this drop was only half what it would have been had the heat been supplied from outside the tube.   The loss of heat from the external surface of the tube was determined and allowed for, though it was nearly negligible.

Three heat flow rates of approximately 4000, 12,000 and 20,000 B.Th.U. per square foot per hour were adopted as the most convenient (save in the case of the $1\frac{1}{2}$-inch tube where the rates had to be reduced to 4000, 8000 and 12,000 on account of the large currents required).   As the bus-bar transformer had two windings which could be connected either in series or parallel, heat flows in the ratio $1 : 3 : 5$ gave very comparable ammeter readings in the ratio $2 : \sqrt{3} : \sqrt{5}$.   In the case of the $\frac{1}{2}$-inch tube the current was so small that it was led twice through the bus-bar transformer.

The alternating supply was from a separate turbo-generator and the heating current was kept constant by continuous hand regulation of a liquid rheostat in the primary circuit of the transformer by an observer watching the bus-bar ammeter.

Nominal or " mass flow " water velocities of 2, 3, 5, 8 and sometimes 1 and 11 feet per second were mostly employed, and were set as near as possible beforehand till the right weight-flow in 5 minutes was obtained.   Correction to these exact velocities was made taking k to vary as $v^{0\cdot8}$.   Inlet water temperatures in steps of about 20° F. (or less) from 40–50° F. to about 140–150° F. were taken, this temperature, and the water velocity, being kept constant while the tests at the three heat flow rates were made.

To obtain greater accuracy in the k's the three values of $\Delta\theta$ (the net tempera-ture difference between the tube and the water) were plotted against the rate of heat flow and a smooth curve—a slightly curved parabola—was drawn through the origin among these three points.   The $\Delta\theta$'s used in calculating

the k's were those taken from this curve. This method was especially valuable at the higher temperatures where it was difficult to get temperature differences correct to the desired $0 \cdot 02$ to $0 \cdot 05°$ F., since it was difficult to keep the temperature of the inlet water constant to anything like this when it was being preheated by 50–100° F.

It is perhaps worth while placing on record here the fact that the potentiometer used with the thermocouples was a Callendar and Griffiths bridge with the connections transposed for potentiometer use. This had the effect of doubling the value of the coils in terms of a centimetre of wire and necessitated the electrical determination of the "end correction" of the wire. With the ordinary student's metre-wire potentiometer first employed very considerable difficulty was experienced owing to the large kicks which the galvanometer gave on making and breaking its circuit owing to the alternating current flowing through the tube to which the thermocouples were soldered. With this excellent instrument these difficulties, which threatened to make the use of an electrically heated tube impossible, disappeared.

9. *A Sample Experiment.*

In order to allow a critical estimate of the accuracy which the experimental method pursued is capable of, we subjoin here particulars of a sample experiment with one of the $\frac{3}{4}$-inch tubes. The experiment selected was with a water velocity of 5 feet per second and an inlet water temperature of about 87° F., and was selected because, corresponding to fairly mean conditions, it gives a better idea of the magnitude of the temperature rises usually obtained than extreme cases.

Sample Experimental Test with a $\frac{3}{4}$-inch Tube.—A length of $178 \cdot 80$ inches held a weight of $2 \cdot 1475$ lbs. of water at 61° F. giving a mean internal diameter of $0 \cdot 6513$ inch ($1 \cdot 654$ cm.), an internal heated area of $1 \cdot 0656$ sq. ft., and a flow of $216 \cdot 4$ lbs. in 5 minutes for a "nominal velocity" of 5 feet per second.

Heat flow.	First.	Second.	Third.
1. Heating current in amperes	840	1458	1876
2. Weight of water in 5 minutes (lbs.)	218	218	$216 \cdot 5$
3. Temperature of outlet water (°F.)........................	$88 \cdot 9$	$92 \cdot 6$	$96 \cdot 3$
4. Temperature rise on inlet (or first) thermocouple ...	$1 \cdot 87$	$5 \cdot 50$	$8 \cdot 90$
5. Temperature rise on outlet (or second) thermocouple	$2 \cdot 91$	$8 \cdot 89$	$14 \cdot 37$
6. Temperature of outside of tube at inlet t.c.	$90 \cdot 77$	$98 \cdot 10$	$105 \cdot 20$
7. Temperature of outside of tube at outlet t.c.	$91 \cdot 81$	$101 \cdot 49$	$110 \cdot 67$
8. Allowed drop through tube wall	$0 \cdot 12$	$0 \cdot 36$	$0 \cdot 60$
9. Temperature of inside tube surface at outlet t.c. ...	$91 \cdot 69$	$101 \cdot 13$	$110 \cdot 07$
10. Resistance of heated portion of tube in milli-ohms	$1 \cdot 913$	$1 \cdot 924$	$1 \cdot 934$

Heat flow.	First.	Second.	Third.
11. C²R heat in B.Th.U. per foot per hour (1 watt = 3·412 B.Th.U. per hour)	4320	13,110	21,800
12. Extra external heat loss due to rise in tube temperature with electric heat	10	40	60
13. Net heat flow rate H	4310	13,070	21,740
14. Calculated water temperature rise from electric heat and water weight	1·75	5·33	8·91
15. Water temperature at outlet t.c. below exit water (= 0·18 of above)	0·33	0·96	1·60
16. Water temperature at outlet t.c. [= (3) — (15)] ...	88·57	91·64	94·70
17. Temperature difference between tube and water at outlet t.c. [= (9) — (16)]	3·12	9·49	15·37
18. Above temperature difference corrected to exact water velocity	3·14	9·55	15·37
19. The same smoothed graphically against H to vanish with it [= Δθ]	3·19	9·47	15·40
20. K = H ÷ Δθ	1352	1380	1410
21. K corrected for conicality of tube by adding 0·83 per cent. (see below)	1363	1391	1421
22. Water temperature for above [= (16) repeated]......	88·6	91·6	94·7

K extrapolated to zero rate of heat flow gives $K_0 = 1348$ at 87·0° F. That is $k_0 = 0\cdot1825$ calories per square centimetre per second per °C. at a water temperature of 30·5° C. Now σ for water at 30·5° C. is 5·25 (see Table II, below, p. 559) while $\tau = v_m d/\mu = 152\cdot4 \times 1\cdot654 \div 0\cdot00792 = 31,830 = \log^{-1} 4\cdot503$. Now Table I, below (p. 556) using the values of A, B and C at $\log \tau = 4\cdot50$, gives $v_m s/k_0 = 152\cdot4/k_0 = 840$ or $k_0 = 0\cdot1815$; while at $\log \tau = 4\cdot503$ Table I gives $k_0 = 0\cdot1818$ in very satisfactory agreement with the above value especially as the table was constructed long before the above tube was bought; and it was bought from a different maker to see, among other things, whether the same value of k could be obtained from different makers' tubes.

A source of error discovered rather late in the work was the conicality of the internal bore of the tubes mentioned in item (21). The outside diameters were remarkably constant and the same was therefore assumed of the internal bore. However, testing the uniformity of potential drops over a measured length showed a variation of resistance per unit length of from 2½ to 13 per cent. for the different tubes. As this resistance varies inversely as the area of the metallic cross section it was possible to find the actual internal diameter at any point from its mean value. This necessitated three corrections to the coefficients, k, (i) a correction to the heat supplied per unit length, (ii) a correction to the area of the internal surface of the tube, and (iii) a correction to the water velocity.*

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* There are certain other small errors which must be considered : (1) The water will not rise at a uniform rate along the tube since its electrical resistance is not constant, the outlet-end being warmer than the inlet end. This was considered and the effect found to be negligible ; (2) owing to the heat flow along the tube in the opposite direction to the water velocity some of the heat will enter the water before the point at which it was generated. If l is the heated length, S the area of metallic cross section, c_0 the thermal conductivity of brass and θ_l the difference of temperature along the tube, the longitudinal heat flow

10. *Method of Reducing the Experimental Results.*

The values of k recorded in this paper depend entirely upon the temperatures given by the thermocouple near the outlet end of the tube ; the other one was chiefly used to obtain the mean temperature of the tube upon which its resistance depended. The values of k, obtained in sets of three as described, were plotted against the water temperature at the position of the outlet-end thermocouple and were then interpolated and extrapolated to give k for the three exact heat flow rates of zero, 10,000 and 20,000 B.Th.U. per square foot per

is $c_b S \theta_d/l$ so that the heat flowing into the water between the outlet end thermocouple and the end of the heated portion is not $2\pi a \times 0\cdot18l \times H$ ($0\cdot18l$ is the distance of the thermocouple from the end) but is less than this by $c_b S \theta_d/l$. The ratio of these quantities is $c_b S \theta_d/0\cdot36\pi al^2 H$. In C.G.S. units this becomes $\theta_d/180,000H$. As θ_d is rarely over $10°$ C. at the highest heat flow rate when $H = 1\cdot5$ calories per square centimetre per second this effect is completely negligible. (3) The *differential* longitudinal heat flow,

$$c_b S \frac{d^2\theta}{dx^2}$$ per unit length, becomes transverse heat flow into the water. Assuming a parabolic distribution of temperature along the tube given by $\theta = 4\theta_m x(l - x)/l^2$, superposed on a uniform temperature gradient (this gives a mid-point temperature in excess of the arithmetical mean of the end temperatures by θ_m) we see that this differential flow is $8c_b S \theta_m/l^2$ per unit length. This bears a ratio of $4c_b S \theta_m/\pi al^2 H$ to the heat flow into the water per unit length. Numerically this ratio is $\theta_m/130,000H$ and so is utterly negligible. (4) The sudden drop of tube temperature which would occur at the point where the heating ceases if there were no longitudinal conduction of heat is smoothed off by such conduction and the effect of this will extend for some length along the tube. Calculation shows that for a thin walled tube of thickness t the reduction of the tube temperature at a distance x before the end of the heated portion is approximately $\frac{1}{2}\Delta\theta . e^{-nx}$ where $n^2 = k/tc_b$. Taking $c_b = 0\cdot26$, $t = 0\cdot125$ and a very low value of k of $0\cdot050$—its value for water at $5°$ C. and 2 feet per second—$n = 1\cdot24$. So that at the position of the outlet thermocouple, $x = 34$ cm., this effect is infinitesimal. (5) Another error is due to the heat flow into or from the copper straps via the lugs. Neglecting the C^2R heat generation in the copper straps this error was compensated for by taking the thermocouple readings on the potentiometer when the heating current was zero and using these readings as a "false zero." This method is almost inevitable when it is required to determine small temperature differences accurately at temperatures considerably above room temperature by means of thermocouples. Owing also to this method of working, the external heat loss to be allowed for in item (12), above, is not the whole heat lost by the outside of the tube, but only the extra heat lost by the rise in temperature of the tube during heat flow. (6) The remaining error, the flow into the tube of the C^2R heat generated in the copper straps, could readily be detected with the largest of the three currents, by a noticeable creep in the thermocouple readings in those experiments, where, owing to the conditions not being sufficiently steady, delay was experienced in getting the readings. Such experiments were repeated another time. We do not think that an error of more than one or two parts in a thousand could have arisen in the majority of experiments from this cause.

hour calculated on the area of the internal surface of the tube. The sets of points for these exact heat flow rates at different water temperatures, but the same velocity, were then connected by smooth curves. From these curves values of k at definite temperatures were selected at the different velocities and log k was plotted against log v_m. These curves were only slightly curved lines of a slope approximately 0.77 to 0.81 and enabled the k's for the intermediate velocities (which were generally 4, 6, 7, 9 and 10 feet per second) to be obtained.

Having done this work for the four diameters of tubes, the velocities divided by the k_0's at 40° F. (which gives $v_m s/k_0$ since $s = 1$ sensibly) were plotted on a single diagram against $v_m d$ for all the different velocities and tube diameters. Similar diagrams were drawn for each of the temperatures 60°, 80°, 100°, 120° and 140° F. By theory all the points on each of these diagrams should lie on a single curve. This was found to be so to within $1\frac{1}{2}$ per cent. ; some tubes giving points consistently high and some consistently low.*

The six mean curves on these six sheets were next replotted on a single large sheet with their abscissæ divided by the viscosity of water for the temperature in question. The abscissa were then the τ's. The quantity $\sigma = \mu s/c$ was next determined for these six temperatures using Kaye and Higgins' recent

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* Some of this systematic deviation between one tube and another seems from later tests to be due to the conicality of the tube : the three known effects of conicality were allowed for as above ; but in a check test in which the same tube was reversed in direction, values of k differing by 2 to 3 per cent. were obtained ; the difference apparently being due to nothing save the reversal. The values of k were greater for a converging than a diverging tube. It is difficult to explain this. We can only suggest that vortices of a certain diameter are formed in the tube, and when moving along a converging tube tend to cut away the viscous film more than they would in a parallel tube and thus decrease the thermal resistance. But even this suggestion seems fantastic considering that the semi-angle of the conical surface was only about 1/40,000 of a radian. Possibly the slight irregularities on the surface were such that they offered less resistance to water flowing towards the larger end than towards the smaller end ; this would make the viscous film thicker in the former case ; and this direction is, of course, the direction in which the mandril was withdrawn from the tube after manufacture ; it was to facilitate this withdrawal that the tubes were made conical. The k's for the $1\frac{1}{2}$-inch tube were among the high ones, but this was thought to be natural as with this tube it was impossible to take the tests as many diameters away from the initial disturbances as with the other tubes.

Some tubes from another manufacturer were obtained and tested after the table of results in Table I below was constructed. These tubes were found to have considerably less conicality than the previous ones, and the values of the heat transfer coefficient obtained from them were in very satisfactory accord with the table, as is seen by the sample experiment given above which was from one of these later tubes.

values for the conductivity of water,* in the absence of which the thermal conductivity of water could not be considered to be known well enough to make the present analysis reliable. Ordinates were next drawn for selected values of τ and the values of $v_m s/k_0$ were replotted as functions of σ for these values of τ. These curves, as previously stated, were found to be slightly curved parabolas for which the constants in the equation

$$v_m s/k_0 = A + B(\sigma - 1) - C(\sigma - 1)^2 \qquad (11)$$

were very easily found. These constants were then plotted against the corresponding value of τ when very satisfactory smooth curves were obtained. In fact the values of k_0 obtained from formula (11) by using the smoothed curves for A, B and C agreed with the values of the two large sheets to within one-third of 1 per cent. practically everywhere.

11. *Table of Results.*

Table I, below, gives the values as above determined for A, B and C together with the values of Z^{-1} from Stanton and Pannell, and the values of α, β and γ defined by $\alpha = AZ$, $\beta = BZ$ and $\gamma = CZ$.

To get regular smoothed values of Z for this analysis the ordinates midway between the two limiting curves marked N.P.L. on Stanton and Pannell's well-known diagram (fig. 5 of their paper) were taken as giving Z. A formula to represent these ordinates has been previously published by one of the present writers.†

The above seven functions are given for regular intervals of log τ that being the most convenient variable to plot them against. The next most convenient variable is $\tau^{-\frac{1}{3}}$ the values of which are also given. Z when plotted against $\tau^{-\frac{1}{3}}$ is practically a straight line. In the letter to 'Nature' above referred to, reason is given for thinking that any correct expression for Z should be intelligible, when τ is negative, which it is when Z is expressed in rational terms of $\tau^{-\frac{1}{3}}$. This follows since there should be no sudden discontinuity in Z when the curvature of the tube wall—which is $2/d$—passes through zero, *i.e.*, when $\tau^{-\frac{1}{3}}$ passed through zero. For negative values of τ, $\rho v^2 Z$ represents the surface friction when a very long cylinder of diameter d is dragged axially through

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* 'Roy. Soc. Proc.,' A, vol. 117, p. 459 (1928). The values of σ taken for the six temperatures were $10 \cdot 60$, $7 \cdot 60$, $5 \cdot 73$, $4 \cdot 49$, $3 \cdot 62$ and $2 \cdot 99$ respectively, the last figure being for $140°$ F.

† 'Nature,' vol. 123, p. 14 (1928).

water with a velocity v, which is thus a matter of considerable theoretical interest.

Table I.

log τ.	$\tau^{-\frac{1}{3}}$.	A.	B.	100 C.	Z^{-1}.	α.	β.	100γ.
3·6	0·0631				195	(1·55)	(0·53)	(0·63)
3·7	0·0584	307	99·$_5$	123	207	1·483	0·480	0·59$_5$
3·8	0·0541	311	97	124·$_5$	220	1·414	0·439	0·56$_5$
3·9	0·0501	315	96	126	234	1·343	0·411	0·53$_5$
4·0	0·0464	321	97·$_5$	127·$_5$	250	1·284	0·390	0·51
4·1	0·0430	329	99	129	266	1·237	0·372	0·48$_5$
4·2	0·0398	341	101·$_5$	130·$_5$	284	1·202	0·358	0·46
4·3	0·0369	356	104·$_5$	132	303	1·175	0·345	0·43$_5$
4·4	0·0341	372	108	133	323	1·152	0·333	0·41
4·5	0·0316	390	111·$_5$	134	344	1·133	0·324	0·39
4·6	0·0293	409	116	135·$_5$	366	1·118	0·316	0·37
4·7	0·0271	429·$_5$	120·$_5$	137·$_5$	388	1·105	0·311	0·35$_5$
4·8	0·0251	450	126	140	411	1·094	0·307	0·34
4·9	0·0233	470·$_5$	132	143	434	1·084	0·305	0·33
5·0	0·0215	491·$_5$	138·$_5$	146·$_5$	457	1·075	0·303	0·32
5·1	0·0199	512·$_5$	145	150·$_5$	480	1·069	0·302	0·31$_5$
5·2	0·0185	535	151·$_5$	155·$_5$	503	1·064	0·302	0·31
5·3	0·0171	555	158·$_5$	160·$_5$	524	1·060	0·301	0·30$_5$
5·5	0·0147				565	(1·053)	(0·301)	(0·30)
6·0	0·0100				650	(1·045)	(0·300)	(0·29$_5$)
∞	0				860 ?	(1·040)	(0·300)	(0·29)

Numbers in brackets are extrapolated.

The coefficients k_0 are given from the above table by the formulæ

$$\rho v s / k_0 = A + B\,(\sigma - 1) - C\,(\sigma - 1)^2$$
$$= \{\alpha + \beta\,(\sigma - 1) - \gamma\,(\sigma - 1)^2\}\,Z^{-1}.$$

Values of σ for water are given in Table II, below, while a curve of log $1/\mu$ or $\mu^{\frac{1}{3}}$ for water (or other fluid) is very convenient for evaluating log $v_m d/\mu$ or $\{\mu/v_m\,d\}^{\frac{1}{3}}$, i.e., $\tau^{-\frac{1}{3}}$.

12. Coefficient of Heat Transfer at Infinite Velocity.

As pointed out, the equation (6) requires that $v_m s/k_0 = Z^{-1}$ when $\sigma = 1$ for all values of τ. That is, it requires that α in Table I should be unity throughout. It is quite possible, and even likely, that this may be so when $\tau = \infty$, for then, and only then, are the conditions necessary to make Reynolds' theory exactly true completely satisfied. Experimentally no values of σ less than 3·0 were employed so that the values for $\sigma = 1$ are a pure extrapolation. But it was shown in §7 that the rate of *decrease* of $v_m s/k_0$ must increase with

increasing rapidity as σ decreases. Thus it is quite likely that some extra term of this behaviour should be added to (10) giving, say, something like

$$v_m s / k_0 = \{\alpha + \beta (\sigma - 1) - \gamma (\sigma - 1)^2 - 0 \cdot 04 \sigma^{-2}\} Z^{-1}. \qquad (12)$$

This expression would then satisfy Reynolds' theory at $\tau = \infty$ and over the range of the experiments would introduce a deviation of less than $0 \cdot 3$ per cent., which is well within the experimental errors. No physical objection can be raised against the σ^{-2} term for giving an infinite value when $\sigma = 0$ since σ cannot be much below unity for any substance. We mention this point chiefly for those who may desire to extrapolate our results below $\sigma = 3 \cdot 0$.

13. *Loss of Hydrostatic Head during Heat Transfer.*

It was one of the original objects of this research to determine the loss of hydrostatic head at the same time as heat transfer was taking place. For this purpose the loss of head over a length of about five-sixths of the electrically heated portion of the tube was measured on a water manometer during each heat-flow test. These determinations gave the extremely simple result, in nearly all cases, that the loss of head during heat transfer was the same as if the water had been at the *tube* temperature and no heat transfer was taking place. When inlet water down to about 40° F. could be obtained it was found that the law was slightly different from this, the loss of head then being the same as if there was no heat flow and the water was at its own temperature plus about 90 per cent. of the difference of temperature between tube and water ; but with rising temperature this fraction soon became sensibly 100 per cent. It is shown in Appendix B that for purely viscous flow this fraction is theoretically $6/11 \approx 54 \cdot 5$ per cent.

14. *Coefficients at Finite Rates of Heat Flow.*

We have already given, in § 5 above, the theoretical form of the expression for the difference between k_H and k_0 at a heat flow rate H. A concrete approximate expression may be obtained theoretically when τ is large. In this case α and β are sensibly constants ; that is, they will not vary with the temperature, which they do generally since τ varies with the temperature through μ. Neglecting the γ term in (10) as being small compared with the other two, we have

$$v_m s / k_0 = (\alpha - \beta + \beta \sigma) Z^{-1};$$

where the $\alpha - \beta$ term and the $\beta \sigma$ term represent the resistance of the core and the film respectively. Now we have just stated that Z is practically

unaltered by heat flow at the same *tube* temperature. Hence for a finite rate of heat flow with a given tube temperature we have merely to change $\beta\sigma$ into $\beta\bar{\sigma}$ where $\bar{\sigma}$ represents the mean value of σ through the film. But

$$\bar{\sigma} = \sigma - \tfrac{1}{2}\frac{d\sigma}{d\theta}\,d\theta,$$

where $d\theta$ is the difference of temperature across the film, and the σ is the value of σ at the tube well temperature. Now the difference of temperature from tube to water is $H/k_H \approx H/k_0$, and the temperature difference across the film is clearly the fraction $\beta\sigma/(\alpha - \beta + \beta\sigma)$ of this amount. Hence we have

$$v_m s/k_H = \left\{\alpha - \beta + \beta\left(\sigma - \frac{\beta\sigma}{2(\alpha - \beta + \beta\sigma)}\frac{H}{k_0}\frac{d\sigma}{d\theta}\right)\right\} Z^{-1};$$

whence

$$\frac{k_H^{-1} - k_0^{-1}}{k_0^{-1}} \approx \frac{k_0 - k_H}{k_0} = -\left(\frac{\beta\sigma}{\alpha - \beta + \beta\sigma}\right)^2 \cdot \left(\frac{1}{2\sigma}\frac{d\sigma}{d\theta}\right)\frac{H}{k_0}. \qquad (13)$$

Now $(k_0 - k_H)/k_0$ is small for ordinary rates of heat flow in practice, so that an approximate evaluation of (13) will suffice. It is seen from fig. 2 that
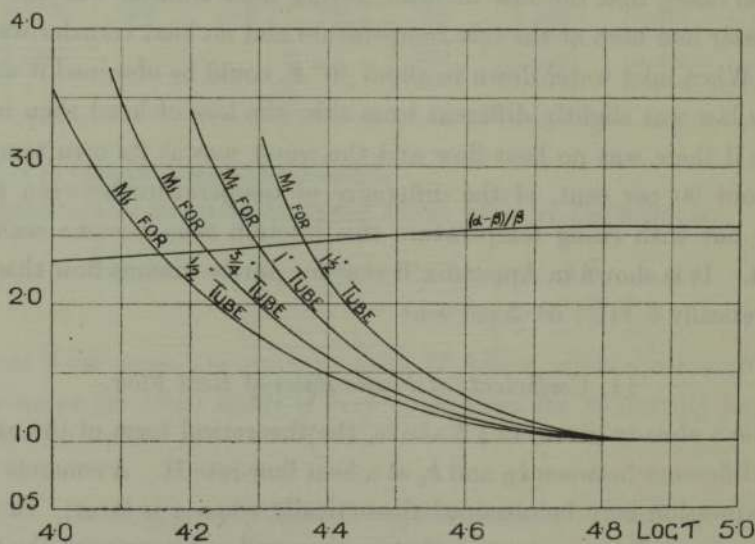

FIG. 2.—Multiplying Factors and $(\alpha - \beta)/\beta$.

$(\alpha - \beta)/\beta$ is remarkably constant over a wide range; from $\tau = 13,000$ to $\tau = \infty$ it only differs from $2\cdot45$ by less than 4 per cent. Hence we have, approximately, for large values of τ,

$$k_H - k_0 = -\left(\frac{\sigma}{2\cdot45 + \sigma}\right)^2 \cdot \left(\frac{-1}{2\sigma}\frac{d\sigma}{d\theta}\right)H. \qquad (14)$$

Table II, below, shows the value of σ and $\dfrac{-1}{2\sigma}\dfrac{d\sigma}{d\theta}$ for water in Centigrade units. The values of σ are required for use in conjunction with Table I; they involve a considerable extrapolation of Kaye and Higgins' linear values of c, but they have been given to 160° C. because values of σ for such temperatures are required for modern feed-water heaters, for which purpose these values of σ are amply accurate enough, as when $\sigma \approx 1$ the percentage change in k_0 for a 1 per cent. change in σ is only about one-third of what it is for cold water; moreover, k_0 being so much greater for hot water, the temperature differences causing the heat flow are so much less that the same accuracy in k_0 is not needed. The values of $\dfrac{-1}{2\sigma}\dfrac{d\sigma}{d\theta}$ were deduced from the values of σ by the finite difference formula

$$D = \Delta - \tfrac{1}{2}\Delta^2 + \tfrac{1}{3}\Delta^3 - \ldots$$

Table II.

°C.	σ.	$q = \dfrac{-1000}{2\sigma}\dfrac{d\sigma}{d\theta}.$	$\left(\dfrac{\sigma}{2\cdot45+\sigma}\right)^2 q.$
0	12·45	18·0	12·6
5	10·46	16·5	11·0
10	8·93	15·1$_5$	9·5
15	7·73	13·9$_5$	8·0
20	6·75	13·0	7·0
25	5·96	12·0$_5$	6·0
30	5·30	11·2$_5$	5·3
40	4·29	9·9	4·0
50	3·55	8·8$_5$	3·1
60	2·99	8·0	2·4
70	2·56	7·3$_5$	1·9
80	2·22	6·7$_5$	1·5
90	1·95	6·2	1·2
100	1·74	5·7	1·0
110	1·56	5·2	0·8
120	1·40	4·8	0·6
130	1·27	4·3	0·5
140	1·17	3·9	0·4
150	1·09	3·4$_5$	0·3
160	1·02	3·0$_5$	0·2

It must be remembered that k_H and the corresponding k_0 are at the same *tube* temperature and that therefore k_H is at a *water* temperature of H/k_H below the water temperature of the corresponding k_0.

At the higher velocities or higher water temperatures the above calculated differences between k_H and k_0 agreed excellently with the experimental results but deviated markedly at the lower temperatures and velocities.

However, on comparing the actual $(k_H - k_0)$'s with the values calculated for them by (14), it was found that the ratios between them (for various water temperatures and velocities) when plotted against τ all lay on the same curve for results on the same tube. This ratio was taken as an empirical multiplying factor. Denoting it by M_f, (14) may be written in the revised form,

$$\frac{k_H - k_0}{H} = -\left(\frac{\sigma}{2 \cdot 45 + \sigma}\right)^2 \cdot \left(\frac{-1}{2\sigma} \frac{d\sigma}{d\theta}\right) M_f. \tag{15}$$

Table II, above, in addition to giving values of $\dfrac{-1}{2\sigma}\dfrac{d\sigma}{d\theta}$ for water also gives

the same quantity multiplied by $\left(\dfrac{\sigma}{2 \cdot 45 + \sigma}\right)^2$; while fig. 2 shows the multiplying

factors arrived at for the four tubes of nominal outside diameters of $\frac{1}{2}$ inch, $\frac{3}{4}$ inch, 1 inch and $1\frac{1}{2}$ inches respectively. In all cases the multiplying factors decrease to their theoretical value for unity for values of τ above 60 to 70 thousand.

For cold water at low velocities $k_0 - k_H$ becomes relatively large ; but if k_H is compared with k_0 for the *same water temperature* (k_0' say) the difference $k_H - k_0'$ (which is then positive) remains of nearly the same magnitude.

The above theory of the difference between the k_H's and the k_0's should be

applicable to all fluids when once the function $\dfrac{1}{2\sigma}\dfrac{d\sigma}{d\theta}$ has been determined for

that fluid. An exception must be made in the case of gases where the change of density consequent upon a change of temperature is so great that entirely fresh convection currents may be set up by this cause alone. This means that

the above theory fails because it does not take into consideration the $\dfrac{\lambda_4}{\rho}\dfrac{d\rho}{d\theta}$

term in (4).

15. *The variation of k at the commencement of heating.*

A crude investigation, perhaps better than nothing, may be made as follows into this difficult problem. Treating γ and f/d as negligible, and identifying β and ϕ, as an approximation, we will represent the temperature, θ, at a distance r from the axis by

$$\theta = \frac{\alpha - \beta}{\alpha - \beta + \beta\sigma} \{1 - \surd(1 - r^2/a^2)\} \theta_w, \tag{16}$$

where θ_w is the temperature of the tube wall, and the temperature at the axis has been taken as zero. This intentionally makes the temperature gradient infinite at the wall to agree with the gradient through the film, which must be

infinite if its thickness is negligible. It also makes the temperature drop across the film and core in the correct ratio $\beta\sigma : (\alpha - \beta)$. Now it is easily found that the above temperature distribution necessitates an amount of heat of at least equal to

$$\frac{\pi}{3} a^2 \rho s \left(\frac{\alpha - \beta}{\alpha - \beta + \beta\sigma} \right) \theta_w$$

per unit length being put into the water before such distribution is possible. But the heat put in per unit length per unit time is $2\pi a H$ which is approximately $2\pi a k_0 \theta_w$. Substituting $\rho v s Z/(\alpha - \beta + \beta\sigma)$ for k_0 we get $a(\alpha - \beta)/6Zv$ as the *time* required to impart this heat. But in this time the liquid flows a distance of $(\alpha - \beta)/12Z$ diameters down the tube. As $Z^{-1} = 8N$, where N is the number of diameters in which the loss of head is equal to the velocity head, this is $\frac{2}{3}(\alpha - \beta) N$; or, roughly, since $(\alpha - \beta)$ is of the order of $0\cdot7$ to $0\cdot9$, say, $\frac{1}{2}N$ diameters. It is fairly evident that the value of k_0 will reach its limiting value, roughly, exponentially, and that in two or three times this distance the initial effect will have become insensible.

As, for ordinary velocities, N varies between about 25–50 this calculation is in no way inconsistent with the experiment which gave values of k at the first thermocouple of anything up to about 5 per cent. greater (when compared for the same water temperature) than those given by the second couple, while one test made with the first thermocouple only 3 inches from the commencement of heating gave values of k about 50 per cent. greater.

If, instead of generating heat at a constant rate along the tube, the tube had been maintained (by a variable heat flow) at a *constant temperature* above the water, which is more the condition in a steam condenser, the effect of this " entrance effect " will be to cause an *extra* heat flow into the water of the amount required to produce the equilibrium distribution of temperature over the cross section. Thus we see that the tube will behave as if its length were somewhere about $\frac{2}{3}(\alpha - \beta) N$ or $\frac{1}{2}N$ diameters longer than it actually is.

The above investigation may be extended to determine roughly the difference in the value of k obtained by taking the water temperature as its axial value and the temperature given by allowing the water at any section to flow into a vessel and to become thoroughly mixed.

We will take the velocity u at a distance r from the axis to be given by

$$u = u_0 \{\beta + (1 - \beta) \sqrt{(1 - r^2/a^2)}\}, \tag{17}$$

where u_0 is the axial velocity. This makes the velocity at the boundary with the film a fraction β of the axial velocity, and agrees approximately with

Stanton and Pannell's determination of the velocity distribution. From (16) and (17) we easily find the " mixing temperature " over the cross section to be

$$\frac{1+\beta}{4+2\beta} \cdot \frac{\alpha-\beta}{\alpha-\beta+\beta\sigma} \cdot \theta_w;$$

so that the temperature difference between tube and water, instead of being θ_w, is less by the above amount. Hence taking the water at its axial temperature would give smaller k's in the ratio

$$\left\{1 - \frac{1+\beta}{4+2\beta} \cdot \frac{\alpha-\beta}{\alpha-\beta+\beta\sigma}\right\} : 1.$$

If, say, roughly, $\alpha = 1\cdot3$ and $\beta = 0\cdot4$ this gives a ratio of $5 + 3\sigma : 7 + 3\sigma$ (approximately) making a difference of about 10 per cent. with water at 80° F. It must not be assumed that with cold water when σ is large this ratio is nearly unity for the thickness of the viscous flow film has been assumed negligible, and that it is only so at high velocities with cold water. It is shown in Appendix A that for purely viscous flow the ratio of the k's given by these two ways of taking the water temperature is 11 : 18.

The authors desire to thank the authorities of the Manchester College of Technology, in the Engineering Department of which this research was carried out ; they also owe a debt of gratitude to Dr. G. Stoney under whose kindly help and supervision the work was commenced.

Summary.

The present paper records the determination of the coefficient of heat transfer, k, from brass tubes to water flowing through them, the tubes being directly heated by large alternating currents. No definite coefficient of heat transfer is obtainable till the temperature distribution over the cross section has attained its limiting form. The coefficient of course varies with what the water temperature at any cross section is considered to be. In these experiments this temperature is taken as the mixing temperature of the water flowing past the section.

It is shown that the limiting coefficient is, apart from a simple factor, a function of Reynolds' number $\tau \equiv \rho vd/\mu$ and $\sigma \equiv \mu s/c$ (*i.e.*, viscosity $\times$ specific heat $\div$ thermal conductivity) only. The results for very small rates of heat flow can be expressed in the form

$$\rho vs/k_0 = A + B(\sigma - 1) - C(\sigma - 1)^2,$$

where A, B and C are functions of τ only given in Table I on p, 556. This holds for all values of σ up to about 15, and for values of τ from about 5000— which is only $2\frac{1}{2}$ times the lower critical velocity and is the lowest at which the flow is stable enough to obtain readings—to infinity.

A theory, agreeing with experiments, is given showing the difference between k_0 and k_H, the coefficient at a heat flow rate H, when τ is large and also a simple empirical rule for the difference when τ is not so large.

An approximate estimate of the distance before k settles down to its limiting value is also given which is not inconsistent with the experiments.

APPENDIX A.

Calculation of k_0 for Viscous Flow.

This problem, though it has been solved before,* contains points to which we wish to draw attention.

When the temperature distribution over the cross section has reached its limiting form we must have

$$\theta = gx + f(r)$$

where g is the gradient along the direction of x, the axis of the tube. The velocity u at any point is known to be given by

$$u = 2v\left(1 - r^2/a^2\right)$$

where v is the mean velocity.

The excess of heat flowing radially into a thin cylinder of unit length and of radii r and $r + dr$ per unit time over that flowing out is clearly

$$\frac{\partial}{\partial r}\left\{c2\pi r\,\frac{\partial\theta}{\partial r}\right\}dr.$$

As the temperature remains steady this is equal to the heat removed by the flow which is

$$(2\pi r\,dr)\,\rho su\,\frac{\partial\theta}{\partial x}$$

per unit time.

Writing g for $\dfrac{\partial\theta}{\partial x}$ and substituting for u, we get on equating,

$$\frac{\partial}{\partial r}\left(r\,\frac{\partial\theta}{\partial r}\right) = \frac{2\rho vsg}{c}\left(r - r^3/a^2\right).$$

The solution of this, calling the axial temperature zero, is

$$\theta = \frac{2\rho vsg}{c}\left(\frac{r^2}{4} - \frac{r^4}{16a^2}\right).$$

* See, *e.g.*, Nusselt's paper cited below.

This gives $3\rho vsga^2/8c$ as the temperature at the tube wall and $\rho vsga/2c$ as the temperature gradient at the tube wall. As the heat flow per unit area is c times the latter quantity we get

$$k_0 = 4c/3a$$

on taking the axial temperature of zero as the water temperature.

Had we taken the static mean temperature over the cross section, viz.,

$$\frac{1}{\pi a^2} \int_0^a 2\pi\theta r \, dr = \frac{5}{24} \frac{\rho vsga^2}{c}$$

as the temperature of the water, we should have obtained

$$k_0 = 3c/a \; ;$$

while if we take the kinematic mean or " mixing temperature " given by

$$\frac{1}{\pi a^2 v} \int_0^a 2\pi u\theta r \, dr = \frac{7}{48} \frac{\rho vsga^2}{c}$$

we get

$$k_0 = 24c/11a \simeq 2\cdot182c/a.*$$

It is interesting to express this last result in the form $\rho vs/k_0$ equal to something with Z^{-1} as a factor. We have

$$\rho v^2 Z = R = \mu \left(\frac{\partial u}{\partial r}\right)_{r=a} \; ;$$

giving $Z = 4\mu/\rho va$. But we have found

$$\rho vs/k_0 = 11a\rho vs/24c,$$

whence, eliminating a, we get

$$v_m s/k_0 = 11\sigma/6Z \; ;$$

so that (10) will hold in this case also if we take $\gamma = 0$ and $\alpha = \beta = 11/6$. This known value of α and β for viscous flow was a useful guide in drawing their graphs.

Appendix B.

Calculation of $(k_H - k_0)$ for Viscous Flow.

This calculation, though quite elementary, is long and tedious so that only the step by step results are given here. We will assume that $d\rho/d\theta$ and $ds/d\theta$

* It should be stated that this result does not agree with the value given by Nusselt in ' Z. Ver. Deut. Ing.,' vol. 54, p. 1156 (1910), where as the result of a more elaborate analysis, during which he also obtains the initial increase in k_0 before it settles down to its limiting value he gives $k = 2\cdot576c/a$ (probably meant for $18c/7a$). There seems, however, no doubt about the correctness of the present result.

are negligible and will denote $d\mu/d\theta$ and $dc/d\theta$ by μ' and c'. We will suppose that the temperature at the axis is taken as zero and that μ and c refer to this temperature. We will further suppose that the velocity is adjusted so that the wall friction is *exactly* the same as when μ and c are constant ; and that θ is the wall temperature in this *latter* case.

With μ and c variable we find that the wall temperature becomes

$$\theta' = \theta \left\{ 1 + \left(\frac{29}{324} \frac{\mu'}{\mu} - \frac{1}{2} \frac{c'}{c} \right) \theta \right\};$$

while the temperature gradient at the tube wall and the heat flow per unit area are

$$1 + \left(\frac{7}{18} \frac{\mu'}{\mu} - \frac{c'}{c} \right) \theta \qquad \text{and} \qquad 1 + \frac{7}{18} \frac{\mu'}{\mu} \theta$$

times their previous values respectively. We also find the total flow of liquid through the tube is

$$1 - \frac{13}{18} \frac{\mu'\theta}{\mu}$$

times its previous value ; the total heat carried away by the liquid is

$$1 - \left(\frac{97}{126} \frac{\mu'}{\mu} + \frac{29}{105} \frac{c'}{c} \right) \theta$$

times its former value ; and the kinematic or " mixing " temperature of the liquid is

$$1 - \left(\frac{1}{21} \frac{\mu'}{\mu} + \frac{29}{105} \frac{c'}{c} \right) \theta$$

times its former value ; thus making the difference of temperature between tube and water

$$1 + \left(\frac{35}{198} \frac{\mu'}{\mu} - \frac{106}{165} \frac{c'}{c} \right) \theta$$

times its former value. Dividing the ratio in which the heat flow has been increased by this, we see that

$$k_{\text{H}}/k_0 = 1 + \left(\frac{7}{33} \frac{\mu'}{\mu} + \frac{106}{165} \frac{c'}{c} \right) \theta.$$

But this is comparing the new k_{H} with k_0 for the same *axial* water temperature ; if it is compared with k_0 for the same tube temperature (which is greater in the ratio $(c + c'\theta) : c$) we get

$$k_{\text{H}}/k_0 = 1 + \left(\frac{7}{33} \frac{\mu'}{\mu} - \frac{59}{165} \frac{c'}{c} \right) \theta.$$

Remembering that by Appendix A, $k_0 = $ H $\div$ 11θ/18 we get

$$k^{\text{H}} - k_0 = \left(\frac{42}{121}\frac{\mu'}{\mu} - \frac{354}{605}\frac{c'}{c}\right)\text{H} \, ;$$

that is

$$k_{\text{H}} - k_0 = (0 \cdot 347 \; \mu'/\mu - 0 \cdot 585 \; c'/c) \, \text{H}.$$

If k_{H} be compared with k_0 at the same kinematic water temperature it will be found that

$$k_{\text{H}} - k_0 = \left(\frac{210}{605}\frac{\mu'}{\mu} - \frac{31c'}{605c}\right)\text{H} = \left(0 \cdot 347 \frac{\mu'}{\mu} - 0 \cdot 051\frac{c'}{c}\right)\text{H}.$$

Since we have assumed such a mean velocity that the wall friction, and therefore the loss of head, is the same as before ; and for viscous flow the loss of head is proportional to the velocity, and, as we have seen, the total flow is increased in the ratio

$$\left(1 - \frac{13}{18}\frac{\mu'}{\mu}\,\theta\right) : 1,$$

it follows that for the *same* total flow the loss of head would be increased in the ratio

$$\left(1 + \frac{13}{18}\frac{\mu'}{\mu}\,\theta\right) : 1.$$

But, as the loss of head is proportional to the viscosity, this is the same as if there were no heat flow (so that the temperature was constant throughout) and the temperature of the liquid was 13θ/18. As 7θ/18 is the kinematic temperature of the liquid, and 11θ/18 is the difference between this and the tube wall temperature, $= \Delta\theta$, say, it follows that the loss of head is the same as if there was no heat flow and the temperature of the liquid was $6\Delta\theta \div 11$; that is, as if the actual kinematic water temperature was increased by 6/11 of the difference between this temperature and the tube temperature and no heat transfer was taking place.