# *On the Coefficient of Viscous Traction and its Relation to that of Viscosity.*

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When experiments are made on the viscous flow of pitch and other substances of similar character, in the form of rods or cylinders, by the torsional method,* it is found that the rate of turning under torsion of these rods is not strictly proportional to the driving couple. Thus the rate of flow of the material under shearing stress cannot be in simple proportion to stress. If it is wished to investigate the exact law connecting the rate of flow with the shearing forces, by means of the torsional method, a complication is at once met with, arising from the fact that the rate of flow in a twisting rod is not of the same value everywhere, but necessarily varies from nothing at the centre to a maximum at the surface of the rod.

With the view of developing a more suitable way of investigating the phenomenon, trials were made with different methods of observing the flow of such bodies, under conditions in which the said objection does not apply. The results obtained in these ways exhibit the same departures from linearity as was suggested by the results obtained by the method of torsion.

The types of flow examined were: (1) the flow produced in a rod or cylinder of the material when under traction; (2) when under axial compression; (3) the flow of a freely descending stream of the material; (4) the rate of bending of a horizontal rod or beam of the material under its own weight when supported only at the ends.

The latter method, however, suffers from the same defect as the torsional method, namely, from giving an integral value through a range from zero up to a certain limit.

### *Traction Experiments.*

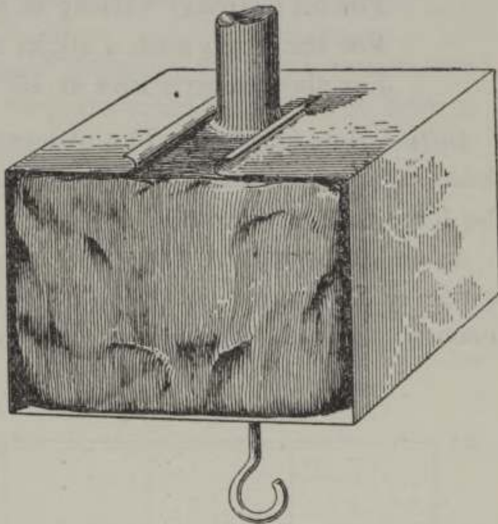
Rods or bars of the material under examination were suspended from one end, while to the other weights were attached. The rod was thus subjected to a force which continuously drew it out. The rate of elongation per centimetre length of the rod could then be determined.

The rods for these experiments were prepared by the method described in the paper already referred to. For the purpose of applying the tractional forces the rod was thickened at each end and worked into an almost cubical block, as shown in fig. 1, so as to fit a metallic box or receptacle open to one

* 'Phil. Mag.,' vol. 19, p. 347, 1904.

side, by which the rod could be hung from a support at the top and a weight attached at the lower end. The opening in the box for the rod was slotted out to one side so as to admit the rod being slid into its place.

FIG. 1.



In estimating the stress in the rod we must add to the weight attached to the lower hook the weight of the box and of the pitch below the lower point of observation on the rod. The weight of the rod under observation itself produces an increase in the stress from the lower end upwards, its amount half way up is obviously half that of the rod between the lower and upper mark. For corresponding points above and below the middle the stress is in excess and defect of the mean value by equal amounts, so that, assuming linearity for the flow, the correction will be half the weight of the portion of the rod in the observation. The rate of elongation of the rod was observed by means of a cathetometer.

In observations with certain materials such as shoemaker's wax the plan was adopted of experimenting under a liquid of the same density as the material itself for the purpose of eliminating the stress due to its own weight, which was much too great to admit of accurate observations being otherwise made on the rate of flow.

The lower end of the rod, in such cases, was made fast while the tractive weight was applied at the top by means of a cord and pulley. The rods were thus drawn upwards. This was done in order to be able the more easily to surround the rod with a liquid, having the same density as its material, and thus to eliminate the action of the rod's own weight. For holding the liquid a wide vertical tube was fitted to the apparatus and surrounded the rod under examination.

Solutions of salt in water were used for the liquids.

*Results of Traction Experiments.*—The results obtained with pitch and other substances showed that the time rate of elongation per centimetre of a rod under tension is approximately proportional to the force of traction per square centimetre cross section or $\frac{F}{A}\Big/\frac{dv}{dx} = \lambda$, where F is the force applied, A the area of the cross section, $v$ the velocity at any point $x$ on the rod, and $\lambda$

a constant for any given material. This we shall term the coefficient of *viscous traction* of the material.

The values found for $\lambda$ for a few of the substances experimented with are given to show the order of the coefficient.

For an ordinary variety of pitch at 15° C. ......... $4{\cdot}3 \times 10^{10}$
For the same with a slight admixture of tar ...... $6{\cdot}7 \times 10^{9}$
For shoemaker's wax at 15° C. ...................... $5{\cdot}9 \times 10^{6}$

*Initial Rate of Flow.*—The observations made on the rate of flow of rods of these substances show that it is faster immediately after the application of the force than afterwards.

As an example, the following table is given, in which the elongation is in arbitrary units and the time is in minutes and seconds to the nearest five seconds. These observations are plotted in fig. 2.

FIG. 2.

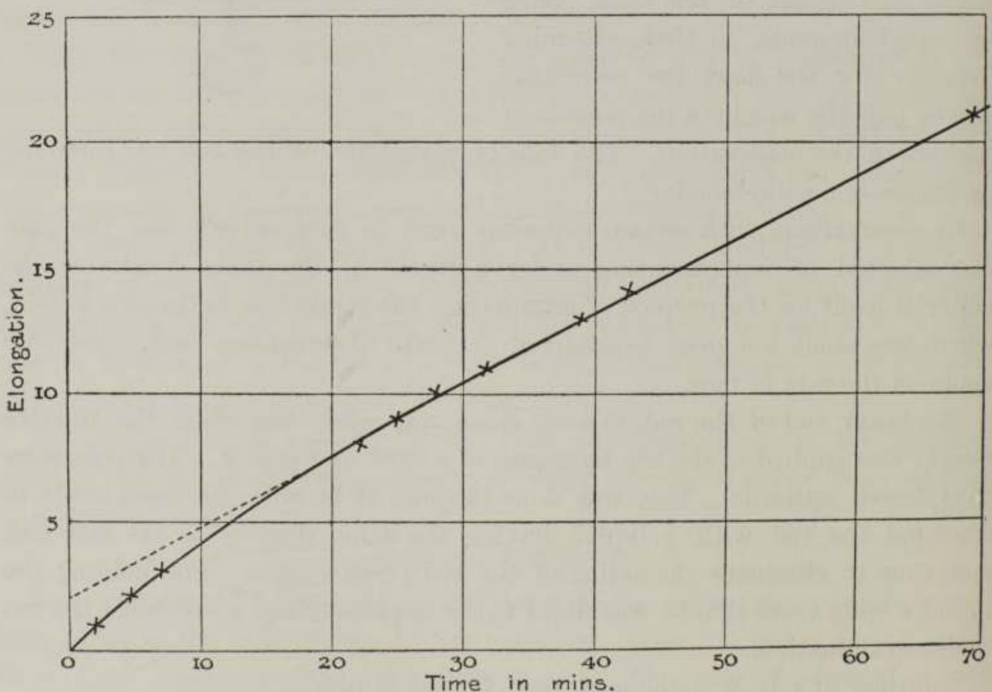



Table I.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\delta l$ ... | 0. | 1. | 2. | 3. | 8. | 9. | 10. | 11. | 13. | 14. | 21. |
| $t$...... | 0 | 2′ 0″ | 4′ 25″ | 6′ 50″ | 22′ 0″ | 24′ 55″ | 28′ 0″ | 31′ 35″ | 39′ 0″ | 42′ 40″ | 69′ 0″ |

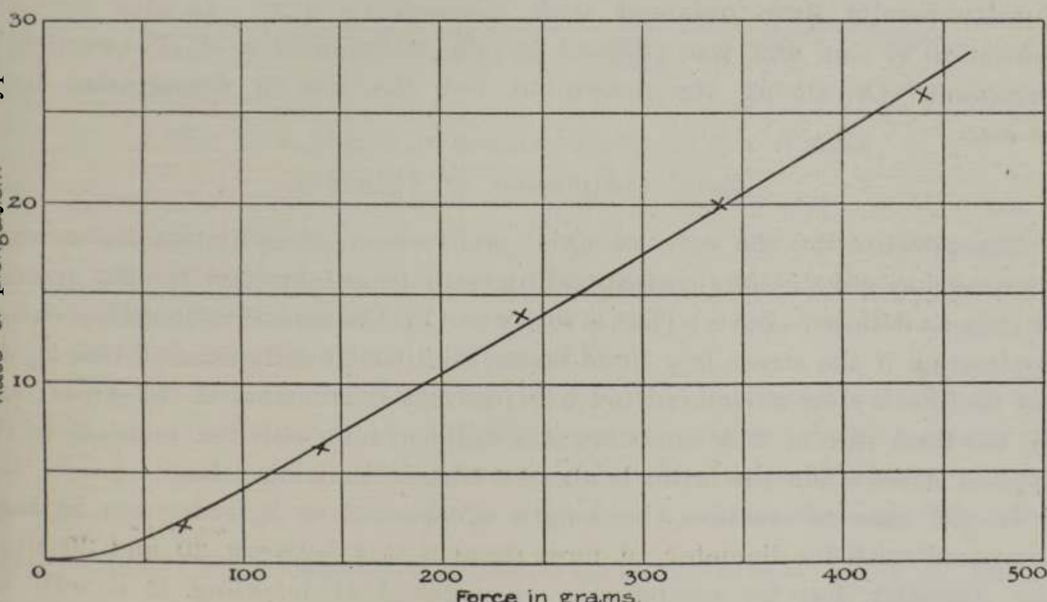
It will be seen that the initial rate of flow is faster than the final rate. This is similar to what was observed in the case of the viscous flow of rods under torsion.* It was also noticed that there was a slow partial movement towards recovery on removal of the force of traction, which gradually fell to zero with time, just as had been previously observed in the case of torsional forces.

*Departure from Linearity with Variation of Tractive Force.*—Experiments made with different values of the tractive force show that the rate of flow is not strictly proportional to the force. The results of determinations made with a variety of pitch given in Table II and shown in fig. 3 are typical.

Table II.

| Force. | 70. | 140. | 240. | 340. | 440. |
|---|---|---|---|---|---|
| Rate of elongation ... | 2·0 | 6·4 | 14 | 20 | 26 |

Fig. 3.




The ordinates represent rate of elongation, while the abscissæ represent the force applied to the rod in grammes. It will be seen from the curve that except near the origin the law is linear. For forces above a certain value the rate of flow may be expressed as $T - T_0 = \lambda dv/dx$. The rate of elongation

* 'Phil. Mag.,' vol. 19, p. 347, 1904.

taken for the curve was in every case that obtaining after the initial stage had been passed.

*The Paths of Flow of the Particles in a Drawn-out Rod.*—The mode of flow of the particles of a rod while being drawn out is of interest. With the view of experimentally ascertaining if the particles lying in a cross section moved out symmetrically on thinning taking place, that is to say, if half of the particles, scattered uniformly throughout a cross section, are left relatively behind, rods were prepared of materials having approximately the same coefficient of viscous traction, but of different colours. Two different coloured rods were united end to end and then drawn out, the surface of demarcation being carefully watched.

Rods of different coloured glass were tried for this purpose and, provided they are approximately of the same fusibility, answered well. Compound rods thus made were warmed and drawn out and the surface where they joined observed. This always remained a plane; though sometimes it did not remain a cross section. No tendency was noticed for the central portion to flow at a different rate to the peripheral parts. The flow shows itself to be quite symmetrical along the axis, as one would naturally expect. Similar results were obtained with shoemaker's wax. In this case the coloration of one end was effected by the addition of a small quantity of vermilion. On slicing the drawn-out rod, the line of demarcation could be seen.

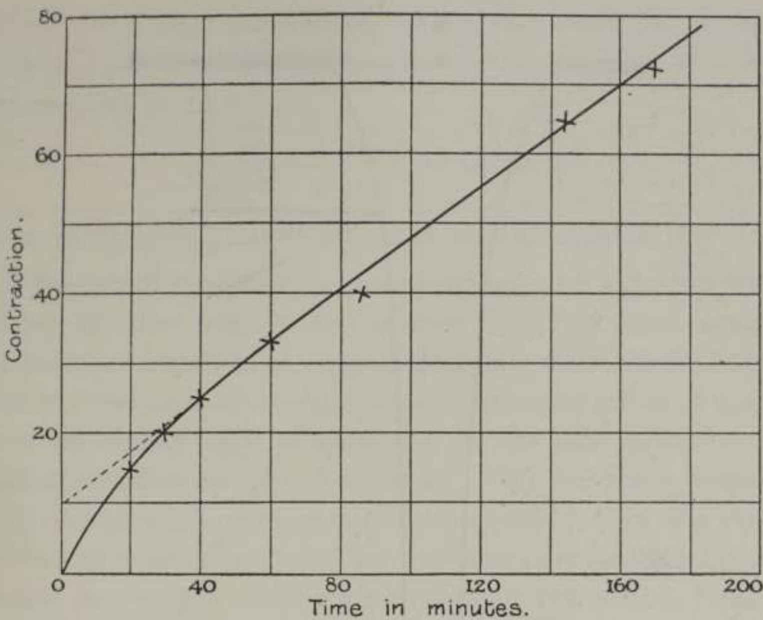
## *Axial Compression of Cylinders.*

Experiments on the rate of axial contraction of cylinders under axial compression gave results corresponding with those obtained for the traction of rods, as detailed above. That is to say:—(1) The rate of contraction on first application if the stress is a little faster than that finally reached (see fig. 4); (2) there is a slow movement towards recovery on removal of the stress; and (3) the final rate of flow increases at a uniform rate with the increase in the applied stress when the latter is above a certain limiting value.

In the case of traction, the length of the rod or cylinder can be large compared with its diameter, in most cases it was between 20 and 30 times the diameter, but for compression on account of buckling, it is well for the length to be not more than about three times the diameter of the cylinder.

The stress was applied by placing weights on a plate which just covered the top of the cylinder, which stood vertically. The rate of depression of the cylinder was observed by means of a cathetometer. The coefficients obtained from experiments on compression made in this way were found to be about the same in magnitude as those obtained from traction. For instance, with a certain specimen of a rather soft pitch, the compressional coefficient was found to be $7 \cdot 6 \times 10^9$, while the coefficient for traction of the same material was found to be $6 \cdot 7 \times 10^9$.

Fig. 4.



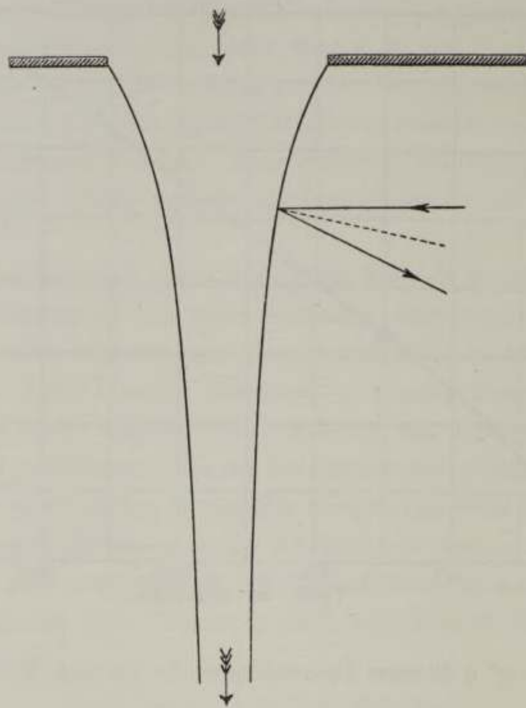

*Flow of a Stream Descending under its own Weight.*

The stream of the material under examination was obtained by allowing it to flow through a circular hole in the bottom of a wide tin vessel. After a steady state is reached, the outline of the stream is of the character shown in fig. 5. The stream gradually tapers down to a very fine thread, which breaks off intermittently from its lower end.

At any point distant $x$ from the top let $v$ be the downward velocity of the material, then the time-rate of elongation per centimetre of the material at this point is $dv/dx$. The tractive force is due to the mass M of the column below this point. Let the cross section be $\pi y^2$, we have then, provided the velocity is small, $Mg = \lambda \pi y^2 dv/dx$. Thus we may write $\lambda = -\dfrac{Mg}{2p}\dfrac{y}{dy/dx}$, where $p$ is the volume of material ($\pi y^2 v$) supplied per second, and $dy/dx$ the slope of the tangent to the surface at the point of the column where $2y$ is the diameter.

*Tangent Method of Determining* λ.—An optical method can be adopted for determining the slope of the surface of the falling column at a definite point. A beam of light from a horizontal slit and lens is allowed to fall on the column and is reflected into a telescope which is pointed upwards to receive it. The tangent of half the angle between the incident and reflected beams is evidently the slope of the surface neglecting curvature.

FIG. 5.



The value of M can be got by cutting off the column at the point in question, collecting and weighing. The value of M varies, it is true, according as the column breaks off at its lower end or otherwise, but only to a negligible extent.

The value found for the coefficient of viscous traction by this method for a mixture of pitch and tar in the ratio of 7 to 1 was $1{\cdot}1 \times 10^8$; and for a mixture of the same materials in the ratio of 3 to 1 about $9{\cdot}3 \times 10^5$. These are about the same values as obtained by the other methods.

*Investigation of Shape of a Stream Descending under its own Weight.*—To determine the shape of the column we have the following considerations. At any point we have seen that the force of traction is

$$F = \lambda A dv/dx, \tag{1}$$

where A is the area of the cross section, so that $vA = \text{const}$. Also

$$\frac{dF}{dx} + g\rho A + A\rho\left(\frac{dv}{dt} + v\frac{dv}{dx}\right) = 0, \tag{2}$$

when $\rho$ is the density of the material.

If the rate of fall is small, the acceleration term may be neglected, and we get

$$\lambda \frac{d}{dx}\left(\mathrm{A}\frac{dv}{dx}\right) = -g\rho\mathrm{A}.$$

Substitute for $v$ from the relation $v\mathrm{A}\rho = \mathrm{M}_1$, where $\mathrm{M}_1$ is the mass of material falling per second; and substitute $\pi y^2$ for A. Then, after differentiating and arranging, we have

$$\frac{1}{y^3}\frac{d^2y}{dx^2} - \frac{1}{y^4}\left(\frac{dy}{dx}\right)^2 - 1/\mathrm{K}^2 = 0,$$

where $\mathrm{K}^2 = 2\lambda\mathrm{M}_1/g\rho^2\pi$. The general solution of this equation is $y = b/\sinh\frac{bx}{\mathrm{K}}$. When $b$ is very small it represents a long filament, such as in the present case. The limiting solution when $b = 0$ is $xy = \mathrm{K}$. This last expression was found, as described below, to fit experimental data with sufficient accuracy.

In order to experimentally examine the question, mixtures of pitch and tar were made sufficiently thick or glutinous for the flow to be slow enough to enable the acceleration term to be neglected. The curvature assumed by the descending column was experimentally determined by observing the diameter of the column at various heights. This was done in some cases by means of a cathetometer, in others by casting the shadow of the column from a distant source of light on a long vertical sheet of paper placed close to the column, and then tracing out the shadow with pencil.

The cathetometer telescope had a scale in the eye-piece, with which the horizontal breadth of the column at the different heights was observed according as the telescope was raised to various positions along the column. From these readings, in conjunction with the height readings, the curve made by the pitch in falling could be plotted. It was then found possible to fit an equilateral hyperbola to it.

The following table (p. 434) gives the results obtained with rather a thick mixture. The first column gives the height in centimetres; the second one half the observed diameter; the third column gives the calculated value of the radius derived from the formula $y(x+m) = \mathrm{K}$.

The values of the constants used were $\mathrm{K} = 1{\cdot}85$ and $m = 1{\cdot}8$. This last is the height above the bottom of the vessel at which the horizontal asymptote is situated.

The divisions of the scale of the eye-piece corresponded to 0·03 cm. This was subdivided by eye, so that the agreement is quite within the limits to be expected. There was some difficulty at times in reading the diameter, especially towards the lower end, as the hanging column would sometimes sway slightly about, even though it was placed inside a tall glass case with front to shelter it from draughts. This was due in large measure to the effect of the end breaking off.

Table III.

| $x$. | $y$ obs. | $y$ calc. | Diff. | $x$. | $y$ obs. | $y$ calc. | Diff. |
|---|---|---|---|---|---|---|---|
| 0 | 1·035 | 1·027 | −0·008 | 8 | 0·190 | 0·188 | −0·002 |
| 0·5 | 0·790 | 0·804 | +0·014 | 9 | 0·165 | 0·171 | +0·006 |
| 1 | 0·635 | 0·660 | +0·025 | 10 | 0·155 | 0·156 | +0·001 |
| 1·5 | 0·550 | 0·560 | +0·010 | 12 | 0·135 | 0·134 | −0·001 |
| 2 | 0·475 | 0·460 | −0·015 | 15 | 0·110 | 0·104 | −0·006 |
| 2·5 | 0·430 | 0·430 | 0 | 20 | 0·090 | 0·089 | −0·001 |
| 3 | 0·370 | 0·385 | +0·015 | 30 | 0·065 | 0·058 | −0·007 |
| 3·5 | 0·340 | 0·349 | +0·009 | 40 | 0·045 | 0·044 | −0·001 |
| 4 | 0·315 | 0·318 | +0·003 | 50 | 0·030 | 0·035 | +0·005 |
| 4·5 | 0·290 | 0·293 | +0·003 | 66·5 | 0·020 | 0·027 | +0·007 |
| 5 | 0·255 | 0·271 | +0·016 | 88 | 0·020 | 0·020 | 0 |
| 6 | 0·225 | 0·237 | +0·012 | 92·5 | 0·015 | 0·019 | +0·004 |
| 7 | 0·200 | 0·210 | +0·010 | | | | |

*The Value of* $\lambda$ *found by Falling Column Method.*—From the value of the constant in $xy = \mathrm{K}$ the coefficient of viscous traction may be calculated. Thus

$$\lambda = \rho^2 g \pi \mathrm{K}^2/2\mathrm{M}_1.$$

Taking as found above $\mathrm{K} = 1{\cdot}85$ $\rho = 1{\cdot}32$ and $\mathrm{M}_1 = 0{\cdot}0000950$, we get $\lambda = 9{\cdot}6 \times 10^7$. This was at a mean temperature of 16° C. A rod made from the same mixture of tar and pitch gave for $\lambda$ by the traction method the value $\lambda = 7{\cdot}8 \times 10^7$ at 17°·5 C. Another rod from the same mixture gave $13 \times 10^7$ at 14° C. The agreement will be considered sufficiently good to confirm the theory when the character of the material is remembered. Unfortunately, though very convenient for making materials of any desired viscosity, these mixtures suffer from the disadvantage that when they are heated for the manufacture of the test rods they lose some of their more volatile constituents, becoming more viscous in consequence.

*Modification Introduced by Inertia.*—The modification from the hyperbolic form, which the falling stream of material undergoes when it is not so viscous as to render the inertia term negligible, may be appreciated by noting that when the shape is hyperbolic the acceleration varies as the third power of the height fallen, so that though at first the inertia term may be quite negligible, it may at lower points become important and sensibly reduce the traction effect. In this way the contour at lower points assumes less of the hyperbolic and approximates more towards the cylindrical shape.

In the limit where viscosity is negligible the shape may be taken as given approximately by $xy^4 = \mathrm{K}$, provided we ignore the tendency to break up into drops due to surface tension. Comparing the two cases, say for points $x_1$ and $x_2$, where $x_2 = 2x_1$, we have for the non-inertia case $y_2 = 0{\cdot}5y_1$, for the non-viscous case $y_2 = 0{\cdot}84y_1$. Where both causes act, some intermediate shape will be assumed by the falling material. Descending columns of viscous liquids, such as the familiar one of honey falling from a spoon, form instances of this.

### *Sagging of a Horizontal Beam.*

If a rod of pitch is laid across between two horizontal supports it will be found to continuously sag downwards. The rate at which this occurs varies with the consistency of the material.

To find how the rate of sagging depends on the coefficient of viscous traction, we can resolve the stresses in the material at any cross section after the manner usual in the cases of stressed beams. This gives compressional forces above and tractive force below a certain point in any cross section. Taking this as being at the central horizontal line of the cross section, we have for the moment of the force about this line $M = \lambda \int \left(\frac{dv}{dx}\right)_y y \, . \, b \, . \, dy$, where $\left(\frac{dv}{dx}\right)_y$ is the rate per unit length of the elongation, or the contraction as the case may be, at any point situated at distances $y$ from the central line, and where $b$ is the breadth there.

This value for M may be approximately expressed in terms of the rate at which a plane in the material at the point rotates at the moment when it is at right angles to the axis; thus $\left(\frac{dv}{dx}\right) = y\,\frac{d\omega}{dx}$, where $\omega$ is this rate of rotation at any cross section, so that $M = \lambda I \frac{d\omega}{dx}$.

Now $u$, the rate of sagging at the centre, is given by $u = \Sigma_0^{\frac{1}{2}L} x\delta\omega$; and, recollecting that $M = \frac{1}{2}gm\left(x - \frac{x^2}{L}\right)$, where $m$ is the mass of the rod between the supports and L its length, we get, after arranging and integrating,

$$u = \frac{5}{384}\,\frac{gmL^3}{\lambda I},$$

where I is the moment of inertia of the cross section of the rod.

One of the methods employed to test this formula was to compare the rate of sagging of rods of different lengths, all other quantities involved being unaltered. A rod of pitch of circular cross section was laid between two supports which could be placed at various distances apart, and the time of sagging through the same distance in each case observed. In all cases the initial rate of sagging was not included in the measurements. To allow this to be done the rod was made with a camber and the observations only begun just before attaining the straight position.

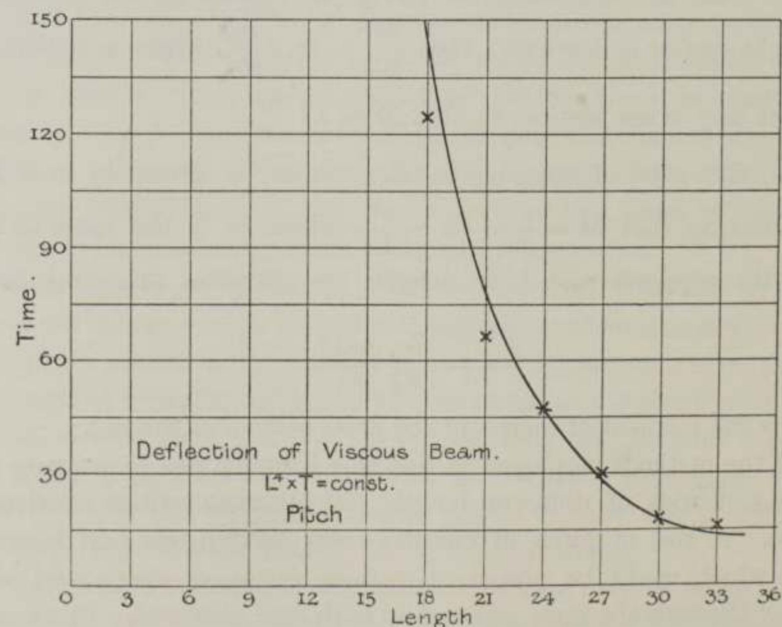
It will be seen on comparing rods of different length, if T represents the time taken, at any given span L, to sag through the standard distance, that $TL^4$ = const. The following table gives in the first column the temperature at the time of the experiment, in the second column the span, in the third the time taken to fall through a standard distance, and in the fourth the value found for this constant in each case.

Table IV.

| °C. | L. | T. | $TL^4$. |
|---|---|---|---|
| 15° | 33 | 14·6 | $1·7 \times 10^7$ |
| 15 | 30 | 18·5 | 1·5 |
| 15 | 27 | 30·4 | 1·6 |
| 15 | 24 | 47·0 | 1·6 |
| 18 | 21 | 66·2 | 1·3 |
| 18 | 18 | 125·0 | 1·3 |

The last two experiments were made on a different day from the others, and were made at a slightly higher temperature. This may in part account for the smaller value found for the constant. The curve obtained by plotting L against T is shown in fig. 6.

Fig. 6.



*The Initial Rate of Sagging.*—As in other cases previously dealt with, so in the case of sagging beams, the initial rate of flow is greater than subsequently. The following table, obtained from experiments with a certain variety of pitch, illustrates this. In the first column are given the distances fallen from zero by the central point of a rod or beam of pitch. In the second column the time taken to each point. The curve obtained by plotting these is shown in fig. 7.

FIG. 7.

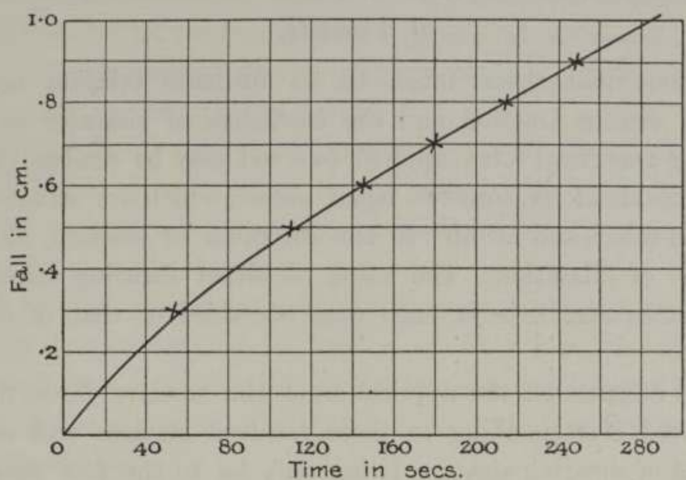



Table V.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *h*......... | 0·0. | 0·3. | 0·5. | 0·6. | 0·7 | 0·8. | 0·9. | 1·0. |
| *t*......... | 0 | 55 | 110 | 145 | 180 | 215 | 250 | 290 |

The dimensions of this rod, which was circular, were as follows: length between supports $L = 25$, mean radius $R = 0{\cdot}47$, mass between supports $M = 21{\cdot}7$, final rate of sagging $U = 0{\cdot}00285$ average. This gives, using above formula, $\lambda = 3{\cdot}96 \times 10^{10}$. The same rod gave by the traction method $3{\cdot}90 \times 10^{10}$.

A number of other mixtures of pitch and tar of varying proportions were experimented with. These gave approximately the same value for the coefficient of viscous traction as that found by either the falling column method or by the direct traction method.

Some of the mixtures of pitch and tar were too soft to deal with as a beam in air, so they were experimented with under water or under strong brine. In this way the downward bending moment could be very greatly reduced, or, if desired, could be even changed in sign, when, of course, the ends of the beam had to be held down. A particular mixture whose density was 1·185 gave the value $\lambda = 3{\cdot}01 \times 10^5$ when operated under water. The value previously found for it by the falling column method was $= 4{\cdot}3 \times 10^5$. The velocity in this case was perhaps too great to neglect the inertia term in the falling column method, and may account for the higher value of the coefficient obtained by it.

### *Connection between the Coefficient of Viscous Traction and the Coefficient of Viscosity.*

It is obvious that there must be an intimate relation between the coefficient of viscous traction and the coefficient of viscosity as ordinarily defined. The tractional force applied to a rod may be resolved, as is usual in questions of elasticity, into two equal shears, which are situated at right angles to each other and at 45° to the direction of traction, along with a uniform force of dilatation. The value of either shearing stress, and also of the dilatation stress, is in each case one-third of that of the tractive stress.

In the first instance on the application of the tractive force, the resolved effects produced corresponding to these resolved stresses will consist of a dilatation and of shearing strain. It can only be to the flow resulting from the latter that the *continued* elongation of the rod is due. Nothing similar can take place in the case of the stress of dilatation, which only can have an initial effect.

The continued application of each shear will produce a corresponding flow, given in each case by $S = \mu\dot{\phi}$, where S is the shearing stress, $\mu$ the coefficient of viscosity, and $\dot{\phi}$ the rate of change of direction of any line in the material in the plane of the shear as it passes through the direction normal to the shearing stress. The resulting flow in the direction of the axis is obtained by adding the resolved components of the two flows in that direction; so that resolving the two effects, adding the components, and reducing the axial elongation to that ($e$) per unit length, we find that $e = \dot{\phi}$.

Since $T = \lambda e$, and $S = \frac{1}{3}T$, where T is the tractive force per square centimetre, we get $\mu = \frac{1}{3}\lambda$, so that the coefficient of viscosity is equal to one-third of the coefficient of viscous traction.*

* In terms of the more usual analysis of viscous flow, with constant stress-modulus, the argument would take the following form :—Consider a viscous cylinder undergoing elongation at rate $e$; if its material is but very slightly compressible, it must at the same time undergo contraction at rate $\frac{1}{2}e$ in all transverse directions. If $\mu$ represent the viscosity of the material, this rate of elongation implies a longitudinal tension of intensity $2\mu e$, and similarly there is transverse tension of intensity $-2\mu \cdot \frac{1}{2}e$. These tractions

In order to compare the coefficient of viscous traction with that of viscosity for the same material, two distinct plans were adopted. One was to select a material sufficiently viscous to allow the coefficient of viscosity to be determined by means of the torsion of a rod made of it,* and also which allowed the coefficient of viscous traction to be found by directly drawing out the rod, or by the method of the sagging horizontal beam. The second plan was to select a material sufficiently fluid to admit of the coefficient of viscosity being determined by the rate of flow through a tube under a pressure head, while at the same time not so fluid but that the coefficient of viscous traction could be observed by the method of the sagging beam or by the method of the column descending under its own weight.

The following are the results obtained for the value of $\lambda$ and $\mu$ in the case of several materials of wide range in the value of the constants. It will be seen that the value of $\lambda$ is, generally speaking, in fair agreement with three times the value of $\mu$, the viscosity.

A variety of pitch which gave by the traction method $\lambda = 4{\cdot}3 \times 10^{10}$ was found by the torsion method to have a viscosity $\mu = 1{\cdot}4 \times 10^{10}$. Another variety of pitch gave $\lambda = 3{\cdot}6 \times 10^{10}$ by the traction method and $\lambda = 3{\cdot}3 \times 10^{10}$ by the sagging beam method, while the viscosity was found to be $= 1{\cdot}0 \times 10^{10}$ by the torsion method.

A material made by adding a little tar to pitch gave by the traction method $\lambda = 12{\cdot}9 \times 10^{9}$ and $\mu = 4{\cdot}2 \times 10^{9}$ by the torsion method. A similar material containing a little more tar gave $\lambda = 6{\cdot}7 \times 10^{9}$ by the traction method and $= 2{\cdot}2 \times 10^{9}$ by the torsion method.

A specimen of shoemaker's wax gave $\lambda = 5{\cdot}9 \times 10^{6}$ by the traction method and $\mu = 2{\cdot}0 \times 10^{6}$ by the torsion method.

For making a comparison by the tube method a mixture of pitch and tar of about three to one was used. This passed sufficiently freely through a tube to enable the coefficient of viscosity to be determined. This was found to be $\mu = 2{\cdot}6 \times 10^{5}$, while the coefficient of viscous traction was found by the sagging beam method to be $= 7{\cdot}6 \times 10^{5}$. Another mixture of somewhat similar proportions, but better filtered, gave $\mu = 2{\cdot}8 \times 10^{5}$ by the tube method and $\lambda = 9{\cdot}3 \times 10^{5}$ by the descending column method.

acting on the surfaces of the cylinder amount in all to a uniform hydrostatic pressure $\mu e$, together with a longitudinal tension of intensity $3\mu e$. Of these the pressure is entirely neutralised by the reaction arising from the slight compression of the materials which it produces; while the longitudinal tension, having an intensity-coefficient $3\mu$, alone remains to operate in other ways, as in the text.

* 'Phil. Mag.,' vol. 19, p. 347, 1904.

These results are collected in Table VI, where it will be seen that the coefficient of viscous traction λ is roughly three times that of viscosity.

Table VI.

| λ. | μ. | λ/μ. | λ. | μ. | λ/μ. |
|---|---|---|---|---|---|
| $4\cdot3 \times 10^{10}$ | $1\cdot4 \times 10^{10}$ | 3·07 | $6\cdot7 \times 10^{9}$ | $2\cdot2 \times 10^{9}$ | 3·04 |
| $3\cdot6 \times 10^{10}$ | $1\cdot0 \times 10^{10}$ | 3·60 | $5\cdot9 \times 10^{6}$ | $2\cdot0 \times 10^{6}$ | 2·95 |
| $3\cdot3 \times 10^{10}$ | | 3·30 | $9\cdot3 \times 10^{5}$ | $2\cdot8 \times 10^{5}$ | 3·25 |
| $12\cdot9 \times 10^{9}$ | $4\cdot2 \times 10^{9}$ | 3·07 | $7\cdot6 \times 10^{5}$ | $2\cdot6 \times 10^{5}$ | 2·91 |

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# *The Vertical Temperature Gradients on the West Coast of Scotland and at Oxshott, Surrey.*

By W. H. Dines, F.R.S.

(Received November 10,—Read December 7, 1905.)

In a paper by Dr. Shaw and the author read before the Royal Society on May 14, 1903,* an account of an investigation into the conditions of the upper air over the sea in the neighbourhood of Crinan, on the West Coast of Scotland, was given. Since that time two fresh series of observations in the same locality have been obtained, the results of which are now submitted. In each case observations of temperature and humidity were made by self-recording instruments sent up by means of one or more kites, which were flown from the deck of a steam vessel.

## *Expenses.*

The expense has been met by a grant of £200 made by the Government Grant Committee, a grant of £50 made by the British Association at the Southport Meeting, and of £40 at the Cambridge Meeting; and also by an anonymous contribution of £25 by a Fellow of the Royal Meteorological Society. These grants have not been used entirely for the observations at Crinan, but have afforded the means of carrying on experimental work at Oxshott; by them, too, apparatus for a separate investigation carried out by Mr. G. Simpson on the North Sea has been provided.† For the observations at Crinan in 1903 a tug was hired, and the Lords Commissioners of the

* 'Phil. Trans.,' A, vol. 202, pp. 123—141.

† 'Met. Soc. Quart. Journ.,' vol. 32, No. 137, pp. 15—25.