On the Connection Assignment Problem of Diagnosable Systems — Source link

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ABSTRACT

This paper treats the problem of automatic fault diagnosis for systems with multiple faults. The system is decomposed into a number of units $u_1, u_2, \ldots, u_n$, where a unit is a well identifiable portion of the system which cannot be further decomposed for the purpose of diagnosis. By means of a given arrangement of testing links (connection assignment) each unit of the system tests a subset of units, and a proper diagnosis can be arrived at for any diagnosable fault pattern.

Methods for optimal assignments are given for instantaneous and sequential diagnosis procedures.

I. INTRODUCTION

During the past decade the complexity of digital systems has increased in enormous proportions; thereby creating an urgent need for the development of automatic diagnosis procedures for the maintenance of systems. Current approaches to the problem include both combinatorial analysis and simulation with emphasis on systems with a single fault.

In this paper we report results which are fundamental to the diagnosis of systems with multiple faults. They are based on a graph-theoretical model of the system constructed for the purpose of diagnosis. With this model it is possible to investigate many questions that are fundamental to any multiple-fault diagnosis procedure. Necessary and sufficient conditions are obtained for any procedure to be multiple-fault diagnosing. A variety of methods are derived for the optimal assignmant of testing links. Both sequential and instantaneous diagnosis procedures have been treated in detail.

The diagnosis philosophy outlined in this paper is a logical extension of current trends in the computer technology as regards the advent of integrated circuitry and the packaging of digital systems. It does not seem unreasonable to expect that the digital systems of the next generation will consist of single-chip units containing several hundreds of active components. While being undecomposable, these units would have the computational capability of testing other units.

II. A GRAPH-THEORETIC MODEL FOR DIAGNOSIS

When a large system is to be diagnosed for faults, even when self-diagnosis is implied, the system is usually partitioned into subunits which, singly or in combination, test another one of the subunits. However, the validity of the test result is often dependent upon the proper functioning of another part of the same system. Let us denote with A the testing part and with B the part being tested. The test outcome can usually be reduced to a binary alternative which is interpreted in either of the following ways:

1) B is defective

2) B is non-defective.

Obviously, the intepretation of the test outcome is meaningful only under the hypothesis that A is non-defective. In the other case the test outcome is unreliable.

If we test each part of the system in some sequence, the combined test outcomes can be exhibited naturally as a directed graph with binary weights. In this graph each part u_i of the system will be a node of the graph, and the presence of a link b_{ij} signifies the fact that there is a test in which u_i evaluates u_j. The weight associated with b_{ij} will be $a_{ij} = \{0, 1\}$. a_{ij} is zero if, under the hypothesis that u_i is non-defective, u_j is also <u>non-defective</u>; a_{ij} is one if, under the same hypothesis, u_j is <u>defective</u>. In the case that u_i is defective, the test outcome is unreliable and a_{ij} can assume any of the values 0, 1, regardless of the status of u_j.

<u>Definition 1</u> - The set of links b_{ij} $(i,j=1,2,\ldots,n)$ represents the so-called <u>connection</u> of the system, and can be conveniently described by a connection matrix $C \equiv \|c_{ij}\|$ defined by

$$c_{ij} = \begin{cases} 1 \text{ if } b_{ij} \text{ exists} \\ 0 \text{ if } b_{ij} \text{ does not exist.} \end{cases}$$

Definition 2 - The set of test outcomes a_{ij} represents the syndrome of the system; obviously a_{ij} can be assigned if and only if the corresponding $b_{ij} = 1$.

We shall illustrate the use of this graph-theoretic model with the following example.

Example: Let us consider a system which consists of five units $u_1, u_2, \ldots, u_5$ with testing links as shown in Fig. 1.

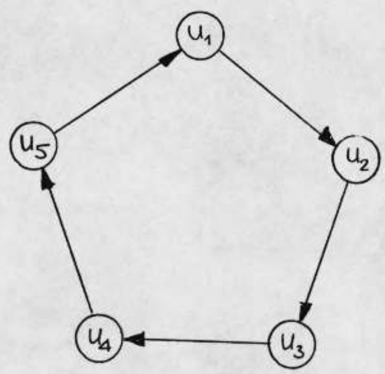

Fig. 1. A system consisting of five units.

Assume u_k is the faulty unit. Then

$$a_{ij} = 0 \quad \text{if} \quad i \neq k,\ j \neq k$$

$$a_{ik} = 1$$

and

$$a_{kj} = x \quad \text{i.e. 0 or 1}$$

since u_k, being faulty, may or may not diagnose unit u_j properly.

For this particular case, i.e. five units containing exactly one faulty unit with the testing link arrangement given in Fig. 1, the syndrome, obtained by reading the values of the five a_{ij}'s in the sequence implied by the directions of the links in Fig. 1, can only be of the form

$$0001x$$

or one of its cyclic permutations. The faulty unit is identified by the 1 following a string of three (in this case) 0's.

We shall first demonstrate that this system can produce a syndrome leading to a unique diagnosis of any single fault: The natural question that follows is whether this system is capable of diagnosing two faults. This question is an ambiguous one as it could be interpreted in two different ways. First, it could mean the capability of locating up to two faults instantly, or it could mean the capability of locating at least one faulty unit if the number of faulty units present does not exceed two. As it might be surmised, these two interpretations actually involve two different situations. These two situations will be carefully distinguished with the definitions below.

<u>Definition 3</u> - A system of n units is one-step t-fault diagnosable if all faulty units within the system can be identified without replacement provided the number of faulty units present does not exceed t.

Definition 4 - A system of n units is sequentially t-fault diagnosable if at least one faulty unit can be identified without replacement provided the number of faulty units present does not exceed t.

It is obvious that every system that is one-step t-fault diagnosable is also sequentially t-fault diagnosable. To demonstrate the existence of systems that are sequentially t-fault diagnosable, but not one-step t-fault diagnosable we shall refer again to the example in Fig. 1.

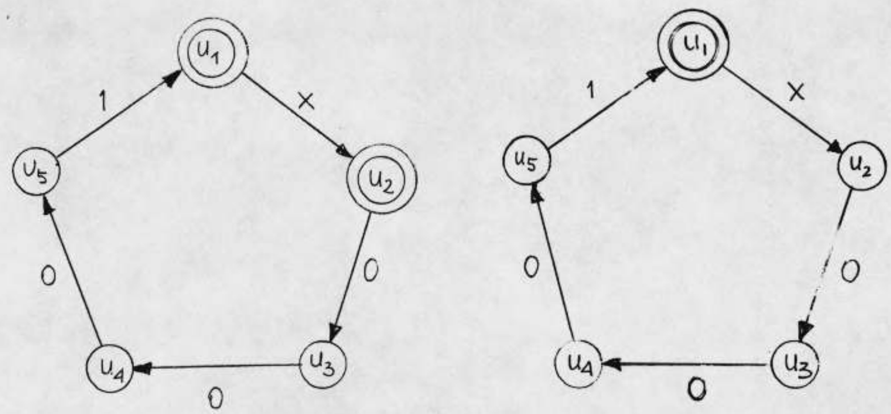

Fig. 2. Two faulty conditions exhibiting the same syndrome. Faulty units are denoted with double circles.

Assume that units u_1 and u_2 are faulty. A possible set of test signals is shown in Fig. 2 (a). An identical set of test signals is shown for the system in Fig. 2 (b) which has only one faulty unit u_1. It is clear that these two situations cannot possibly be differentiated. Hence the system shown in Fig. 1 is not one-step two-fault diagnosable. It will be shown in Section IV using a systematic approach that the system shown in Fig. 1 belongs to a class of systems that are in fact sequentially two-fault diagnosable.

For this specific example, however, some elementary observations will also suffice. All fault patterns, up to a cyclic permutation of the subscripts of the units, are shown in Fig. 3 along with their test signals (syndrome). It is easy to see that if the cyclic test pattern contains the sequence 001 then the unit at the arrowhead of this sequence is faulty. If the sequence 001 is not present then the sequence must be of the form 01x11. In this case the unit at the arrowhead of the cyclic sequence 1101 is faulty.

<u>Fault pattern</u> None	u_1	u_1, u_2	u_1, u_3
Syndrome 00000	0001x	001xx	01x1x

Fig. 3. All possible faulty patterns of the system in Fig. 1.

III. ONE-STEP DIAGNOSABLE SYSTEMS

A. Necessary and Sufficient Conditions

As stated before, a diagnosable system S is entirely specified by its nxn connection matrix C, the order of which is the number of units into which S is decomposed. In this section we investigate the relationship between n and t, the number of defective units, for one-step diagnosable systems.

We can state the following fundamental theorem.

Theorem 1: Let a system S be one-step t-fault diagnosable. Then $n \geq 2t+1$. Conversely, if $n \geq 2t+1$ it is always possible to provide a connection to form a system S such that S is one-step t-fault diagnosable.

Proof: To prove the converse we construct a maximally connected graph, that is, we make a connection among all possible pairs of these n units in both directions. One characteristic of such a graph is that there exists a loop connecting any subset of the n units. It is easily verified that given any loop connecting s units with all testing signals in the loop exhibiting the value "0," then the s units in the loop are either all faulty or all non-faulty. In particular if $s \geq t + 1$ all units in the loop must not be faulty for otherwise this would violate the hypothesis on the maximum number of faulty units. The location of a loop of $t + 1$ or more fault-free units will essentially have completed the diagnosis process as any identified fault-free unit will immediately locate all faulty units through direct connections. Finally, since S can have at most t faulty units, it must contain at least t + 1 fault-free units; hence the existence of a loop of t + 1 or more fault-free units is guaranteed. For the necessity part it suffices to exhibit, for a system S with $n < 2t+1$ units and arbitrary connections, the existence of two distinct allowable fault patterns that may result in exactly the same syndrome. An allowable fault pattern for our specific system S is any fault pattern with at most t faulty units. It is convenient to treat separately the cases of n odd or even; however, since the treatments of the two cases are perfectly analogous, we restrict ourselves to $n = 2t_o$, with $t_o \leq t$. We partition system S into two parts P_1 and P_2 each with t_o units in it. Suppose all units in P_1 are faulty and all units in P_2 are not faulty. Then all connection between units within P_2 will have a value zero and all connections pointing from units in P_2 to units in P_1 will have a value one. Since the units in P_1 are faulty, many possible combinations of values will occur. One such possible combination is for all connections between units in P_1 to have a value zero and for all connections pointing from units in P_1 to units in P_2 to have a value one. The situation is illustrated in Fig. 4.

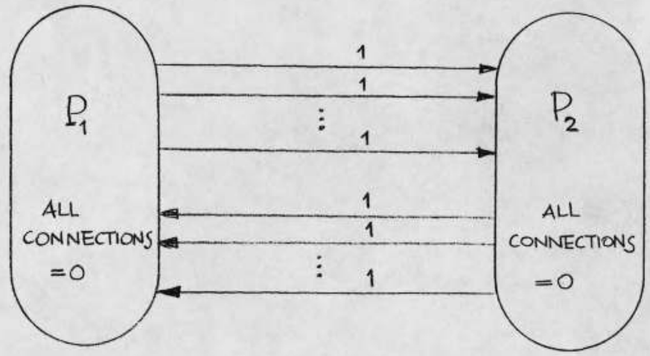

Fig. 4. An Undiagnosable Situation

From the symmetry that is evident in Fig. 4 it is seen that, when all units in P_1 are not faulty and all units in P_2 are faulty, the same pattern of test signals may result. Hence, it is not always possible for S to differentiate between the two allowable fault patterns and S is not one-step t-fault diagnosable.

B. Optimal Designs for One-Step t-Fault Diagnosis

It has been proved that for a system S which is one-step t-fault diagnosable $n \geq 2t+1$. We shall now derive a lower bound on the number of units that concurrently test a particular unit.

<u>Theorem 2</u>: In a one-step t-fault diagnosable system S, a unit is tested by at least t other units .

<u>Proof</u>: On the hypothesis that S is one-step t-fault diagnosable, we may assume that $u_1, u_2, \ldots, u_k$ are all the units in S which test a certain unit u_o, and $k < t$. Consider the case in which $u_1, u_2, \ldots, u_k$ are faulty. As remarked in Section II, the outcomes of the tests performed by units of this set do not depend only upon the

actual status of the units they test, but may assume arbitrary values. Hence there is no reliable test being performed on u_o, and the two legitimate fault patterns $(u_1, u_2, \ldots, u_k)$, $(u_o, u_1, u_2, \ldots, u_k)$ are not distinguishable under all circumstances. Hence, according to definition 1, S is not one-step t fault diagnosable. Since a contradiction has been arrived at, the assertion stated in the theorem is proved.

Definition 3 - A one-step t-fault diagnosable system S is said to be optimal if n = 2t + 1 and each unit is tested by exactly t units.

As it turned out there are many optimal designs for S. To describe these families of designs it is convenient to designate the n units to be u_o, u_1, u_2,..u_{n-1}. We shall consider a class of designs in which the testing connection at each unit is identical. In fact, whether there is a testing link from u_i to u_j depends entirely upon the value of j-i (modulo n).

Definition 4 - A system S is said to belong to a design $D_{\delta t}$ when a testing link from u_i to u_j exists if and only if $j-i = m\delta$ (modulo n) and m assumes the values 1,2,...,t.

In Fig. 5 two designs are illustrated for n = 5. D_{12} is shown in Fig. 5(a) and D_{22} is shown in Fig. 5(b).

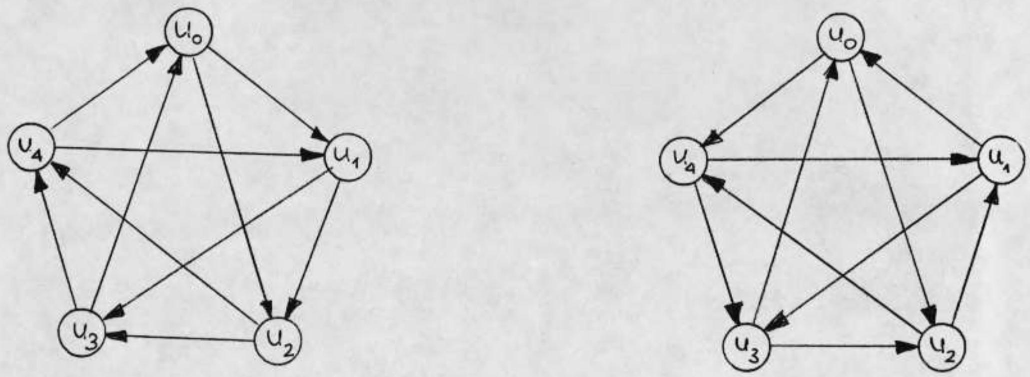

Fig. 5. Two designs for one-step t-fault diagnosis.

We shall prove that a system is one-step t-fault diagnosable if it employs a design $D_{\delta t}$ with $(\delta, n) = 1$, i.e. δ and n are relatively prime. But first we need the following lemma.

Lemma 1: If a system S employs design D_{1t} then S is one-step t-fault diagnosable.

Proof: With reference to the cyclic connecting loop of u_o, $u_1, \ldots, u_{n-1}$ a faulty string of length b is a sequence of b faulty units preceded and followed by a non-faulty unit. Consider a faulty string of length $b < t$, u_i, $u_{i+1}, \ldots, u_{i+b-1}$: the design D_{1t} provides a bypassing connection from u_{i-1} to u_{i+b} both not faulty. Hence, we will always be able to locate a loop of s fault-free units with $s \geq t + 1$ as long as the t-fault pattern does not form a single string of t consecutive faulty units. For the same reason each faulty unit is tested by at least one fault-free unit, and therefore, the system is one step diagnosable.

Let us now focus our attention on the case of t consecutive faulty units. Let the faulty units be u_i, $u_{i+1}, \ldots, u_{i+t-1}$. This implies that u_{i+t}, $u_{i+t+1}, \ldots u_{i-1}$ form a string of t + 1 consecutive fault-free units. (All subscripts are to be considered modulo n). Furthermore, $a_{j,j+1} = 0$ for $j = i+t, i+t+1, \ldots i-2$ and $a_{i-1,i} = 1$. Suppose not that u_{i-1} is faulty. In this case all $u_j (j = i+t, i+t+1, \ldots, i-2)$ would be faulty, for a total of $(i + 2t + 1) - i+t =$ t+1 faulty units, which contradicts the hypothesis on the maximum number of faulty units. Hence u_{i-1} is fault-free. Since S employs design D_{1t}, u_{i-1} tests all faulty units u_i, $u_{i+1}, \ldots, u_{i+t-1}$. This completes the proof of the lemma.

<u>Theorem 3</u>: If a system S employs design $D_{\delta t}$ such that $(\delta,n) = 1$ then S is one-step t-fault diagnosable. $D_{\delta t}$ is an optimal design.

<u>Proof</u>: The fact that $D_{\delta t}$ is an optimal design follows from definition and the fact that there are exactly t testing connections terminating at each unit. To demonstrate that S is one-step t-fault diagnosable it suffices to show that $D_{\delta t}$ is equivalent to D_{1t} for $(\delta,n) = 1$. Let us start with relabeling the units u_o, $u_1, \ldots, u_{n-1}$ of a design D_{1t} in the following manner.

$$u_o \rightarrow u'_{(o \times \delta)} = u'_o$$

$$u_1 \rightarrow u'_{(1 \times \delta)} = u'_{\delta}$$

$$u_2 \rightarrow u'_{(2 \times \delta)} = u'_{2\delta}$$

$$u_{n-1} \rightarrow u'_{\left((n-1) \times \delta\right)} = u'_{(n-1)\delta}$$

All subscripts are computed modulo n. It is obvious that each u_i is mapped to a unique $u'_{i\delta}$. To see if it is possible to map two distinct units to the same one we proceed as follows. Assume u_i and u_j both mapped to u'_k, then

$$i\delta \equiv k \equiv j\delta. \qquad \text{modulo } n$$

Since $(\delta, n) = 1$ there exists a and b, both integers such that $a\delta + bn = 1$. Hence

$$ai\delta \equiv aj\delta \qquad \text{modulo } n$$

implies

$$i(1-bn) \equiv j(1-bn) \qquad \text{modulo } n$$

and

$$i \equiv j \qquad \text{modulo } n.$$

Thus the mapping between the set of "u_i"s and the set of "u'_k"s is a one-to-one mapping. Let us rearrange the units u_i according to the mapping and examine the new connection patterns as a result of the rearrangement. It is seen that originally in D_{1t} there are t testing connections coming out from an arbitrary unit u_i and they go to $u_{i+1}, u_{i+2}, \ldots, u_{i+t}$, respectively. In the new arrangement u_i becomes $u_{i\delta}$ and testing connections coming from $u'_k = u'_{i\delta}$ are going to $u'_{(a+1)\delta} = u'_{k+\delta}$, $u'_{(i+2)\delta} = u'_{k+2\delta}, \ldots$ $u'_{(i+t)\delta} = u'_{k+t\delta}$. This testing connection pattern is easily identified as that of $D_{\delta t}$. Hence $D_{\delta t}$ is a rearrangement of D_{1t}. Since the set of all possible fault patterns with at most t fault is invariant with respect to this rearrangement by Lemma 1 the system S with design $D_{\delta t}$ is also one-step t-fault diagnosable. Q.E.D.

IV. SEQUENTIALLY DIAGNOSABLE SYSTEMS

It was shown in the last section that even for optimal designs nt links are required for a system S to be one-step t-fault diagnosable. The investigation of sequential systems is motivated by the expectation that fewer test links are required in such systems. It can be easily verified that theorem 1 is valid for sequential systems also. Hence for any sequentially t-fault diagnosable system $n \geq 2t + 1$.

Let us denote the number of test links in a design by N. It can be easily shown that designs exist for sequential t-fault diagnosis with $N \leq 2n-2$. However, the following theorem presents a stronger result.

<u>Theorem 4</u>: There exists a class of designs with $N = n+2t-2$ that are sequentially t-fault diagnosable.

<u>Proof</u>: Consider the following design. First connect all units $u_o, u_1, \ldots, u_{n-1}$ in a loop such that for every i there is a link from u_i to u_{i+1} modulo n. Secondly we select a subset S_1 of the 2t-2 units from the set $(u_1, u_2, u_3, \ldots, u_{n-2})$ and establish a link from each unit of S_1 to u_o. Let the number of testing signal from S_1 and u_{n-1} to u_o having the value zero (one) be $n_o(n_1)$. The following cases are possible:

1) <u>$n_1 > t$</u>. The assumption (u_o not faulty) implies that $n_1 > t$ units are faulty, thus violating the hypothesis on the maximum number of faulty units. Therefore, $n_1 > t$ implies u_o to be faulty.

2) $\underline{n_1 < t}$. The assumption (u_o faulty) implies that $n_o > t-1$ more units are faulty (for a total of $n_o + 1 > t$ faulty units); but this also violates the aforestated hypothesis. Therefore, $n_1 < t$ implies u_o to be not faulty.

3) $n_1 = t$. Let us consider the set S_1 plus u_o which is composed of 2t units. If u_o is not faulty, the set contains $n_1 = t$ faulty units; if u_o is faulty, S contains u_o and $n_o = t-1$ additional faulty units, for a total of t. In both cases the set contains t faulty units. We conclude that all units of S not contained within the set S_1 plus u_o are not faulty and since $n \geq 2t + 1$, at least one fault-free unit can be identified. Therefore, $n_1 = t$ implies the existence and identification of at least one fault-free unit. To locate at least one faulty unit we proceed as follows. In case 1), u_o is a faulty unit. In cases 2) and 3) we have located at least one fault-free unit. To locate a faulty unit we simply travel along the loop in the direction of the arrows. We follow the test signals until we see a one for the first time: the unit at the terminal of the arrowhead is faulty.

It is easily verified that $N = n + 2t-2$ for this class of designs.

Example: In Fig. 6 we exhibit a sequential diagnosis connection designed according to theorem 4 for $t = 6$ and $n = 14$.

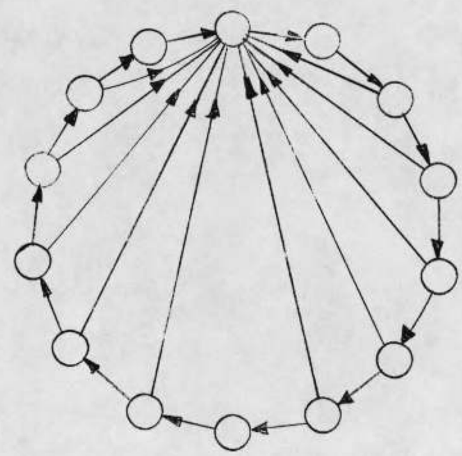

Fig. 6. An example of sequential diagnosis connection for n = 14 and t = 6.

V. SINGLE LOOP SYSTEMS

For designs presented in the last section n could be as low as $2t + 1$. In that case $N = 2t + 1 + 2t - 2 = 4t - 1$. Although we do not have any proof, it seems true that to reduce significantly the number of test links, we have to have $n > 2t + 1$. In what follows we investigate the class of designs with n test links in the form of a loop such that for every i there is a link from u_i to u_{i+1} modulo n. The number of units n required for single-loop systems to be sequentially t-fault diagnosable is derived. As a plan of attack we shall first find a relation between n and t for single loop systems, and then find the minimum n possible.

<u>Lemma 2</u>: In a single-loop system with $n \geq 2t + 1$ the value zero always appears as a test signal.

<u>Proof</u>: Since there are at most t faulty units in the system, which consists of at least $2t + 1$ units, there is at least one pair of units u_i and u_{i+1} that are not faulty. Hence $a_{i,i+1} = 0$. Q.E.D.

We shall now introduce an algorithm to partition the loop into sequences:

1) Choose a zero test signal followed by a one. The existence of a zero is guaranteed by lemma 2, the existence of a one is assured if the number of faults present is between 1 and t. The all-zero condition would mean either no faulty unit or all n units faulty. Go to step 2).

2) Mark with an X the test link following the one whose test signal is "1." Proceed to step 3) if the test link was not previously marked; otherwise, the algorithm terminates.

3) Proceed inspecting the following test link in the direction indicated by the arrowheads. If the value of its test signal is "0," repeat the present step (on the following link). If the value of the test signal is "1," return to step 2).

Four typical situations are illustrated in Fig. 7.

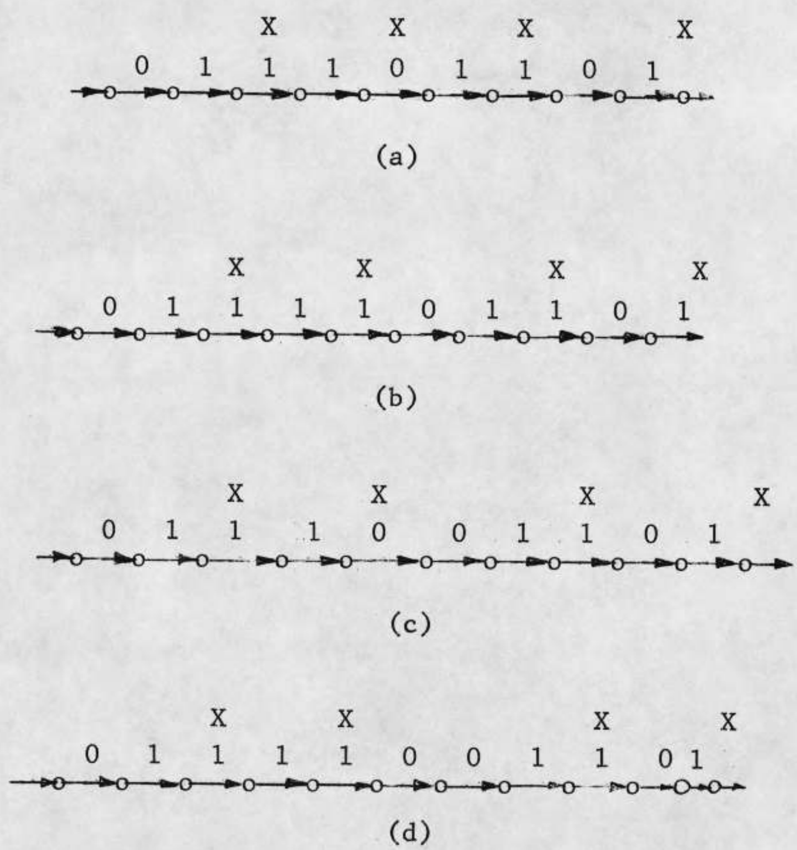

Fig. 7. Four typical situations in segmentation of the loop.

The segmentation or partition of the loop into sequences is performed by removing the links marked with X: a <u>sequence</u> consists of the units comprised between two successive X-marked links and of their connecting links. It is almost superfluous to remark that the signal pattern pertaining to a sequence is always of the form

$$0\ldots01$$

with possibly no 0 preceeding the 1.

We must still show that the segmentation provided by the given algorithm is unique. The only variable element of the algorithm is the choice of the pattern 01 to which step 1) applies, if there is more than one such pattern. It suffices to show that, given any such pattern $a_{i,i+1} = 0$, $a_{i+1,\ i+2} = 1$, whichever the starting point may be, the link $b_{i+2,\ i+3}$ will always be marked with X. In fact $b_{i+1,\ i+2}$ cannot be X-marked because $a_{i,i+1} = 0$ (see step 2). Then, if $b_{i,i+1}$ is X-marked, we apply steps 3) and 2) and mark $b_{i+2,\ i+3}$ with X; if $b_{i,i+1}$ is not X-marked, we apply step 3) twice and step 2), arriving at the same conclusion. The loop segmentation is therefore unique.

<u>Definition 5</u> - The <u>length</u> of a sequence is the number of units it contains.

Obviously, the length of a sequence is equal to or larger than 2.

To obtain the lower bound on n for single-loop systems we need the two following lemmas, which easily follow from the definitions.

<u>Lemma 3</u>: A sequence contains at least one faulty unit.

<u>Lemma 4</u>: If the last unit of a sequence of length $(r+1)$ is fault-free, then the sequence contains r faulty units.

We can now state the following theorem.

<u>Theorem 5</u>: A single-loop system is sequentially t-fault diagnosable if the number n of units it contains satisfies the following bound

$$n \geq \nu = 1 + (m+1)^2 + \lambda(m+1)$$

with $t = 2m + \lambda$, m integer and $\lambda = 0,1$.

<u>Proof</u>: As mentioned before, the proof consists of two parts. First we obtain a relation between n and t, taking as parameter the maximum sequence length r_{max}; secondly, we maximize n with respect to all possible choices of r_{max}, to assure that sequential diagnosis can always be performed $(2 \leq r_{max} \leq t+2)$.

For a given syndrome, let s be the number of sequences into which the loop has been segmented and let r_{max} be the length of the longest sequence (or sequences, in case of ties). Consider a maximum length sequence and assume that its last unit u_j is fault-free: in this case, by lemma 4, the sequence contains $(r_{max} - 1)$ faulty units. Furthermore, by lemma 3, each of the remaining $(s-1)$ sequences contains at least one faulty unit, for a total of at least $(r_{max} + s - 2)$ faulty units. If now we meet the condition

(1) $$r_{max} + s - 2 \geq t + 1$$

the hypothesis on the number of faulty units is violated and therefore u_j cannot be fault-free. The above stated condition (1) can be met if the number n of units cannot be partitioned in less than s sequences of the given maximum length. It is easily recognized that this happens if and only if

$$(2) \qquad n \geq r_{max}(s-1) + 1.$$

By eliminating s between (1) and (2) we obtain

$$(3) \qquad n \geq r_{max}(t+2 - r_{max}) + 1$$

and the first part of the proof is completed.

To prove the second part we must distinguish the case of even t from the case of odd t.

A) $t = 2m$. Relation (3) takes the form

$$n \geq (m+1)^2 + 1 - \left[r_{max} - (m+1)\right]^2 .$$

It is obvious that the most demanding requirement on n is posed by $r_{max} = m+1$, resulting in

$$(4) \qquad n \geq (m+1)^2 + 1 .$$

B) $t = 2m+1$. In this case relation (3) takes the form

$$n \geq (m+1)(m+2) + 1 - \left[r_{max} - (m+1)\right]\left[r_{max} - (m+2)\right].$$

It is easily recognized that n is maximum either when $r_{max} = m+1$ or $r_{max} = m+2$. In fact, in either case the product $\left[r_{max} - (m+1)\right]\left[r_{max} - (m+2)\right]$ is 0. When $r_{max} < m+1$, both factors are negative. When $r_{max} > m+1$, both factors are positive. Hence

$$(5) \qquad n \geq (m+1)(m+2) + 1 = (m+1)^2 + 1 + (m+1).$$

By comparing (4) and (5), the assertion of the theorem follows.[1]

We close this section with a table of the values of ν as functions of increasing values of t.

TABLE

t	2t + 1	ν
2	5	5
3	7	7
4	9	10
5	11	13
6	13	17
7	15	21

[1] The same result could be arrived at by a simple geometric argument.

V. CLOSING REMARKS

In this paper we have presented a new approach to the diagnosis problem of digital system, which has provided several interesting results and may help gain further insight into this general area.

The obtained results are of a rather fundamental type, and only a class of special connections, i.e. single-loop systems, has been extensively investigated. The outlined approach has disclosed several other questions, such as the investigation of the diagnosis capabilities offered by particular topologies of units, which we present as open research problems.

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This paper treats the problem of automatic fault diagnosis for systems with multiple faults. The system is decomposed into a number of units $u_1, u_2, \ldots, u_n$, where a unit is a well identifiable portion of the system which cannot be further decomposed for the purpose of diagnosis. By means of a given arrangement of testing links (connection assignment) each unit of the system tests a subset of units, and a proper diagnosis can be arrived at for any diagnosable fault pattern.

Methods for optimal assignments are given for instantaneous and sequential diagnosis procedures.

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