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Abstract

This paper revisits the role of prospect theory in explaining the pull-to-center effect that has been consistently observed in laboratory experiments addressing the newsvendor decision problem. Our results contrast with the extant literature, most of which uses a zero-profit reference point in prospect theory models. We use that same literature to establish the salience of mean demand in newsvendor decisions and show that, when one uses the outcome associated with that decision as a reference point, prospect theory cannot be ruled out as an explanation for the pull-to-center effect.

Key words: Prospect Theory; Newsvendor Problem; Pull-to-center Effect; Stochastic Reference Point

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1. Introduction

The classic newsvendor problem studied in the operations management literature has received increased attention in the context of individual decision making under uncertainty. In their seminal work, Schweitzer and Cachon (2000) identified the existence of a *pull-to-center* (PTC) effect in newsvendor decisions: for products with high (low) profit margins, the average order quantity is greater (less) than the mean demand but less (greater) than the expected profit-maximizing quantity.[1] This effect was consistently reported in many subsequent studies; see, for example, Bolton and Katok (2008), Rudi and Drake (2014), and the references therein. Lau et al. (2014) have shown that the PTC effect does not represent the heterogeneity in the subject pool. Yet they show also that (i) the aggregate data is consistent with this effect and (ii) a small portion of the subject pool exhibits behavior that is not inconsistent with this effect.[2]

The literature on behavioral experiments with the newsvendor model has discussed various explanations for the observed PTC effect; see Nagarajan and Shechter (2014) for a comprehensive summary of this literature. Following Kahneman and Tversky (1979) and Tversky and Kahneman (1992), prospect theory (PT) has figured prominently in the literature on individual decision making. It is hardly surprising, then, that PT has been much discussed as a possible explanation for the PTC effect.

[1] The interval between the mean demand and the expected profit-maximizing quantity is known as the *PTC region*.

[2] Lau et al. (2014) report that 15–35% of subjects exhibit behavior that accords with the PTC effect when such accordance is defined as "at least half of the subject's orders are within the PTC region".

Schweitzer and Cachon (2000) argue that PT cannot explain the observed behavior when the decision maker can only make gains (i.e., earn positive profits) as a result of his quantity decisions. Hence these authors reject PT as an explanation of the PTC effect by conducting experiments in the "gains" domain only—that is, when no quantity decision can lead to a negative profit—and showing the PTC effect's persistence in these experiments. A serious limitation in their study is that the analysis does not account for nonlinear weighting of probabilities by the decision makers. Nagarajan and Shechter (2014) address this limitation and demonstrate that the insights from Schweitzer and Cachon remain unchanged. Together these studies seem to indicate that PT is not consistent with the PTC effect and thus should be ruled out as a possible explanation of that effect.

Contrary to these two studies, we find that PT actually could explain the PTC effect. We obtain this result in large part because, unlike previous studies, we assume that the individual decision maker's *reference point* is the profit associated with ordering a quantity equal to the mean demand.[3] In a parallel study, Long and Nasiry (2014) also show consistency between PT and the PTC effect. However, different than us, they assume that the reference point is a function of the chosen order quantity; whereas, the reference point in our paper is independent of the chosen order quantity and is a function of an exogenous salient quantity. Later in this section we will provide theoretical arguments as well as supporting data for our choice of such a reference point. We believe that Long and Nasiry (2014) and this paper complement each other, and rigorously establish that PT cannot be ruled out as a potential explanation of the PTC effect. In the rest of this section we briefly discuss the assumption made in Schweitzer and Cachon (2000) and Nagarajan and Shechter (2014) regarding the reference point and then contrast it—and the corresponding insights—with this paper.

The analysis in Schweitzer and Cachon (2000) and Nagarajan and Shechter (2014) implicitly assumes that the reference point for the decision maker is zero-profit. That reference point may be a reasonable assumption in some contexts—as when the decision maker, when offered a set of prospects to choose from, also has the option of not choosing any. The prospect of choosing nothing is salient and represents the status quo for the decision maker with an associated profit equal to zero. However, in the previously cited literature on newsvendor experiments, the subjects do *not* have the option of ordering a quantity of zero (i.e., the quantity that would entail a zero profit). Moreover, Schweitzer and Cachon as well as Nagarajan and Shechter, when arguing that PT is

[3] The reference point is an important feature of prospect theory. In this theory, value is assigned to gains and losses with respect to the reference point and not to the final asset (a feature known as *reference dependence*), and the probabilities associated with different outcomes are replaced by corresponding decision weights (which account for probability distortion on the decision maker's part). The value function under PT is concave for gains and convex for losses, and it is generally steeper for losses than for gains—that is, losses "loom larger" than do gains of the same size (a phenomenon known as *loss aversion*).

inconsistent with the PTC effect, restrict their analysis to a setting in which the decision maker is guaranteed a positive profit irrespective of the order quantity chosen. So given this absence of zero profit as a possible outcome, we propose that a zero-profit reference point may not yield an accurate characterization of PT in the analysis presented by those two studies.

The question that naturally arises is: What *could* serve as a reasonable reference point? We draw from the literature on PT to motivate such an alternative. First, Kahneman (1992) suggests that a reference point is a *salient point*, within the cognitive norm of the decision maker, at which his value function changes abruptly. Second, studies in decision theory and behavioral economics (Sugden 2003, Kőszegi and Rabin 2006, De Giorgi and Post 2011) indicate that a reference point need not be a fixed outcome and instead may be the outcome associated with a fixed decision that is subject to an exogenous uncertainty (such a reference point is called *reference lottery* or *stochastic reference point* in this literature). Therefore, if it can be established that a certain order quantity is a salient point for the decision maker faced with a newsvendor problem, then one can reasonably argue that the outcome associated with this quantity serves as a reference point for that decision maker.

Of all the prospects presented to a decision maker in the newsvendor experimental setting,[4] the prospect of ordering a quantity that equals mean demand (henceforth referred to as the *mean prospect*) is arguably the most salient prospect. In all of the experimental studies that we have cited, demand follows either the normal distribution or the uniform distribution. In experiments that assume the normal distribution, subjects are presented with the distribution's mean and standard deviation. By the *focusing effect* (Kahneman et al. 1982), such a representation makes the mean more salient than any other order quantity. In contrast, if the uniform distribution is assumed then experimental subjects are presented with the two end points of the support of that distribution. The two end-points seem an unreasonable choice for subjects who are faced with a decision of balancing a trade-off between over- and under-ordering. Although in these experiments the mean is not explicitly salient, it is implicitly a reasonable salient quantity because it is mid-point of the two reported end points. Another argument in favor of the mean's salience is its status as the modal decision made by subjects across most experiments reported in several independent studies (Bolton and Katok 2008, Lau et al. 2014, Rudi and Drake 2014). For example, Figure 1 presents the histograms of orders placed by decision makers in an experiment reported in Lau et al. (2014). Hence, we rationalize that the profit associated with the mean prospect is a reasonable reference point for characterizing the PT value function for newsvendor experiments. In contrast, Long and Nasiry (2014) assume that the reference point, for each prospect for the decision maker, is the

[4] Because the "prospects" in this context are the order quantities, we shall use these terms interchangeably.

weighted average of the maximum and the minimum possible payoff. As the role of the reference point is *critical* in both Long and Nasiry (2014) and this paper to establish the potential consistency between PT and the PTC effect, we believe that the difference in this assumption makes these two papers significantly distinct and complementary to each other.

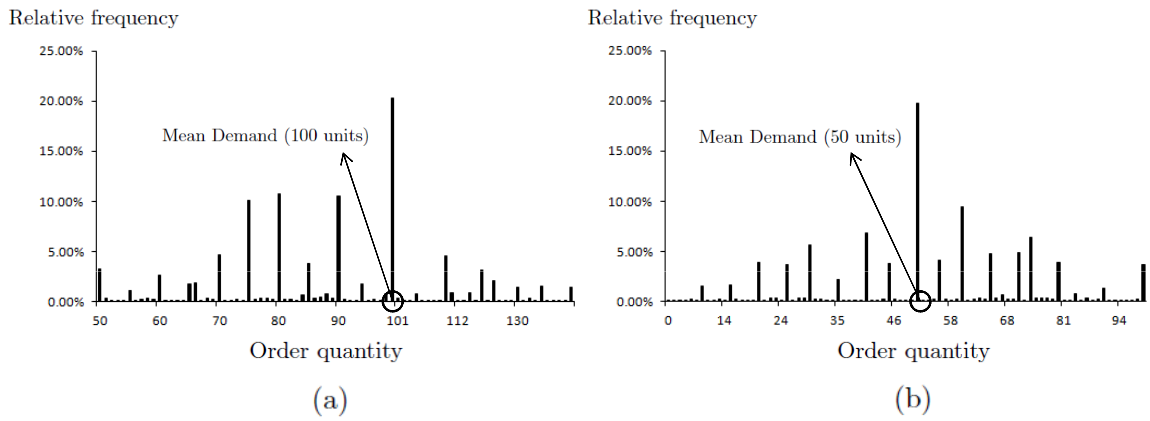

Figure 1 Histograms of order quantities from Lau et al. (2014) for (a) the low-margin case and (b) the high-margin case.

This paper shows that, when the outcome associated with the mean prospect is used as the reference point, prospect theory is consistent with the pull-to-center effect. We therefore conclude that PT cannot be ruled out as an explanation for the PTC effect. Interestingly, we find that PT is consistent with the PTC effect when a set of inequalities reflecting the individual characteristics of decision makers (e.g., degree of loss aversion) are satisfied. Therefore, our findings are theoretically consistent with Lau et al.'s (2014) demonstration that not all decision makers exhibit an ordering behavior that is consistent with the PTC effect.[5]

2. Model and Analysis

In this section we describe the mathematical model. The decision maker is assumed to face a newsvendor problem with unit cost equal to c, unit revenue equal to r, and no salvage value. Demand distribution is represented by F, which is known to the decision maker, with support ranging from $d_{\min}$ to $d_{\max}$. Let μ represent the mean of this distribution, and define $\alpha = c/r$. It is well known that the expected profit-maximizing quantity q_{EP} satisfies $F(q_{\text{EP}}) = 1 - \alpha$. The product is said to be a high-margin (HM) product if $\alpha < 0.5$ or a low-margin (LM) product if $\alpha > 0.5$.

We assume that the newsvendor is a PT value-maximizing individual who is characterized by the value function v and by the probability weighting functions w^+ for gains and w^- for losses.[6] We

[5] If our results showed that PT and PTC are unconditionally consistent, then theoretically our results would *not* be consistent with Lau et al. (2014).

[6] See Nagarajan and Shechter (2014) for a detailed explanation of the newsvendor problem as a choice among prospects.

assume that the newsvendor is loss averse and that the degree of this loss aversion is characterized by the parameter $\lambda > 1$ such that $v(-x) = -\lambda v(x)$ for $x > 0$.[7] As is the convention with PT models, we assume that v is concave and increasing with $v(0) = 0$ and $v'(0) > 0$ and also that w^+ and w^- are both increasing with $w^+(0) = w^-(0) = 0$ and $w^+(1) = w^-(1) = 1$. We do not assume any particular shape (e.g., an inverse S-shape) for either w^+ or w^-.

As mentioned in Section 1, we assume the outcome associated with the mean prospect as the reference point. We use the framework of De Giorgi and Post (2011) to derive the PT value function for newsvendor decisions.[8] Using their terminology, we define the relative gain–loss utility of an order quantity as the relative gain or loss experienced by the newsvendor when he orders that quantity as compared with the case when he orders the quantity that is equal to mean demand. We can then write this relative gain–loss utility, contingent on the realized demand x, as follows. Let the order quantity q be less than the mean μ of the demand distribution. Then the following statements hold.

(i) For $d_{\min} \le x < q$, the relative gain–loss $= (rx - cq) - (rx - c\mu) = c(\mu - q) > 0$. Hence we are in the gains domain and the corresponding value is $v(c(\mu - q))$.

(ii) For $q \le x < \alpha\mu + (1-\alpha)q$, the relative gain–loss $= (rq - cq) - (rx - c\mu) > 0$. We are still in the gains domain, with value $v((rq - cq) - (rx - c\mu))$, but now this value is less than $v(c(\mu - q))$ and decreases monotonically to zero as x approaches $\alpha\mu + (1-\alpha)q$.

(iii) For $\alpha\mu + (1-\alpha)q \le x < \mu$, the relative gain–loss $= (rq - cq) - (rx - c\mu) \le 0$. Thus we are in the losses domain with corresponding value $-\lambda v(-(rq - cq) + (rx - c\mu))$. This value decreases monotonically from zero to $-\lambda v((r-c)(\mu - q))$ as x approaches μ.

(iv) For $\mu \le x \le d_{\max}$, the relative gain–loss $= -(r-c)(\mu - q) < 0$. Again we are in the losses domain, and the corresponding value is $-\lambda v((r-c)(\mu - q))$.

The relative gain–loss for $q < \mu$ is plotted in Figure 2(a). Under PT, the decision weight in the gains domain is the marginal probability weight of obtaining a better or equal outcome; in the losses domain, it is the marginal probability weight of obtaining an equal or worse outcome (Tversky and Kahneman 1992, Tversky and Wakker 1995). Figure 2(a) shows that in the gains domain, a better or equal outcome is obtained with lower demand, whereas in the losses domain,

[7] Assuming that $\lambda > 1$ ensures that losses loom larger than do equal-sized gains. The higher the value of λ, the more loss averse is the newsvendor.

[8] Formally, the model in this paper is obtained by setting $m(x) = x$, $\eta_1 = 0$, and $\mu(\cdot) = v(\cdot)$ in the model of De Giorgi and Post (2011).

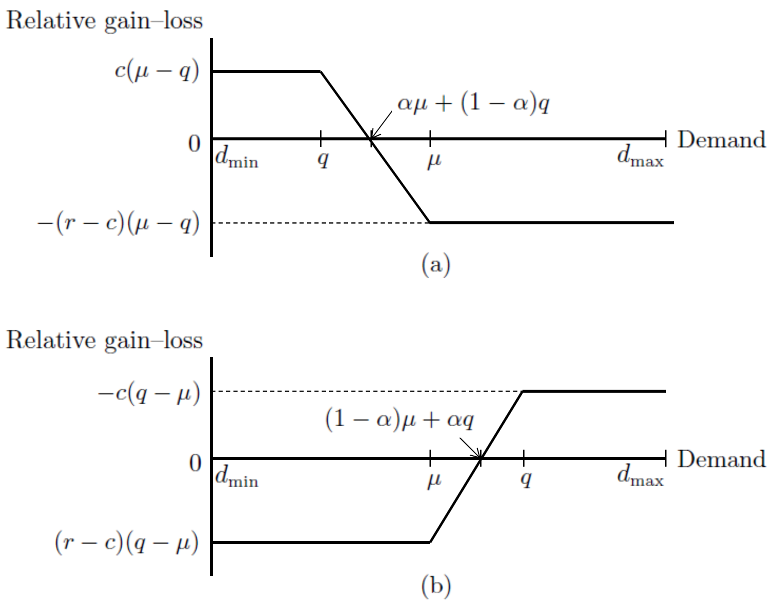

Figure 2 **The relative gain–loss plotted for (a)** $q<\mu$ **and (b)** $q>\mu$**.**

an equal or worse outcome is obtained with higher demand.[9] We can therefore write the PT value obtained from ordering a quantity $q<\mu$, denoted $L_{q<\mu}(q)$, as

$$\begin{aligned}L_{q<\mu}(q) = &\int_{d_{\min}}^{q} v(c(\mu-q))\, d[w^+(F(x))] + \int_{q}^{\alpha\mu+(1-\alpha)q} v(r(q-x)-c(q-\mu))\, d[w^+(F(x))]\\ &-\lambda \int_{\alpha\mu+(1-\alpha)q}^{\mu} v(r(x-q)-c(\mu-q))\, d[-w^-(\bar{F}(x))]\\ &-\lambda \int_{\mu}^{d_{\max}} v((r-c)(\mu-q))\, d[-w^-(\bar{F}(x))].\end{aligned}$$

We can similarly obtain the relative gain–loss for the case $q>\mu$ that is shown in Figure 2(b). The corresponding PT value function, denoted $L_{q>\mu}(q)$, is

$$\begin{aligned}L_{q>\mu}(q) = &-\lambda\int_{d_{\min}}^{\mu} v(c(q-\mu))\, d[w^-(F(x))] - \lambda\int_{\mu}^{(1-\alpha)\mu+\alpha q} v(r(\mu-x)-c(\mu-q))\, d[w^-(F(x))]\\ &+\int_{(1-\alpha)\mu+\alpha q}^{q} v(r(x-\mu)-c(q-\mu))\, d[-w^+(\bar{F}(x))]\\ &+\int_{q}^{d_{\max}} v((r-c)(q-\mu))\, d[-w^+(\bar{F}(x))].\end{aligned}$$

Since the outcome associated with the mean prospect is the reference point, it follows that $L_{q<\mu}(\mu)=L_{q>\mu}(\mu)=0$. The corresponding first-order conditions are

$$L'_{q<\mu}(q) = -cv'(c(\mu-q))w^+(F(q)) + (r-c)\int_{q}^{\alpha\mu+(1-\alpha)q} v'(r(q-x)-c(q-\mu))\, d[w^+(F(x))]$$

[9] We remark that the models of Sugden (2003), Kőszegi and Rabin (2006) and De Giorgi and Post (2011) do not account for nonlinearly weighted probabilities. Here we do use nonlinear probabilities—so that the analysis will be general. The results do not change if we instead assume linearly weighted probabilities.

$$
\begin{aligned}
&+\lambda(r-c)\int_{\alpha\mu+(1-\alpha)q}^{\mu} v'(r(x-q)-c(\mu-q))\,d[-w^-(\bar F(x))] \\
&+\lambda(r-c)v'((r-c)(\mu-q))w^-(\bar F(\mu)) \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
L'_{q>\mu}(q) = &-\lambda c\left\{v'(c(q-\mu))w^-(F(\mu))+\int_{\mu}^{(1-\alpha)\mu+\alpha q} v'(r(\mu-x)-c(\mu-q))\,d[w^-(F(x))]\right\} \\
&-c\int_{(1-\alpha)\mu+\alpha q}^{q} v'(r(x-\mu)-c(q-\mu))\,d[-w^+(\bar F(x))] \\
&+(r-c)v'((r-c)(q-\mu))w^+(\bar F(q)). \qquad (2)
\end{aligned}
$$

If $\lambda = 1$ and if v, w^+ and w^- are linear, then $L'_{q<\mu}(q) = L'_{q>\mu}(q) = -rF(q) + (r-c)$, which coincides with the expected profit maximization. Before we state formally the main result of this paper, we first note a few technical details.

It is clear that each PT value function $L_{q<\mu}$ and $L_{q>\mu}$ is an infinite summation of positively weighted convex and concave functions. Hence these functions need not necessarily be concave. Moreover, at μ the overall value is continuous but not necessarily differentiable (because $L'_{q<\mu}(\mu)$ and $L'_{q>\mu}(\mu)$ need not be equal). Hence we do not prove any concavity or uniqueness results in the following theorem; we only establish the existence of the optimal solution in the PTC region and no such existence outside that region. This is sufficient to demonstrate the PTC effect. It is important to note also that the analysis remains the same even if the newsvendor cannot incur any monetary losses (i.e., if $rd_{\max} - cd_{\min} > 0$), as in the second study of Schweitzer and Cachon (2000) and the analysis of Nagarajan and Shechter (2014). Provided the newsvendor's reference point is the outcome associated with the mean prospect and that he is sufficiently loss averse (i.e., provided the Theorem 1 constraints are satisfied), he will exhibit the PTC effect.

For notational convenience, we define $q^\alpha = (1-\alpha)\mu + \alpha q$ and $q^{1-\alpha} = \alpha\mu + (1-\alpha)q$ for any feasible q. More specifically, define q_i^α and $q_i^{1-\alpha}$ as q^α and $q^{1-\alpha}$ evaluated at $q = q_i$, where $i \in \{\mathrm{LM}, \mathrm{HM}\}$.

THEOREM 1. *Let $\alpha_i = c_i/r_i$ for $i \in \{\mathrm{LM}, \mathrm{HM}\}$, and let q_i be the corresponding expected profit-maximizing quantity. If the loss aversion parameter λ of a prospect theory value-maximizing newsvendor satisfies*

$$\max\left\{\frac{1-\alpha_{\mathrm{LM}}}{\alpha_{\mathrm{LM}}}\frac{w^+(\bar F(\mu))}{w^-(F(\mu))}, \frac{\alpha_{\mathrm{LM}}}{1-\alpha_{\mathrm{LM}}}\frac{w^+(1-\alpha_{\mathrm{LM}})}{w^-(\bar F(q_{\mathrm{LM}}^{1-\alpha}))}\right\} < \lambda < \frac{\alpha_{\mathrm{LM}}}{1-\alpha_{\mathrm{LM}}}\frac{w^+(F(\mu))}{w^-(\bar F(\mu))}, \qquad (3)$$

$$\max\left\{\frac{\alpha_{\mathrm{HM}}}{1-\alpha_{\mathrm{HM}}}\frac{w^+(F(\mu))}{w^-(\bar F(\mu))}, \frac{1-\alpha_{\mathrm{HM}}}{\alpha_{\mathrm{HM}}}\frac{w^+(\alpha_{\mathrm{HM}})}{w^-(F(q_{\mathrm{HM}}^{\alpha}))}\right\} < \lambda < \frac{1-\alpha_{\mathrm{HM}}}{\alpha_{\mathrm{HM}}}\frac{w^+(\bar F(\mu))}{w^-(F(\mu))}, \qquad (4)$$

then the newsvendor exhibits the pull-to-center effect.

We now discuss the constraints on λ in Theorem 1. All experiments in the aforementioned newsvendor literature use a symmetric demand distribution, so here we assume that $F(\mu) = 0.5$.

By definition, $\alpha_{\mathrm{LM}} > 0.5$ and $\alpha_{\mathrm{HM}} < 0.5$. If we assume that $w^+(0.5) \approx w^-(0.5)$ (which is empirically true), then the constraints

$$\lambda > \frac{1-\alpha_{\mathrm{LM}}}{\alpha_{\mathrm{LM}}} \frac{w^+(\bar{F}(\mu))}{w^-(F(\mu))} \quad \text{and} \quad \lambda > \frac{\alpha_{\mathrm{HM}}}{1-\alpha_{\mathrm{HM}}} \frac{w^+(F(\mu))}{w^-(\bar{F}(\mu))}$$

are redundant because $\lambda > 1$. Moreover, if α_{LM} and α_{HM} are symmetric—that is, if $\alpha_{\mathrm{LM}} = 1 - \alpha_{\mathrm{HM}}$, as was the case in these experiments ($\alpha_{\mathrm{LM}} = 0.75$ and $\alpha_{\mathrm{HM}} = 0.25$)—then the two constraints are identical. Using Tversky and Kahneman's (1992) probability weighting functions $w^+(p) = p^\gamma / (p^\gamma + (1-p)^\gamma)^{1/\gamma}$ and $w^-(p) = p^\delta / (p^\delta + (1-p)^\delta)^{1/\delta}$ with the median values of γ and δ ($\gamma = 0.61$ and $\delta = 0.69$), we obtain $w^+(0.5) = 0.4206$, $w^-(0.5) = 0.4540$, $w^+(0.25) = 0.2907$, and $w^+(0.75) = 0.5683$. Given these values, we can now evaluate the constraints with the following demand distributions.

1. *Uniform distribution.* For $i \in \{\mathrm{LM}, \mathrm{HM}\}$, $F(q_i^{1-\alpha}) = \alpha F(\mu) + (1-\alpha)F(q_i) = 0.5\alpha + (1-\alpha)^2$ and $F(q_i^\alpha) = (1-\alpha)F(\mu) + \alpha F(q_i) = (0.5+\alpha)(1-\alpha)$, from which it follows that $w^-(\bar{F}(q_{\mathrm{LM}}^{1-\alpha})) = w^-(F(q_{\mathrm{HM}}^\alpha)) = w^-(0.5625) = 0.4936$. Both constraints are then equivalent to $1.7668 < \lambda < 2.7793$.
2. *Normal distribution.* For $\mu = 1000$ and $\sigma = 400$ (Rudi and Drake 2014) we have $q_{\mathrm{LM}} = 730$ and $q_{\mathrm{HM}} = 1270$; then $w^-(\bar{F}(q_{\mathrm{LM}}^{1-\alpha})) = w^-(F(q_{\mathrm{HM}}^\alpha)) = w^-(0.5670) = 0.4965$. In this case, both constraints are equivalent to $1.7565 < \lambda < 2.7793$.

Tversky and Kahneman (1992) report the median value of λ as 2.25, which falls within the range for both the above examples. In fact, 2.25 is approximately the mid-point of each interval and is safely within them. If we define the median person as the one with the median values of γ, δ, and λ, then the *median person exhibits the PTC effect.*[10]

3. Conclusion

In this paper we reexamine the inconsistency, promulgated in the literature, between prospect theory and the pull-to-center effect. Our analysis shows that, if we use the outcome associated with the mean prospect as a reference point, then PT cannot be ruled out as an explanation for the PTC effect. We also present support from the literature on PT—as well as data from past experiments—to support our assumption about the reference point. We then show that PT is consistent with the PTC effect when a set of inequalities reflecting the individual characteristics of decision makers are satisfied. Our results are thus theoretically in accord with Lau et al. (2014), who show that not all

[10] This result remains unchanged when we use the two-parameter weighting function from Gonzalez and Wu (1999), which is of the form $w^j(p) = \delta_j p^{\gamma_j} / (\delta_j p^{\gamma_j} + (1-p)^{\gamma_j})$ for $j \in \{+, -\}$. Using the median values of γ_j and δ_j elicited by Abdellaoui et al. (2011) in a description-based decision context, we obtain $1.5831 < \lambda < 2.8193$ for the uniform distribution and $1.5724 < \lambda < 2.8193$ for the normal distribution. The median values of λ elicited using two different methods in their setting (i.e., 2.47 and 2.62) fall within these intervals.

decision makers exhibit ordering behavior that is consistent with the PTC effect. Using values of decision-maker characteristics from the literature, we show that an individual with median values satisfies the inequalities required to establish the consistency between PT and the PTC effect.

In short, this paper rigorously demonstrates that—contrary to previous claims—prospect theory cannot be ruled out as a possible explanation for the empirically observed pull-to-center effect.

Appendix

Proof of Theorem 1. Let q^* be the PT value-maximizing quantity. Since v is concave, v' is decreasing. Therefore, when $q<\mu$, $v'(c(\mu-q)) \le v'(r(q-x)-c(q-\mu))$ for $q \le x \le \alpha\mu+(1-\alpha)q$, and $v'((r-c)(\mu-q)) \le v'(r(x-q)-c(\mu-q))$ for $\alpha\mu+(1-\alpha)q \le x \le \mu$. Using these inequalities in (1), we obtain

$$\begin{aligned} L'_{q<\mu}(q) \ge\, & rv'(c(\mu-q))\big\{(1-\alpha)w^+(F(q^{1-\alpha}))-w^+(F(q))\big\} \\ & + rv'((r-c)(\mu-q))\lambda(1-\alpha)w^-(\bar F(q^{1-\alpha})). \end{aligned} \tag{5}$$

In a similar manner, equation (2) yields

$$\begin{aligned} L'_{q>\mu}(q) \le\, & -rv'(c(q-\mu))\lambda\alpha w^-(F(q^{\alpha})) \\ & - rv'((r-c)(q-\mu))\big\{\alpha w^+(\bar F(q^{\alpha}))-w^+(\bar F(q))\big\}. \end{aligned} \tag{6}$$

Let us now analyze the low-margin and high-margin cases separately.

Low-margin. To avoid complex notation, set $\alpha=\alpha_{\mathrm{LM}}$, $c=c_{\mathrm{LM}}$, and $r=r_{\mathrm{LM}}$. By definition of low-margin, we have $r-c<c$. Since v' is decreasing, it follows from (5) that

$$L'_{q<\mu}(q) \ge rv'(c(\mu-q))\big\{(1-\alpha)w^+(F(q^{1-\alpha}))-w^+(F(q))+\lambda(1-\alpha)w^-(\bar F(q^{1-\alpha}))\big\}.$$

For $q \le q_{\mathrm{LM}}$, we have $w^-(\bar F(q^{1-\alpha})) \ge w^-(\bar F(q_{\mathrm{LM}}^{1-\alpha}))$ and $w^+(F(q^{1-\alpha})) \ge w^+(F(q))$ because $q \le q^{1-\alpha} \le \mu$. Therefore,

$$\begin{aligned} L'_{q<\mu}(q) &\ge rv'(c(\mu-q))\big\{-\alpha w^+(F(q))+\lambda(1-\alpha)w^-(\bar F(q_{\mathrm{LM}}^{1-\alpha}))\big\} \\ &\ge rv'(c(\mu-q))\big\{-\alpha w^+(F(q_{\mathrm{LM}}))+\lambda(1-\alpha)w^-(\bar F(q_{\mathrm{LM}}^{1-\alpha}))\big\}. \end{aligned}$$

As is customary, we use LHS and RHS to denote (respectively) "left-hand side" and "right-hand side". Using the LHS inequality of (3) yields $L'_{q<\mu}(q)>0$. We also have

$$L'_{q<\mu}(\mu) = rv'(0)\big\{\lambda(1-\alpha)w^-(\bar F(\mu))-\alpha w^+(F(\mu))\big\}.$$

Using $v'(0)>0$ and the RHS inequality of (3) yields $L'_{q<\mu}(\mu)<0$. Moreover, for $q \ge \mu$, again using concavity of v and $r-c<c$, it follows from (6) that

$$\begin{aligned} L'_{q>\mu}(q) &\le -rv'(c(q-\mu))\big\{\lambda\alpha w^-(F(q^{\alpha}))+\alpha w^+(\bar F(q^{\alpha}))-w^+(\bar F(q))\big\} \\ &\le -rv'(c(q-\mu))\big\{\lambda\alpha w^-(F(\mu))-(1-\alpha)w^+(\bar F(q))\big\} \\ &\le -rv'(c(q-\mu))\big\{\lambda\alpha w^-(F(\mu))-(1-\alpha)w^+(\bar F(\mu))\big\} \end{aligned}$$

because $q > q^{\alpha} > \mu$. By the LHS inequality of (3), $L'_{q>\mu}(q) < 0$.

Since $L'_{q<\mu}(q) > 0$ for $q \le q_{\mathrm{LM}}$, $L'_{q<\mu}(\mu) < 0$, and $L'_{q>\mu}(q) < 0$ for $q \ge \mu$, it follows that there exists an optimal q^* satisfying $q_{\mathrm{LM}} < q^* < \mu$.

High-margin. Set $\alpha = \alpha_{\mathrm{HM}}$, $c = c_{\mathrm{HM}}$, and $r = r_{\mathrm{HM}}$. Using the definition of high-margin that $r - c > c$ and concavity of v in (6), we obtain

$$L'_{q>\mu}(q) \le -rv'((r-c)(q-\mu))\big\{\lambda\alpha w^-(F(q^{\alpha})) + \alpha w^+(\bar{F}(q^{\alpha})) - w^+(\bar{F}(q))\big\}.$$

For $q \ge q_{\mathrm{HM}}$, we have $w^-(F(q^{\alpha})) \ge w^-(F(q^{\alpha}_{\mathrm{HM}}))$ and $w^+(\bar{F}(q^{\alpha})) \ge w^+(\bar{F}(q))$ because $\mu \le q^{\alpha} \le q$. As a result,

$$\begin{aligned} L'_{q>\mu}(q) &\le -rv'((r-c)(q-\mu))\big\{\lambda\alpha w^-(F(q^{\alpha}_{\mathrm{HM}})) - (1-\alpha)w^+(\bar{F}(q))\big\} \\ &\le -rv'((r-c)(q-\mu))\big\{\lambda\alpha w^-(F(q^{\alpha}_{\mathrm{HM}})) - (1-\alpha)w^+(\bar{F}(q_{\mathrm{HM}}))\big\}. \end{aligned}$$

Using the LHS inequality of (4) yields $L'_{q>\mu}(q) < 0$. We also have

$$L'_{q>\mu}(\mu) = rv'(0)\big\{-\lambda\alpha w^-(F(\mu))) + (1-\alpha)w^+(\bar{F}(\mu))\big\}.$$

Using the RHS inequality of (4) yields $L'_{q>\mu}(\mu) > 0$. Moreover, for $q \le \mu$, again using the concavity of v and $r - c > c$, it follows from (5) that

$$\begin{aligned} L'_{q<\mu}(q) &\ge rv'((r-c)(\mu-q))\big\{(1-\alpha)w^+(F(q^{1-\alpha})) - w^+(F(q)) + \lambda(1-\alpha)w^-(\bar{F}(q^{1-\alpha}))\big\} \\ &\ge rv'((r-c)(\mu-q))\big\{(1-\alpha)w^+(F(q)) - w^+(F(q)) + \lambda(1-\alpha)w^-(\bar{F}(\mu))\big\} \\ &\ge rv'((r-c)(\mu-q))\big\{-\alpha w^+(F(\mu)) + \lambda(1-\alpha)w^-(\bar{F}(\mu))\big\} \end{aligned}$$

because $q < q^{1-\alpha} < \mu$. Using the LHS inequality of (4), we have $L'_{q<\mu}(q) > 0$.

Since $L'_{q>\mu}(q) < 0$ for $q \ge q_{\mathrm{HM}}$, $L'_{q>\mu}(\mu) > 0$, and $L'_{q<\mu}(q) > 0$ for $q \le \mu$, it follows that there exists an optimal q^* satisfying $\mu < q^* < q_{\mathrm{HM}}$. $\square$

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