ON THE CONSTRUCTION OF A MULTI-STAGE, MULTI-PERSON BUSINESS GAME

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P-1056

Revised November 21, 1957

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SUMMARY

The purpose of this paper is to discuss a multi-stage, multi-person, business game which will be used for executive training purposes by the American Management Association.

We begin with a brief discussion of the basic philosophy of game play and of the many analytic, computational and conceptual difficulties encountered in the construction of business games.

Following this, we present four features of the particular game which we have constructed and have played which we feel merit consideration:

1. Absence of an explicit criterion function
2. Principle of marginal change
3. Hidden formulas
4. Minimal computation

The game is described in some detail and it is shown how it circumvents or overcomes a number of the obstacles described above.

In a number of preliminary plays of the game using top management and academic personnel, the game has met with a favorable reception.

CONTENTS

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Introduction

In this paper we propose to discuss in greater and lesser detail a number of questions connected with the interesting and significant problem of constructing games portraying various aspects of economic and industrial interaction. We shall combine a general discussion with a detailed study of a multi-stage, multi-person game with which we have had some experience over the past year.

Although a certain amount of effort has been devoted in recent years to the subject of economic games, and an even greater effort has been allocated to the closely allied domain of military games, there has been little attention given to careful analyses of the basic philosophy motivating the construction of these models, with due regard to the numerous difficulties of analytic, computational and conceptual nature that crop up in expected, and sometimes unexpected, places.

Since many more groups are entering this field of activity which shows definite signs of assuming a role in business planning and executive training, we feel that it will be a contribution to the development of this young art first to point out some of the pitfalls that may trap the unwary, secondly to indicate some of the methods that can be used, and have been used, to circumvent these snares, and finally to describe some of the rewards at the end of the trail.

1. The RAND Corporation, Consultant Booz, Allen and Hamilton
2. Booz, Allen and Hamilton
3. American Management Association
4. Booz, Allen and Hamilton
5. American Management Association

Although in the sense that misery loves company, it is comforting to be assured that certain difficulties are well-recognized, it is even better to know how to overcome these obstacles. We feel that we have had some degree of success in this direction and that we have some positive contributions to make to the solution of a number of basic problems involved in the formulation of these games.

Since the very idea of a business game mimicking actual business practice is so attractive and beckoning to the imagination, there is a real difficulty that the eager amateur may become so disillusioned upon experiencing unforeseen setbacks as to abandon the entire project.

It is the consensus of opinion that business games are extremely valuable tools in the hands of experienced practitioners. As in the case of other scientific tools, such as, for example, digital computers, they neither solve significant problems by themselves, nor do they in any way replace the need for intelligent interpretation. These remarks, of course, carry over in obvious fashion to the general study of simulation processes, of which the processes described below constitute only a small part.

While in the discussion that follows, it is our purpose to consider the more general aspects involved in the construction of business games, it is perhaps well to present a brief description of the particular game which has proved the inspiration for this paper. This will serve the useful purpose of creating a frame of reference for the ensuing discussion.

The end-product of the research to date has been a game which simulates a competitive industrial situation wherein five companies are competing in a growing economy. The usual objectives concerning the desired share of the market and growth in total assets provide the motivations, and also a rough measure of success, to the participants who are considered to be presidents of the five competing forms.

Figure 1 is a reproduction of the form that is given to each company at the beginning of each "play" of the game, and represents one quarter of a year of operation. As is shown, all companies begin the game with identical positions in regard to inventory, assets, price, plant capacity, etc. A single play consists of each company president, (assisted by his staff), making decisions concerning the allocation of company money to the production, marketing, research and development and plant investment programs of the company for the ensuing quarter. The price of the product must also be specified and it is possible to buy market information. Each company makes these decisions by circling one of the possible choices for each category as shown in the section "Operating and Decision Information" in Figure 1. The sum of the monies committed by these decisions must be equal to or less than the Total Funds Available shown in the box on the bottom of the form.

The completed forms for all five companies (prepared in the usual traditions of industrial security) are submitted to a control group which prepares the necessary information for submission to an IBM 650 computer. The computer has been programmed to calculate the effect of decisions made by each company upon the

STATEMENT OF ASSETS

YEAR 0 QUARTER 0

		TOTAL	NET CHANGE
CASH		$ 4,425,000	$25,000
INVENTORY	150,000 units @ $ 4.50	$ 675,000	$ 0
PLANT INVESTMENT	1,010,000 units @ $ 5.00	$ 5,050,000	$50,000
TOTAL ASSETS		$10,150,000	$75,000

ANNUAL STATEMENTS

YEAR 0

	COMPANY 1	COMPANY 2	COMPANY 3	COMPANY 4	COMPANY 5
CASH	$ 4,425,000	$ 4,425,000	$ 4,425,000	$ 4,425,000	$ 4,425,000
INVENTORY	$ 675,000	$ 675,000	$ 675,000	$ 675,000	$ 675,000
PLANT INVESTMENT	$ 5,050,000	$ 5,050,000	$ 5,050,000	$ 5,050,000	$ 5,050,000
TOTAL ASSETS	$10,150,000	$10,150,000	$10,150,000	$10,150,000	$10,150,000

INCOME STATEMENT

SALES INCOME	900,000 units @ $ 5.00		$4,500,000
COST OF GOODS SOLD & OPERATING EXPENSES			
COST OF GOODS SOLD		$4,050,000	
MARKETING & RESEARCH AND DEVELOPMENT		$ 300,000	
OTHER (MARKET RESEARCH)		$ 0	$4,350,000
			$ 150,000
OTHER INCOME (PLANT DISPOSAL)			$ 0
INCOME BEFORE TAXES			$ 150,000
TAXES			$ 75,000
NET INCOME			$ 75,000

MARKET INFORMATION

	COMPANY 1	COMPANY 2	COMPANY 3	COMPANY 4	COMPANY 5
PRICE	$ 5.00	$ 5.00	$ 5.00	$ 5.00	$ 5.00
SHARE OF MARKET	20.00%	%	%	%	%
TOTAL MARKET	4,500,000				
POTENTIAL SALES	900,000				

MARKET RESEARCH REPORT

TOTAL INDUSTRY MARKETING EXPENDITURE	$	
TOTAL INDUSTRY RESEARCH & DEVELOPMENT EXPENDITURE	$	
POTENTIAL SHARE OF MARKET – MAXIMUM MARKETING		%
POTENTIAL SHARE OF MARKET – MAXIMUM PRICE		%

OPERATING AND DECISION INFORMATION
(for next period)

	1	2	3	4	5 (DECISIONS LAST PERIOD)	6	7	8	9
UNIT COST OF PRODUCTION	$ 4.65	$ 4.61	$ 4.57	$ 4.54	$ 4.50	$ 4.49	$ 4.48	$ 4.46	$ 4.45
UNITS OF PRODUCTION	720,000	765,000	810,000	855,000	900,000	918,000	936,000	954,000	972,000
DECISION ALTERNATIVES									
COST OF PRODUCTION	$3,348,000	$3,526,700	$3,701,700	$3,881,700	$4,050,000	$4,121,800	$4,193,300	$4,254,800	$4,325,400
MARKETING		$ 170,000	$ 180,000	$ 190,000	$ 200,000	$ 210,000	$ 220,000	$ 230,000	
RESEARCH & DEVELOPMENT		$ 85,000	$ 90,000	$ 95,000	$ 100,000	$ 105,000	$ 110,000	$ 115,000	
ADDITIONAL PLANT INVESTMENT	$ 0	$ 10,000	$ 20,000	$ 30,000	$ 40,000	$ 50,000	$ 60,000	$ 70,000	$ 80,000
MARKET RESEARCH INFORMATION									
S=COMPETITORS' SHARE OF MARKET M=TOTAL INDUSTRY MARKETING EXPENDITURE R=TOTAL INDUSTRY RES. & DEVELOP. EXPENDITURES A=POTENTIAL MARKET SHARE–MAX. MARKETING P=POTENTIAL MARKET SHARE–MAX. PRICE	NONE	S $ 5,000	M $ 10,000 NONE	R $ 10,000 A $ 22,500	S&M $ 15,000 P $ 22,500	S&R $ 15,000 A&P $ 45,000	M&R $ 20,000	S,M&R $ 25,000	
PRICE	$ 4.80	$ 4.85	$ 4.90	$ 4.95	$ 5.00	$ 5.05	$ 5.10	$ 5.15	$ 5.20
PLANT DISPOSAL (in units)				NONE	5,000	10,000			

IBM 650 REPORT — TOTAL FUNDS AVAILABLE → $4,425,000 — COMPANY 1 — PERIOD 1 — GAME 1

Fig. 1. Present game report form.

share of the market it obtains and also the unit cost of production it sustained for the period being played. The formulas programmed in the computer are, of course, the heart of the game from the game designer's point of view and will be the subject of detailed discussion in the sections that follow.

The computer takes four or five minutes to make the necessary computations, perform the bookkeeping as indicated by the sections "Statement of Assets" and "Capital Income Statement" in Figure 1 and prints out a new form, with new decision levels for the next quarter or play of the game.

This process is repeated to "simulate" as many years of operation as is desired — forty periods or ten years having been the most commonly used length of game to date.

Information concerning competitor performance is available in much the same way as it is in the real world. For example, price information is given free in each period and "Annual Statements" are given free every fourth quarter. Other information such as competitor share of the market, total industry marketing expenditures, etc., must be budgeted for the purchaser as desired. Only then is this information printed out on the form of the particular company purchasing the information.

With these general remarks concerning the nature of the game with which we are concerned let us turn to a discussion of the contents of the paper. We have divided the paper into a number of parts which may to some extent be read independently.

In the first part, we consider some of the variegated reasons for constructing not only business games, but multi-person, multi-stage games, in general, beginning with the immediate

objective of solving problems which arise in the business world. We point out some of the various ways in which games can contribute in this direction.

We then turn to a discussion of the many fascinating mathematical problems forced upon us in the course of studying games. As is usual in the mathematical world, the study of significant problems in the physical world automatically yields significant mathematical problems.

Following this, we sketch in turn the ways in which these games can be useful to the economist, to the industrial engineer, to the psychologist, and in the university in the teaching of courses in mathematics, engineering, economics, and business administration.

The second part of the paper is devoted to a partial enumeration and critical analysis of some problems that confront one at the very outset. Since a number of these questions are closely related, there is a certain amount of repetition of essential points. This, however, is not a serious scientific crime.

In the third and fourth parts, we leave the realm of generalities and discuss a particular game we have recently constructed and played extensively, attempting at each step to indicate the principles guiding us in the selection and rejection of various features.

The fifth part presents the mathematical structure of one of the early games. Since then, although a number of modifications have been made, the basic structure remains unchanged.

The sixth and final part contains some results taken from typical plays of the game in various stages of evolution.

In a number of places our discussion is rather brief in order not to increase the size of the paper unduly. It will be clear to the reader that in many of these places we are encountering fundamental questions common to any mathematical treatment of the physical world. Tempting as it is to consider these in some detail, we feel that the result might well be to obscure the particular features of business games, the topic of primary interest to us here.

PART I

OVER-ALL OBJECTIVES

Introduction.

In this part of the paper, we wish to indicate some of the reasons why we are interested in the construction and play of business games. Although it is immediately clear that there is a reasonable chance of an immediate application of these devices to business planning and executive training, it may not be as clear that there are rewards equal in their own way awaiting the mathematician, the economist, the industrial engineer, and the psychologist.

Furthermore, there seem to be as yet untapped pedagogical possibilities in the use of games as classroom tools in courses in economics, business administration and operations research.

A. Applications to the Business World.

1. Simulation. The basic hypothesis governing scientific research is that we can construct mathematical models of physical phenomena which will yield results in significant agreement with

experimentally observed results.

Once we have a mathematical model in good agreement with observation in some directions, the further consequences of this model can be used to predict and to guide further experimentation.

With the advent of modern computing machines which permit the consideration of processes of magnitudes undreamt of a decade or two ago, this same hypothesis has been gradually penetrating the business world.

Despite the success of these mathematical methods in the physical world, there remain a host of problems particularly in the engineering field, which defy the present level of mathematical ability. To overcome this, we use a very simple idea. In place of constructing mathematical models of a physical process, we construct actual models and proceed to determine the behavior of systems by direct experimentation. Wind tunnels and towing tanks are two well-known examples of this simulation technique.

Although there are a variety of classical techniques such as the theory of differential equations, the calculus of variations, and many other theories, and a number of newly developed techniques such as linear programming, dynamic programming, and sequential analysis which can be applied to a cross-section of questions, many of the most important business problems appear hopelessly beyond these techniques at the moment.

It is tempting then to contemplate the application of simulation to the business world. A significant new factor, conveniently absent from most of the problems of the engineer appearing in these models is the decision process. The simulation processes we shall discuss below will involve the use of human beings and machines, rather than machines alone.

Many of the problems encountered are so involved that no simple simulation suffices. We must first construct a mathematical model, then construct a simulation process based upon this model. Many more problems arise to plague us in the construction of these business models than ever confronted an engineer. The result is that the mathematician plays an essential role in designing the games and interpreting the results.

Once we have constructed a satisfactory model, we are in the same position as an aeronautical engineer in possession of a wind tunnel. Quite simply and rapidly we can observe the effects of parameter changes, test the outcomes of various policies, submit the system to random shocks, and generally perform the type of experimentation that is usually either costly or impossible in real life.

2. Abstraction.

In reality, just as almost all significant problems in the economic sphere are too complex for complete mathematical analysis, so, as we have mentioned above, are they too complex for complete simulation. It follows that we must content ourselves with making various types of approximations, or abstractions, in either our analytic or simulation techniques. Our hope, and again it should be stressed that this is an article of faith, is to learn enough from the study of various combinations of analytic and simulation methods to be able to handle the actual problems.

Making models, mathematical or otherwise, of complex systems is an art with a small amount of science to guide one. We shall discuss some of the scientific features below; the artistic features can be appreciated only on the basis of experience and intuition. Immediate success in this field is not to be expected, and a certain amount of failure is almost predestined.

3. Training in Decision-Making.

If we are concerned with training executives to make decisions, or in determining which are superior to others in this skill, there are several methods we can employ, separately or jointly.

In the first place, we can train and test candidates in the specific areas in which they are going to operate. In the second place we can train them in general techniques of decision-making by means of processes which have no direct connection with their future field of expertry.

On the whole, there seem to be a number of advantages to this latter approach as far as turning out versatile executives who have an over-all perspective of the art of decision-making. It would seem that once general principles have been grasped, it is relatively easy to adjust to any particular environment.

Naturally, this principle can be carried to absurd lengths. There is no substitute for a certain bedrock of knowledge and experience.

B. Feedback to the Research Mathematician.

In the foregoing sections we have touched lightly upon the applications of game processes, and, simulation processes in general, to the business world, a point of such obvious character that we have not felt it necessary to belabor it. Let us now turn to a discussion of the value of this research to the mathematician.

1. Variational Problems.

The problem of determining efficient operation of an economic system leads immediately to a variety of variational problems arising from a desire to maximize profit, minimize cost, or do both. Some of these can be resolved by means of classical techniques of calculus and the calculus of variations. The majority, however, possess novel features requiring extensions of classical theory and the development of new techniques, cf. [1], [2], for further discussion. Many of these problems, particularly those of combinatorial type involved in scheduling theory, are complete beyond present-day mathematics and stand as challenges to our ingenuity, cf. [3], [5].

In any case, it is as true here as in other aspects of the physical world that significant physical problems give rise to significant mathematical problems. Conversely, research on significant mathematical problems will pay off in the solution of significant physical problems, which, in turn, will trigger further mathematical research, and so on.

2. Multi-Person Game Theory.

The study of one-person processes such as those involved in the allocation of resources or the scheduling of activities leads to classes of variational problems similar in general structure to classical variational questions arising in mathematical physics. The problems of maximizing profit or minimizing cost in many cases lead to mathematical questions of identical form with those arising from minimizing energy or maximizing volume. In more modern applications of mathematical techniques, we deal with more complicated functions, but the basic principles of these one-person processes are the same, maximize or minimize.

When, however, we begin the study of multi-person processes involving individuals engaged in competitive endeavor, the questions that arise are conceptually different from those of classical theory.

The von Neumann-Morgenstern theory of games, superseding an earlier attempt of E. Borel, represents a tremendous advance in this field in laying down the foundations of two-person games. The theory, however, is only satisfying in the treatment of two-person zero-sum games. The most important applications involve non-zero sum games, games in which the competitors have different utility functions, and in many cases, three or more competitors.

Most likely, despite a number of highly ingenious efforts, cf. von Neumann-Morgenstern, [10], Nash, [7], Shapley, [9], there will never be a unified theory, but rather a set of incompatible theories adjusted to a variety of particular situations; cf. in this regard, Mc Kinsey, [6].

It is to be hoped then that a thorough study of particular multi-person games, beginning with an analysis of their formulation and culminating in a study of actual play, will furnish clues leading to more satisfactory theories of n-person games.

3. Learning and Prediction Theory.

The study of multi-stage decision processes leads in a natural way to the study of realistic processes where we must simultaneously make decisions and explore the partially unknown structure of the underlying system.

The problem area is so huge and unexplored, that the study of particular processes serves a vital focussing role. This is, of course, a basic virtue of all mathematical models of significant processes occurring in the real world.

In regard to prediction theory, despite the enormous advances contained in the work of Wald, [11], and Wiener-Kolmogoroff, [12], quite basic problems defy successful analysis at the moment. In the field of learning theory, a theory of paramount interest to the psychologist, the statistician, and through them to the mathematician, even the simplest-appearing problems baffle us, and even seem to escape precise formulation. A discussion of some aspects of learning processes will be found in Robbins, [8]; cf. also Bush-Mosteller, [4].

Here again, particular processes will light the way to a coherent theory.

4. Computational Techniques.

Once we have constructed a mathematical model of a process, to be treated by some combination of analytic and simulation techniques, we are well on our way in our investigation of a system. However, the final objective of interpretation and understanding cannot be achieved until we have methods for obtaining output numbers from input numbers. Furthermore, in order to perform successful experimentation, we must be able to perform calculations quickly and accurately.

Problems of this type are highlighted in the study of multi-stage processes, and, particularly, in the study of multi-stage, multi-person processes with their combinatorial overtones. The experience gained in the successful treatment of one process can usually be carried over to the treatment of others. Again, particular processes serve a useful triggering role.

In most cases, the use of a digital computer is assumed.

5. Policies.

One way to avoid the multi-dimensional wilderness of multi-stage processes is to focus upon the concept of a "policy" or "strategy". Games and other types of simulation processes serve as excellent proving-grounds for the study of the effects of various policies. Reciprocally, the study of these games yields information concerning efficient policies.

One of the main purposes of the game we shall describe below is to force the players to think in terms of policies and long range effects.

6. Interaction Theory.

Finally let us repeat what we have said in the foregoing sections in various fashions. There is a science to model-building, which is to say to the construction of mathematical idealizations of physical processes. It follows that the ability gained in one domain in translating such elusive concepts as information, competition, efficiency and learning into clean-cut mathematical formulas permitting quantitative evaluation can readily be carried over into other domains. We thus gain a foothold on the terrain of general interaction theory.

The study of multi-stage, multi-person decision processes draws heavily upon these skills, and the outside world is capable of furnishing an unlimited quantity of interesting and important processes, each with its special features to intrigue the mathematician.

Our purpose in writing this paper is to furnish some sign-posts along the road to guide the newly arrived. However, the road is not a royal one, and it cannot be too emphatically stated that there is no substitute for experience and a "do-it-yourself" kit.

C. Feedback to the Economist.

It is apparent that the study of business games can be of the utmost importance to the economist. Consequently, we shall content ourselves with some brief remarks.

1. Interaction Study.

The construction of a mathematical model in any field serves the essential role of crystallizing thinking. Vague, general statements must be reduced to precise quantitative statements whose consequences can be tested and evaluated. In particular, we are led to studies of cause and effect, and of payoff and motivation.

2. Data Collection.

The construction of a mathematical model tells us what information is required for further study, and in what form data should be collected. This is one of the major purposes of a study of this type, and sometimes its sole purpose.

It may also help us dispose of unnecessary accumulations of data, a problem which is becoming of greater and greater significance.

3. Key Variables.

In the course of experimentation with these games, we can hope to observe the relative importance of various factors. If we note that some variables play an unimportant role, we can eliminate them from the model and thus considerably simplify the mathematical and economic analysis. On the other hand, unsatisfactory behavior may force us to the conclusion that we may have uncritically lumped some variables or neglected others.

Questions of this type are very difficult to resolve analytically.

D. Feedback to the Psychologist.

Let us now discuss briefly the point that these games can make a valuable contribution to psychological research.

1. Decision-Making.

A problem of prime concern to the psychologist is the study of the whys and wherefores of various types of decision-making. Although the standard way to learn about these processes is to carry out experiments, a common difficulty is that the motivation in these experiments is never powerful enough to ensure that the subjects act in a manner reflecting their behavior under actual situations.

In the play of the game we shall discuss below, we have been in a unique position insofar as the selection of players is concerned. Choosing top management, strongly actuated by prestige considerations and intellectual curiosity, we have an excellent opportunity to perform significant psychological observations of the decision-making processes of a very important stratum of American life.

By changing the information pattern, the duration of play, and so forth, we can carry out psychological experimentation, as well as gain in mathematical and economic interpretation.

2. Group Interaction.

In a multi-person game of the type we shall describe below, the information pattern is such that the kind of game that is actually played depends to a considerable extent upon the particular players selected.

We thus have a means of observing group interaction, controllable to some degree by varying the information given each player of the other player's positions.

3. Learning Processes.

Another vital problem to the psychologist is that of learning, as we have mentioned above. In the study of the behavior of the players as they attempt to understand the structure of the game, and to predict the moves of their competitors, we have a ready-made psychological laboratory.

A point to stress is that we are dealing with people of stature and responsibility engaged in the solution of problems similar to those that confront them in their actual lives rather than volunteers engaged in make-believe situations playing for pennies.

It need not be emphasized that there will be the usual difficulties of reproducibility and statistical analysis attendant upon all psychological studies. It is to be hoped that the records of thousands of games of this type will disclose certain structural properties. This would seem to offer the blueprint of an ambitious program of psychological research.

E. Feedback to the Industrial Engineer.

The industrial engineer must embody within himself something of the mathematician, something of the economist and something of the psychologist. Among his many other duties, it will be the industrial engineer's job to criticize, implement, or install, the new procedures, controls, and so forth, that may be the operational changes suggested by the results of such research. It then follows that the study of business games furnishes an excellent means of tying these threads together and of furnishing a certain amount of synthetic experience.

F. Feedback to the Classroom.

Without entering into any discussion of actual classroom procedures, let us merely point out that games of the type we shall discuss below furnish excellent motivation and material for courses in mathematics, operations analysis, economics and business administration.

PART II

SNAGS IN THE YARN

In this part we wish to discuss a number of unpleasant problems that arise as soon as we descend from the qualitative to the quantitative, and approach the problem of constructing actual business games.

A. What Constitutes Optimal Play?

Two immediate difficulties confront us in attempting to define optimal play. The first is common to all types of decision processes and arises in connection with the construction of any mathematical model, while the second is characteristic of two-person, and even more so, N-person decision processes.

At first glance, it would appear to be an easy task to assess the degree of success or failure of a business enterprise, relying upon such well-known indicators as total assets, profit, share of market, and so forth. A little thought, however, will reveal the fact that these are static data, and therefore not entirely trustworthy in evaluating a dynamic process. In addition to the above pieces of information which describe the state of the process, we would like to know something about the rate of change of total assets, the rate of change of profit, and so on. In other words, the history of the process may be important, and very often is. Granted that we possess all the desired information, there is still the problem of evaluation.

Many of the factors mentioned above are incommensurable, or quasi-incommensurable. By this we mean that there is no convenient yardstick for converting a statistic for one into a statistic for the other. This, combined with a distinct lack of unanimity among mathematicians, economists, industrial engineers and such as to how to weight these factors, renders the construction of a criterion function which serves the purpose of evaluating policies a matter of great difficulty.

As a matter of fact, one of the functions of a business game of this type is to furnish these criteria.

A further difficulty that arises is due to what may be called an "end effect". Starting an economic process at this time, it may seem reasonable to consider maximization of total profit over the next ten years as a sensible goal. However, after nine years have passed, we certainly do not wish to continue as if the only goal were to maximize profit over the coming year.

There are several commonly used ways of circumventing this "end effect", but they smack more of mathematical convenience than any intuitive operational concept. The basic idea is to discount the future in some systematic fashion. This is closely connected with prediction theory and shares the usual difficulties.

Let us now consider a characteristic difficulty of multi-person games.

In the one-person process, arising from a simulation or programming problem, once a criterion function has been decided on, we possess a simple means of determining optimal play—it is play which maximizes the criterion function.

In the multi-person case, in general no such simple-minded optimization is possible. To illustrate this, let us consider the two-person process first. Further, let us take the simplest case where the players are in direct competition, so that one's loss is the other's gain. Assume, as is natural, that there is an interaction between the players, which means that the return to each is dependent upon the actions of both. It follows that neither side can maximize without paying attention to the decisions of the competitor.

In this case, the theory of games of von Neumann-Morgenstern shows how to resolve this apparent circularity.

It is shown that a certain value, a number, can be attached to the game, having the property that one player can guarantee at least that return, if he plays properly and the other player can guarantee not losing more than that quantity.

The concept of "proper play" is not a simple one, involving the ideas of randomization and average return. The classic work on the subject is von Neumann-Morgenstern, [10], while the most entertaining and readable account of the fundamental ideas is contained in Williams, [13].

Let us now add two realistic features:

1. The players are not in direct competition.
2. The players possess different utility scales, i.e., they have different estimates of what constitutes optimal play.

There exist several proposed theories, to treat these more general processes, none, however, of any universal acceptance. It follows that we have no unique way of determining optimal play for these realistic processes, even if we can decide on an appropriate criterion function.

The situation for N-person games of any type, $N \geq 3$, is chaotic. Again there are several proposed theories, due to von Neumann-Morgenstern [10], Nash [7], and Shapley [9], but none are uniformly satisfactory.

It is essential to emphasize the significance of what has just been said, since it is not at all intuitive, and contradicts a number of beliefs many of us hold dear.

In a one-person process, with a definite criterion for measuring the effect of a sequence of decisions, there is a <u>unique</u> set of policies which we can call optimal. These are the policies which maximize the criterion function.

In various types of two-person games, as we have mentioned above, each player possesses a certain set of strategies which guarantee a certain average return, regardless of what the other player does. Furthermore, deviation from these strategies may be expensive.

In the general two-person game, and multi-person games in general, sets of strategies of the above type <u>need not</u>, and in the majority of cases, <u>do not</u>, exist. The consequence of this is that a set of policies which work well in a game involving one set of players may be disastrous in a game played under the same rules, but involving a different set of players with different philosophies as to optimal play.

This is a well-known fact as far as card games are concerned. Optimal play must be a combination of certain basic principles and information concerning the psychology of the opponents. Interestingly enough, this is even true of a game like chess.

The fact that we cannot determine optimal play does not destroy the usefulness of these games. It actually makes them more valuable. They can be used to explore the effects of various classes of policies. Generally speaking, this is far more important information than a knowledge of optimal play, which in many cases may be far too complicated in structure ever to use.

This last is an important point which must continually be kept in mind. Although it is not sensible for an optimal policy to possess a higher degree of complication than the structure of the model, it is not easy to make this concept precise.

B. What are the Effects of Decisions?

Let us now consider the next difficulty.

As we shall discuss in the section devoted to a general description of the game, in the course of play each player is required to make decisions concerning the allocation of money for advertising purposes, for research and development, for production, determining both output and increase or decrease in plant capacity, and finally to determine the price of the item.

The choice of advertising budget, research and development budget, and price, to some extent determine, as we know, the share of the market. But how? Certain qualitative features can be seen easily such as the fact that increasing the first two allocations and decreasing the price increase the share of the market, and conversely. Quantitative knowledge in this area simply does not exist. There are no experts to refer to, no reports to read.

At first sight, this depressing state of affairs would seem to militate against the whole idea of a business game. But, there is no reason to be too discouraged. After all, decisions are continually being made in actual business operation without precise knowledge of these effects. As we shall see below, we have actually turned this very real difficulty into an advantage.

C. How Detailed and How Realistic Should the Model Be?

We start out with the knowledge that the only accurate model of a process is the process itself, and hence that any mathematical or simulation model is an approximation of a process which is itself an approximation. We see then that we are always faced with the difficult question of determining the degree of realism, and thus complication, that we wish our model to possess.

The type of model to be constructed should obviously depend upon the depth of the answers we wish to obtain. Frequently, however, a certain amount of trial and error is required in order to determine the proper level of complexity. Here it is necessary to tread a very narrow path between the danger of over-simplification and the morass of overcomplication. If the model is over-simplified, it lacks sensitivity and we cannot distinguish between large classes of policies; if the model is over-complicated, we will not be able to isolate cause and effect.

Let us further note that complication in the model usually increases computational labor in a nonlinear way. In other words, what appears to be one additional factor may increase computing time by an appreciable factor such as a fourth or a third.

Finally it should be emphasized that increase in complexity does not necessarily entail increase in accuracy, and that actually the reverse may occur. If, for example, we set up a model involving ten different activities requiring, say, the solution of systems of ten linear equations in ten unknowns, we obtain certain estimates of parameters which determine optimal policies. Carried away by the success of this operation, we may attempt to set up a model involving a hundred different activities involving the solution of a system of one hundred linear equations in one hundred unknowns. The estimates for the parameters obtained from this more sophisticated model may be completely nonsensical due to the fact that the large number of numerical operations involved in the solution of large systems, each involving round-off error, may completely overwhelm the input data, accurate to only a few significant figures. Furthermore, the model can very easily become <u>less stable</u> as its size is increased rather than <u>more stable</u>.

All these factors must be carefully considered before large scale models are constructed and the computing machines set grinding away.

A particular case of the problem of isolating cause and effect is the question of deciding between <u>deterministic</u> versus <u>stochastic</u> models. By a deterministic model, we mean one in which the outcome of every decision is uniquely determined by the decision, although the mechanism may perhaps not be known to the player. By a stochastic model, we mean one in which the outcome of a decision is a random variable, with a distribution function which may be known or unknown to the player. The temptation in many cases is to construct stochastic model processes in the hope that they reflect the actual business operation more accurately.

The difficulty, however, arises in evaluating the outcomes. Is a good result the product of a superior policy, or due merely to a fortuitous chain of events; is a poor result the product of an inferior policy or due merely to a run of hard luck? Unless a stochastic process is run a sufficient number of times, these questions may not be able to be readily answered. Consequently, if we are interested in testing policies, it may be more efficient to use deterministic processes initially.

Let us note in passing, that if the underlying process is genuinely stochastic, there is the usual difficulty of choosing a criterion function for a process that may only be carried out once, or at best a few times. This is one of the fundamental difficulties of the Theory of Games. The resolution of this problem depends upon one's personal philosophy.

As a general rule of thumb, it is far better to start with apparently simple models, and gradually to increase the complexity of the model on the basis of experience in actual play than to mire oneself in an unwieldy complicated model at the outset. It is quite amazing to see the depth of optimal policies possessed by apparently simple processes.*

D. <u>How Difficult Should It be to Make Decisions?</u>

In constructing a game of this type, which is intended for

* The Japanese Game of Go is a good example of this.

a fairly large audience, a great deal of thought must be devoted to the problem of learning and playing the game. Let us consider three aspects of this:

1. Learning the Game.
2. Information Pattern.
3. Decision-Making.

These three are, of course, closely interrelated, but we find it convenient to categorize our discussion in this way.

1. Learning the Game.

There are two parts to learning the game. The first consists of understanding the basic structure of the game and its objectives, while the second consists of the knowledge of the individual moves. It should be repeatedly stressed that simplicity must be the principal theme. If not, it is very difficult, if not impossible, to pursue policies and observe their outcomes. Since simple decisions at each stage of a multi-stage process can combine to yield exceedingly complicated policies, there is no need for games of intricate local structure.

2. Information Pattern.

The decisions of the players will depend, once the rules of the game have been detailed, upon the information available to the players concerning the state of the process. Thus, the type of game that is played depends to a large extent upon this information pattern. This is a valuable fact to keep in mind since it furnishes the game-maker a fairly simple way of modifying and altering the game. There are three aspects to the information pattern:

a. The player's own position.

b. The position of the other players.

c. The structure of the game, i.e. what makes the wheels turn.

It is quite realistic to have concealed information concerning any or all of these factors, and as we shall point again below, quite desirable.

3. Decision-Making.

Once we have covered the first two points above, learning the game and the nature of the information pattern, we are faced with the problem of deciding upon the time to be allocated to the player in making decisions, and the tools to be employed for this purpose. Since we wish to train the player to think in terms of fundamentals and to take long-term and over-all views, we must prevent him from being bogged down in arithmetic computation, prevent him from being distracted by detail, and force him to think in terms of essentials.

This is not an easy objective to attain, since it is not at all simple to distinguish between inessential detail and significant information, particularly at the beginning of the game. We shall discuss some techniques we have employed to this end below.

In closing this section, let us note that in designing a game of this type we wish to be careful to avoid putting a premium upon mathematical training or upon ability in rapid arithmetic. Generally, the emphasis is to be upon correctness of principles rather than rapidity of action. There is no reason, of course, why particular games should not be designed to test these rapid-fire abilities, but it should be understood that in the majority of decision processes, there is no shortage of time for decision-making.

E. Stability, Elasticity and Gimmicks.

Let us now discuss three major points which are crucial in deciding the success of a game.

1. Stability.

In the course of a football season, a superior team can be upset by an inferior team, or lose a close game on a fluke play, a blocked kick, a fumble, a desperation pass, and so on. However, examining the records of games over a season, it is readily seen that the teams with better trained players and superior teamwork have a consistently better record.

The same principle holds in the construction of these games. The purpose is to emphasize the worth of sound principles and long-term planning. Consequently, we want to be sure that the state of the game cannot vary wildly from stage to stage as a consequence of fluke moves on the parts of the players. This is what we mean by stability.

This is not to say that the real world does not contain examples of coups and brilliancies which have rescued seemingly hopeless situations. We do maintain, nonetheless, that this is the exception rather than the rule. Before one looks for brilliances, one must know sound moves. A foundation of training in basic principles is the proper springboard for innovations.

2. <u>Elasticity</u>.

If we circumscribe decisions in such a way as to ensure stability, we may run into the difficulty that the game has become insensitive to even gross changes of policy. The Scylla of extreme sensitivity or unstability is balanced by the Charybdis of extreme insensitivity or <u>inelasticity</u>. We must pursue a careful path between these. Although this is not an easy task, a great deal can be done upon observing the play of the game.

We shall discuss some techniques we have employed to follow a straight and narrow path below. Trial and error, and patient, if profane, calculation play essential roles.

3. <u>Gimmicks</u>.

Since the purpose of the game is training, or the solution of actual problems, we must make sure that the players act as if they were in the actual business situation, rather than seize upon special features of the game. In particular, there must not be extreme policies, obviously unrealistic in the actual process, which are successful. In other words, there must be no <u>gimmicks</u>.

Unfortunately, it is not as easy to guard against gimmick policies as one might think. Checks and balances against extreme policies, which exist in the real world, may very often be omitted in the necessarily small scale model we build. Quite often it is easier to build in an artificial guard against extreme policies than to include the actual mechanism which performs this vital role in the realistic process.

F. Caveat Ludeator.

Finally it must be emphasized that the game must not be taken too seriously. No matter how accurately it reflects the above principles, and no matter how realistic its optimal policies may seem, we must constantly remember that it is after all only an approximation to reality.

Let us point out also that there are many areas in which top management must make crucial decisions which this game does not touch. Some of these are:

1. Innovation and technological advances.
2. Environmental and, in particular, governmental influences.
3. Catastrophes.
4. Substitution of other products by the customer.
5. Mergers and coalitions.
6. Labor-management problems.

PART III
GENERAL DESCRIPTION OF A MULTI-PERSON, MULTI-STAGE GAME

Let us now describe in general terms the particular multi-person, multi-stage game we shall discuss in greater detail below.

The basic situation is taken to be that of a number of firms producing a single item competing for a known consumer market. Each firm, which may be represented by a team of one or more players, possesses the following information concerning its own position at each stage of the process:

(1) a. Total sales in units and dollars over the preceding time period.

b. Price of the item over the preceding time period.

c. Inventory.

d. Maximum productive rate over the next time period, and actual productive rate over the preceding time period.

e. Unit cost of production.

f. Share of market over preceding time period.

g. Allocation of the total budget to marketing, research and development and increase in productive capacity over the last period.

h. Total funds available for allocation over the next period.

In addition, it possesses a certain amount of information concerning its competitors, such as their prices and perhaps their marketing budgets.

On the basis of the above information, and past history which the players are allowed and encouraged to keep, a number of decisions must be made governing the play over the next period. These involve the determination of

(2) a. Price

b. Marketing Budget

c. Research and Development Budget

d. Rate of Production

e. Increase or decrease in Productive Capacity.

A basic restriction on allocations is that no borrowing is allowed, which means that all budgetary allocations must be covered by capital on hand. We shall discuss a quite important subsidiary restriction below.

Concerning the effects of those decisions, the players are given only the following obvious qualitative information:

(3) a. Increase in productive capacity incrrases maximum production rate.

b. Increase or decrease in actual productive rate and productive capacity involves a change in unit cost.

c. Increase in research and development decreases unit cost and increases the attractiveness of the product, and vice versa.

d. Increase in marketing expenditure increases the attractiveness of the product.

e. Increase in price decreases the attractiveness of the product.

f. Attractiveness, which depends upon price, marketing and research and development, determines the share of the market.

The market consists of a known total demand for the item which increases at a certain known rate per stage.

This process continues for a fixed number of stages, with each team making its decisions so as to optimize. What we mean by this, and other features of the game which we have deliberately left obscure, will be discussed in the next part.

PART IV

DISCUSSION OF SPECIAL FEATURES OF THE GAME

In this part of the paper, we wish to discuss a number of special features which we have built into our game, indicating how they circumvent or resolve, partially or wholly, some of the traditional difficulties pointed out in the foregoing sections.

These special features, which we feel represent our contribution to the art, are

I. Absence of an explicit criterion function.

II. Principle of marginal change.

III. Hidden formulas.

IV. Minimal computation.

A. <u>Absence of an explicit criterion function</u>.

As we pointed out in A of Part II, there is a certain amount of confusion and contradiction present in the statement of any explicit criterion function, such as, for example, maximum profit over a fixed number of stages. Consequently, it was decided to cut the Gordian knot, and eliminate not only any analytic definition of the criterion function, but even any explicit mention of a particular goal.

Instead, the players are told to play the game as if it were an actual business operation. The burden is then doubly shifted: The game must be realistic enough to motivate this behavior, and the players must be mature enough, and sufficiently interested to pursue this course.

We might say that in our experience with teams drawn from top management and academic ranks, we have had no difficulty in this direction.

It must also be emphasized that this imprecisely defined situation is, after all, realistic. In any actual competitive situation the payoff is not well-defined and there exists no clean-cut evaluation of a policy.

Observe that this type of criterion, implicit rather than explicit, avoids the "end effect" described in A of Part II, and to some extent diminishes the possibility of optimal policies of gimmick variety.

B. Principle of Marginal Change.

In realistic processes occurring in the economic sphere there is very often a time interval between a decision, and the effect of a decision. Thus, construction of new capacity may take a year, planning an advertising campaign may take six months, and the results of money devoted to research and development may take years to show up. These time-lags necessitate long-term planning.

If we attempt to take this retardation into account, we encounter a formidable difficulty as far as the information pattern is concerned, and a further difficulty in connection with the stability of the process. Let us discuss the effect upon the information pattern first. At the present time, we have as state variables a certain set of quantities. If we allow a time-lag, the information pattern must contain not only the present state of the system, but also a good deal of the past history, the amount being dependent upon the length of the lag. This greater amount of detail is not only more difficult for the player to store and assimilate, but it also greatly increases the computational effect required to determine the effects of decisions at each stage and hence the time required per stage. This difficulty is by no means insurmountable, particularly with modern digital computers. However, careful thought should be given as to whether the degree of complication introduced by time-lags is worth the gain in realism, considering the many costs involved to both the players and the directors of the game.

In addition to the disagreeable features described above, there is the discontinuity in the play of the game introduced by a sudden increase in productive capacity, a sudden spurt of advertising activity, and similar behavior.

To counter both these difficulties, we have introduced the constraint of marginal change. By this we mean that none of the state variables can be affected by any decision at any stage by more than a certain percentage of its value. This percentage depends upon the individual state variable, and upon the stage of the game.

This constraint possesses a number of desirable features:

1. It automatically ensures a certain degree of stability.
2. It forces long-term planning, since major changes require a large number of stages.
3. It simplifies decision-making on the part of the player.

4. Combined with the idea of discrete change which we shall discuss below, it greatly simplifies decision-making, computation, and tabulation of results.

Since realistic business processes do contain discontinuous features of the type described above, it is important to know whether the absence of time-lag is a major defect. A mathematical analysis which we shall not present here shows that in a process of sufficient length the effect of time-lag washes out.

C. <u>Hidden Formulas</u>.

As we have mentioned above, in business operations there are no precise relationships connecting allocations and monetary return. This may be taken to be due either to ignorance or to the stochastic mechanism of the actual processes involved. Consequently, we decided not to disclose the formulas used by the computer to determine the outcomes of decisions, and to give the teams only the structural information concerning the process outlined in Part III.

Apart from its realistic aspects, this decision has the following desirable characteristics:

1. It forces the players to think in terms of general concepts and policies.
2. It equalizes the mathematical level of the teams.
3. It simplifies decision-making and reduces the time required per stage.
4. It prevents gimmick policies to some extent.

D. Minimal Computation.

As mentioned above, we feel that if a great deal of value is to be derived from the play of the game, it is essential that the maximum effort of the team be devoted to thinking in terms of basic policies, and a very minimum of effort be devoted to inessential calculation.

In particular, the amount of arithmetic work should be kept at a very minimum.

Particularly helpful in this context is the concept of marginal change, which we have discussed above, which automatically forces each team to consider at each stage only a small set of decisions associated with its current position.

Thus, for example, if the price of the item chosen by a particular team is $5.00, and we allow a maximum change of at most 3% in the price over any period, there is room for a choice of a price between $4.85 and $5.15. The choice of 31 different prices, which we obtain upon permitting changes of a penny, is still too free, particularly in view of the fact that the team possesses slight rational basis for a choice between prices of $5.11 and $5.12, say, as a consequence of the hidden formulas discussed above.

Hence, we introduce as an additional constraint the condition that within the 3% range, the price can only vary by multiples of 5¢. The team thus has a choice of seven new prices:

(1) 4.85, 4.90, 4.95, 5.00, 5.05, 5.10, 5.15.

Furthermore, this range of prices is tabulated for the team at each stage by the digital computer, together with a table of other admissible allocations and decisions.

This further principle of discrete steps combined with a tabulation of possibilities for each team at each stage by the computer considerably reduces the arithmetic labor of each team and thus almost completely eliminates the possibility of trivial computational errors on the parts of the players.

E. Deterministic, yet Quasi-Stochastic.

In order to spotlight the effects of policies, and prevent to some extent the onus of failure from being shifted from the player to the fickle goddess Fortuna, it was decided to use a deterministic model.

Despite this, the game possesses stochastic features in the following fashion. As we have already pointed out in our discussion of the difficulties associated with multi-stage games, since there are no guiding rules for optimal play, the type of game that develops depends to an overwhelming extent upon the composition of the teams, which is to say, the philosophies and psychologies of the individual players. It follows that the chance element is actually present in the form of the random choice of players.

The same player pursuing the same policy can do very well in one run of the game, and very poorly in the next. The game thus possesses the desirable quality of forcing the player to be flexible, and adjust his policies to meet actual competitive conditions. There would be little difficulty in converting the game from a deterministic to a stochastic model.

PART V

MATHEMATICAL STRUCTURE OF GAME

Although the players do not need the analytic form of the equations describing the interaction of the decisions, the digital computer does.

The two basic quantities are the <u>attractiveness</u>, which determines the share of the market, and the <u>unit cost of production</u>.

A. <u>Attractiveness</u>.

As we have described above, we are very much concerned with the stability of the process. We begin then by converting two allocations into ratios in order to decrease sensitivity.

Let

(1) a_i = advertising budget of the $i^{\underline{th}}$ team at a particular stage.

We then compute the new quantities

(2) $$a_i' = a_i \,/\, \sum_i a_i .$$

Let

(3) r_i = total allocation of the $i^{\underline{th}}$ team for research and development over the last four stages.

Then compute

(4) $$r_i' = r_i \,/\, \sum_i r_i .$$

We now define the attractiveness, denoted by A_i, of the item produced by the $i^{\underline{th}}$ team as a function of the three quantities a_i', r_i' and p_i', the price of the item set by the $i^{\underline{th}}$ team,

(5) $A_i = f(a_i', r_i', p_i)$.

Once A_i is determined, the fraction of the market purchasing the item of the i^{th} team is given by

(6) $f_i = A_i / \sum_i A_i$.

Hence if N is the size of the market, the return from sales to the i^{th} player is given by

(7) $R_i = p_i f_i N$,

provided that $f_i N \leq q_i$, the quantity of items produced over the last period plus those in inventory.

It remains to determine the function $f(a_i', r_i', p_i)$. As pointed out above, we have little basis for any analytic description of the function. Consequently we decided to choose the simplest function possessing the correct qualitative features.

To begin with, we want $A_i > 0$ for all values of a_i', r_i', p_i. Consequently, f takes the form

(8) $f(a_i', r_i', p_i) = e^{g(a_i', r_i', p_i)}$.

The simplest form for g is a linear function

(9) $g(a_i', r_i', p_i) = c_1 a_i' + c_2 r_i' - c_3 p_i$,

where $c_1, c_2, c_3 > 0$. The function f now has the correct qualitative behavior.

We found, however, that this function was still too sensitive to changes. Consequently, we added a buffer constant c_4 and let f have the form

C. Discussion.

We have presented the simplest versions of these functions here to indicate the ideas guiding our choices. Actually in more recent plays of the game we have modified these functions in a number of ways both to preserve stability in the beginning of the process and to preserve elasticity in the middle and end of the process.

We feel that there is little point going into the fine detail here.

Once having seen the construction of this game, it is relatively easy to construct more complex games involving the production, sale and distribution of a number of products. We shall discuss some of these games elsewhere.

PART VI

SOME TYPICAL PLAYS

In all, the game has been played some twenty-five times; however, four "bench-mark" plays will suffice to illustrate the way in which the presentation of information has evolved and give some idea of the results. These four plays will be referred to by the following code:

Play	Location	Date	Participation	Persons per Side
I	Endicott	Oct. 16-17, 1956	Middle Management	1
IIA,B	Los Angeles	Dec. 3-4, 1956	Company Vice-presidents	2 and 3
IIIA,B	New York	March 6-7, 1957	Company Vice-presidents	3
IVA,B	New York	May 2, 1957	Company Presidents	4 and 5

Figures 2, 3, and 4 (applicable to Plays I, III, and IV, respectively) indicate the advances made in form design. While the basic structure of the game remained essentially the same, the inclusion of income taxes in Play III reduced the growth in assets. It is also to be noted that the nature of information, and the manner in which it was obtained, changed in the direction of giving added realism; e.g., a change in share-of-the-market information was incorporated. Also, the information was presented more nearly in the form to which top management is accustomed.

Control Room

The results of each company's actions were charted in the control room. These afforded information to the observers and provided the basis for a critique at the end of the game. Figure 5 is a typical chart taken from Play IIB, company 1.

Total assets (cash and inventory and plant investment), also plotted in the control room, gradually seemed to become the best over-all measure of performance. Figures 6-11, inclusive, represent the plot of total assets for various games as indicated.

Critique

At the end of each game, each company was handed the control charts, similar to those presented in Figs. 12-17; and a team member was asked to give an explanation of the strategy used during the game and to state what difficulties the group had experienced in its decision making.

At this point in the research, it is too early to draw any conclusions from the observed performance. However, those who have played the game feel that it has great possibilities for improving judgment and reasoning capacity and that it vividly demonstrates the complexity of running a modern business. As one participant put it, "The game's great merit lies in reminding the players of the complex interlocking nature of the factors that affect most decisions--management by rule of thumb is no longer possible."

Some participants remarked that the experience suggested ideas to consider in their own businesses; others felt that it would provide a most efficient way of training second-level management in the problems of obtaining a balanced program. A comment often heard indicated that the game gave an insight into just what information—that is, what data and reports—one should have to facilitate his own decision-making. Other typical comments are: "This game really brings out the importance of having facts in decision making. It also forceably demonstrates the need for keeping a company's operations in balance." ... "The game is not very different from real life. It may not be like any particular consumer goods or industrial goods market, but there is a great similarity between the basic things you do in the game and the results that you get. They may not happen at the time you think they will or they may happen a lot quicker than you think they should, but they do seem to happen."

As has been seen, the game provides the researcher with a laboratory tool to use in observing the decision-making process. For example, it is interesting to note the use which players in Plays IIIA and IIIB made of the opportunity to purchase market information. Table I indicates the percentage of times that a given company purchased "Market Research Information" of various types—with the various companies arrayed in order of decreasing total assets at the end of the game.

Many other types of statistical analysis can be carried out through the use of gaming techniques under controlled conditions involving many plays. It is hoped that the abbreviated results here presented will be suggestive of the potential research uses of simulation techniques.

TABLE I

ANALYSIS INFORMATION USAGE GAME III

Total Assets at End of Game	o/o of Plays in which Information Purchased	
	Combination of S M R *	Combination of A P
Game III A		
14,931,000	33 %	0
14,898,000	46 %	2 %
14,497,000	37 %	9 %
13,475,000	46 %	7 %
12,955,000	19 %	7 %
Game III B		
14,137,000	33 %	2 1/2 %
13,675,000	60 %	12 1/2 %
13,012,000	13 %	7 1/2 %
11,384,000	50 %	10 %
10,506,000	55 %	12 1/2 %

* See Figure 1 for greater detail

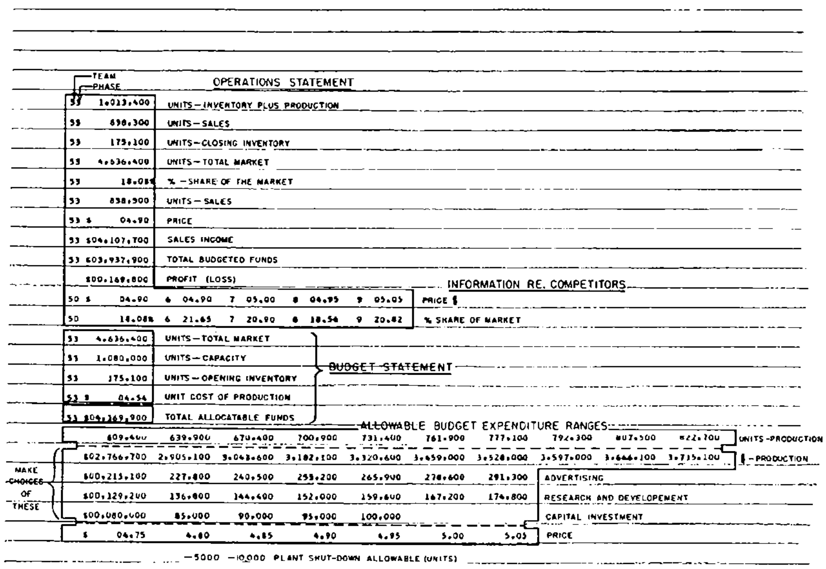
S — Competition Share of the Market

M — Total Industry Marketing Expenditure

R — Total Industry Research and Development Expenditure

A — Potential Market Share — Max Marketing

P — Potential Market Share — Max Price



TEAM / PHASE

OPERATIONS STATEMENT

53	1,013,400	UNITS—INVENTORY PLUS PRODUCTION
53	838,300	UNITS—SALES
53	175,100	UNITS—CLOSING INVENTORY
53	4,636,400	UNITS—TOTAL MARKET
53	18.08%	% —SHARE OF THE MARKET
53	838,900	UNITS—SALES
53 $	04.90	PRICE
53	$04,107,700	SALES INCOME
53	$03,937,900	TOTAL BUDGETED FUNDS
	$00,169,800	PROFIT (LOSS)

INFORMATION RE. COMPETITORS

50 $	04.90	6 04.90	7 05.00	8 04.95	9 05.05	PRICE $
50	18.08%	6 21.65	7 20.90	8 18.54	9 20.82	% SHARE OF MARKET

BUDGET STATEMENT

53	4,636,400	UNITS—TOTAL MARKET
53	1,080,000	UNITS—CAPACITY
53	175,100	UNITS—OPENING INVENTORY
53 $	04.54	UNIT COST OF PRODUCTION
53	$04,369,900	TOTAL ALLOCATABLE FUNDS

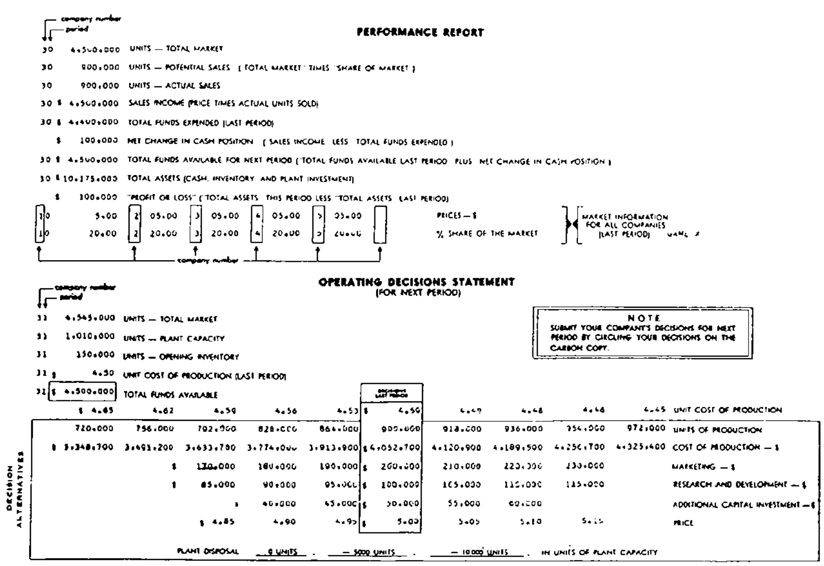
ALLOWABLE BUDGET EXPENDITURE RANGES

MAKE CHOICES OF THESE

609,400	639,900	670,400	700,900	731,400	761,900	777,100	792,300	807,500	822,700	UNITS-PRODUCTION
$02,766,700	2,905,100	3,043,600	3,182,100	3,320,600	3,459,000	3,528,000	3,597,000	3,666,100	3,735,100	$-PRODUCTION
$00,213,100	227,800	240,500	253,200	265,900	278,600	291,300	ADVERTISING			
$00,129,200	136,800	144,400	152,000	159,600	167,200	174,800	RESEARCH AND DEVELOPEMENT			
$00,080,000	85,000	90,000	95,000	100,000			CAPITAL INVESTMENT			
$ 04.75	4.80	4.85	4.90	4.95	5.00	5.05	PRICE			

—5000 —10,000 PLANT SHUT-DOWN ALLOWABLE (UNITS)

Fig. 2. Game report form, Endicott, Oct. 16–17, 1956.



company number / period

PERFORMANCE REPORT

30		4,500,000	UNITS — TOTAL MARKET
30		900,000	UNITS — POTENTIAL SALES (TOTAL MARKET TIMES SHARE OF MARKET)
30		900,000	UNITS — ACTUAL SALES
30	$	4,500,000	SALES INCOME (PRICE TIMES ACTUAL UNITS SOLD)
30	$	4,400,000	TOTAL FUNDS EXPENDED (LAST PERIOD)
	$	100,000	NET CHANGE IN CASH POSITION (SALES INCOME LESS TOTAL FUNDS EXPENDED)
30	$	4,500,000	TOTAL FUNDS AVAILABLE FOR NEXT PERIOD (TOTAL FUNDS AVAILABLE LAST PERIOD PLUS NET CHANGE IN CASH POSITION)
30	$	10,175,000	TOTAL ASSETS (CASH, INVENTORY AND PLANT INVESTMENT)
	$	100,000	"PROFIT OR LOSS" ("TOTAL ASSETS" THIS PERIOD LESS "TOTAL ASSETS" LAST PERIOD)

1	0	5.00	2	05.00	3	05.00	4	05.00	5	05.00	PRICES — $
1	0	20.00	2	20.00	3	20.00	4	20.00	5	20.00	% SHARE OF THE MARKET

company number

MARKET INFORMATION FOR ALL COMPANIES (LAST PERIOD) GAME #

OPERATING DECISIONS STATEMENT
(FOR NEXT PERIOD)

company number / period

31		4,545,000	UNITS — TOTAL MARKET
31		1,010,000	UNITS — PLANT CAPACITY
31		150,000	UNITS — OPENING INVENTORY
31	$	4.50	UNIT COST OF PRODUCTION (LAST PERIOD)
31	$	4,500,000	TOTAL FUNDS AVAILABLE

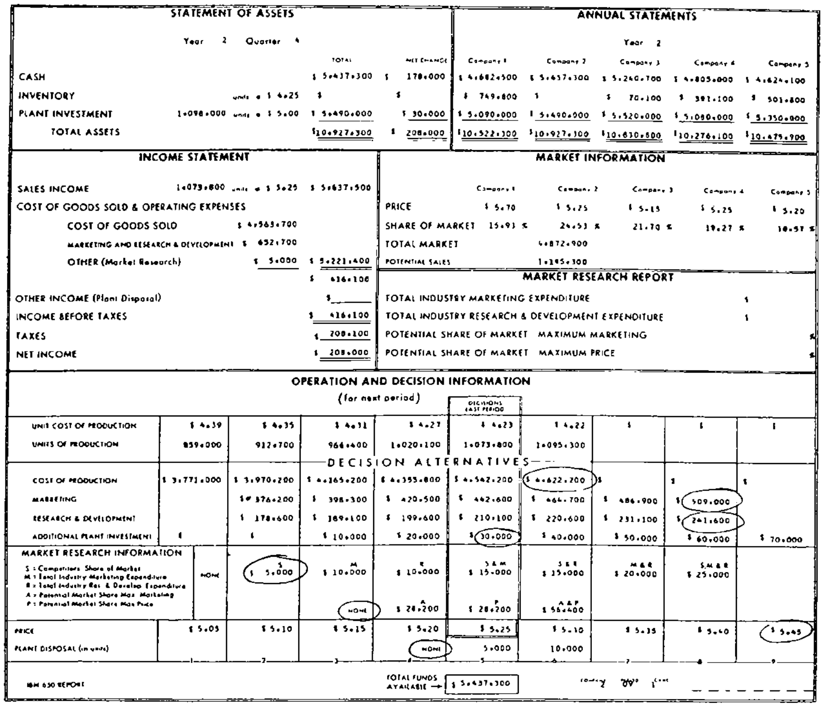
NOTE
SUBMIT YOUR COMPANY'S DECISIONS FOR NEXT PERIOD BY CIRCLING YOUR DECISIONS ON THE CARBON COPY.

DECISION ALTERNATIVES

					DECISIONS LAST PERIOD					
$ 4.65	4.62	4.59	4.56	4.53	$ 4.50	4.49	4.48	4.46	4.45	UNIT COST OF PRODUCTION
720,000	756,000	792,000	828,000	864,000	900,000	918,000	936,000	954,000	972,000	UNITS OF PRODUCTION
$ 3,348,700	3,491,200	3,633,700	3,774,000	3,913,900	$ 4,052,700	4,120,900	4,189,500	4,256,700	4,325,400	COST OF PRODUCTION — $
		$ 170,000	180,000	190,000	$ 200,000	210,000	220,000	230,000		MARKETING — $
		$ 85,000	90,000	95,000	$ 100,000	105,000	110,000	115,000		RESEARCH AND DEVELOPMENT — $
		$	40,000	45,000	$ 50,000	55,000	60,000			ADDITIONAL CAPITAL INVESTMENT — $
		$ 4.85	4.90	4.95	$ 5.00	5.05	5.10	5.15		PRICE

PLANT DISPOSAL 0 UNITS, — 5000 UNITS, — 10,000 UNITS, IN UNITS OF PLANT CAPACITY

Fig. 3. Game report form, New York, March 6–7, 1957.



STATEMENT OF ASSETS

Year 2 Quarter 4

			TOTAL	NET CHANGE
CASH			$ 5,437,300	$ 178,000
INVENTORY		units @ $ 4.25	$	$
PLANT INVESTMENT	1,098,000	units @ $ 5.00	$ 5,490,000	$ 30,000
TOTAL ASSETS			$10,927,300	$ 208,000

ANNUAL STATEMENTS

Year 2

Company 1	Company 2	Company 3	Company 4	Company 5
$ 4,682,500	$ 5,437,300	$ 5,240,700	$ 4,805,000	$ 4,624,100
$ 749,800	$	$ 70,100	$ 391,100	$ 501,800
$ 5,090,000	$ 5,490,000	$ 5,520,000	$ 5,080,000	$ 5,350,000
$10,522,300	$10,927,300	$10,830,800	$10,276,100	$10,475,900

INCOME STATEMENT

SALES INCOME	1,073,800 units @ $ 5.25		$ 5,637,500
COST OF GOODS SOLD & OPERATING EXPENSES			
COST OF GOODS SOLD		$ 4,563,700	
MARKETING AND RESEARCH & DEVELOPMENT		$ 652,700	
OTHER (Market Research)		$ 5,000	$ 5,221,400
			$ 416,100
OTHER INCOME (Plant Disposal)			$
INCOME BEFORE TAXES			$ 416,100
TAXES			$ 208,100
NET INCOME			$ 208,000

MARKET INFORMATION

	Company 1	Company 2	Company 3	Company 4	Company 5
PRICE	$ 5.70	$ 5.75	$ 5.15	$ 5.25	$ 5.20
SHARE OF MARKET	15.93 %	24.53 %	21.70 %	19.27 %	18.57 %
TOTAL MARKET		4,872,900			
POTENTIAL SALES		1,195,300			

MARKET RESEARCH REPORT

TOTAL INDUSTRY MARKETING EXPENDITURE	$
TOTAL INDUSTRY RESEARCH & DEVELOPMENT EXPENDITURE	$
POTENTIAL SHARE OF MARKET MAXIMUM MARKETING	%
POTENTIAL SHARE OF MARKET MAXIMUM PRICE	%

OPERATION AND DECISION INFORMATION

(for next period)

	1	2	3	4	5 (DECISIONS LAST PERIOD)	6	7	8	9
UNIT COST OF PRODUCTION	$ 4.39	$ 4.35	$ 4.31	$ 4.27	$ 4.23	$ 4.22	$	$	$
UNITS OF PRODUCTION	859,000	912,700	966,400	1,020,100	1,073,800	1,095,300			
DECISION ALTERNATIVES									
COST OF PRODUCTION	$ 3,771,000	$ 3,970,200	$ 4,165,200	$ 4,355,800	$ 4,542,200	$ 4,622,200	$	$	$
MARKETING		$ 376,200	$ 398,300	$ 420,500	$ 442,600	$ 464,700	$ 486,900	$ 509,000	
RESEARCH & DEVELOPMENT		$ 178,600	$ 189,100	$ 199,600	$ 210,100	$ 220,600	$ 231,100	$ 241,600	
ADDITIONAL PLANT INVESTMENT	$	$	$ 10,000	$ 20,000	$ 30,000	$ 40,000	$ 50,000	$ 60,000	$ 70,000
MARKET RESEARCH INFORMATION S = Competitors Share of Market M = Total Industry Marketing Expenditure R = Total Industry Res. & Develop. Expenditure A = Potential Market Share Max. Marketing P = Potential Market Share Max Price	NONE	S $ 5,000	M $ 10,000 NONE	R $ 10,000 A $ 28,200	S & M $ 15,000 P $ 28,200	S & R $ 15,000 A & P $ 56,400	M & R $ 20,000	S,M & R $ 25,000	
PRICE	$ 5.05	$ 5.10	$ 5.15	$ 5.20	$ 5.25	$ 5.30	$ 5.35	$ 5.40	$ 5.45
PLANT DISPOSAL (in units)				NONE	5,000	10,000			

IBM 650 REPORT — TOTAL FUNDS AVAILABLE → $ 5,437,300 — COMPANY 2 YEAR 09 CARD 1

Fig. 4. Game report form, New York, May 2, 1957.

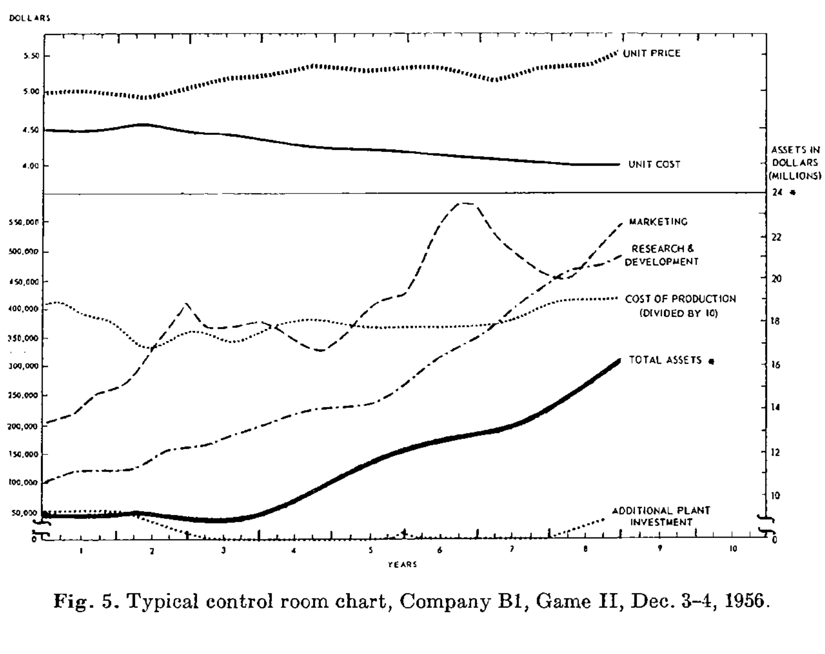


Fig. 5. Typical control room chart, Company B1, Game II, Dec. 3-4, 1956.

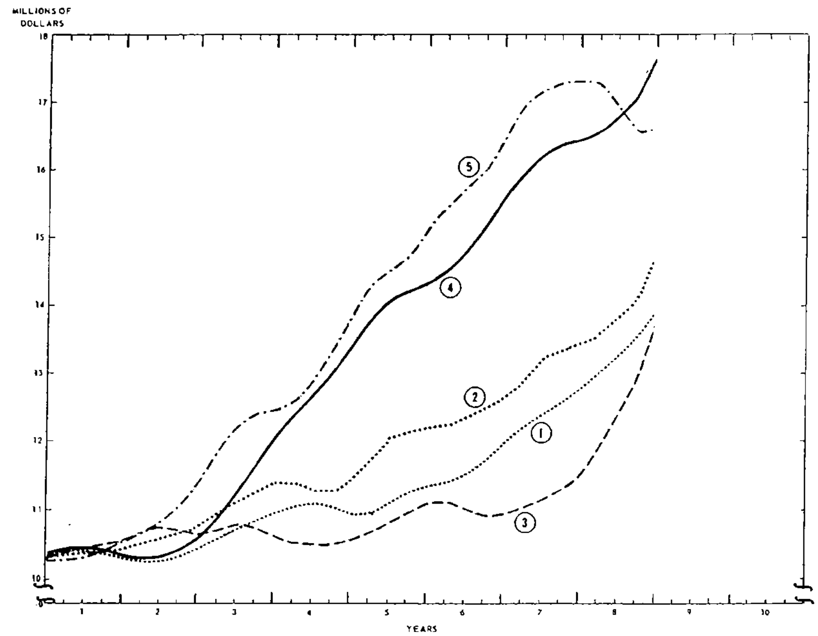


Fig. 6. Total assets, Game I, Endicott, Oct. 3-4, 1956.

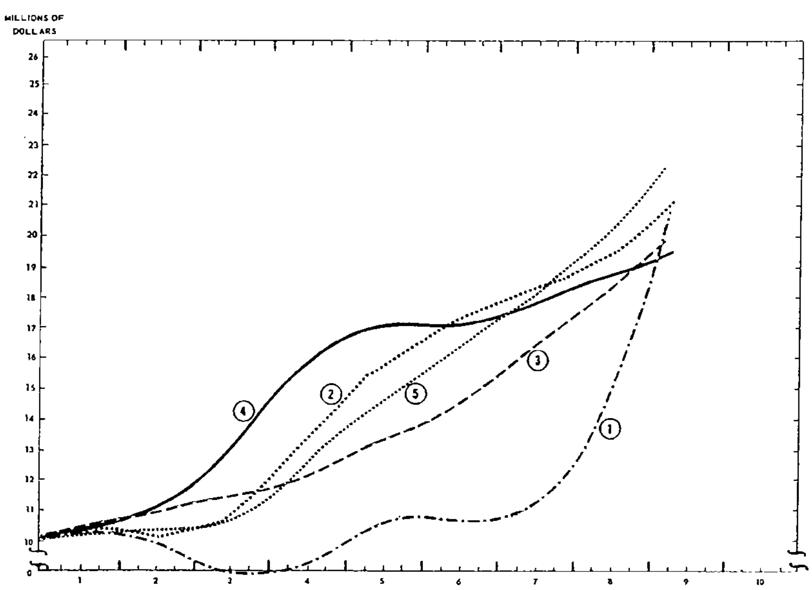


Fig. 7. Total assets, Game IIA, Los Angeles, Dec. 3-4, 1956.

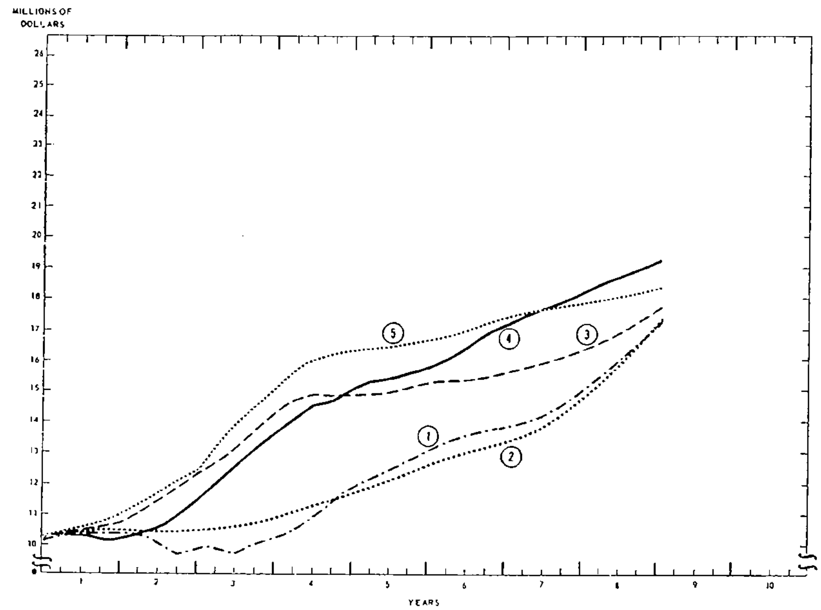


Fig. 8. Total assets, Game IIB, Los Angeles, Dec. 3-4, 1956.

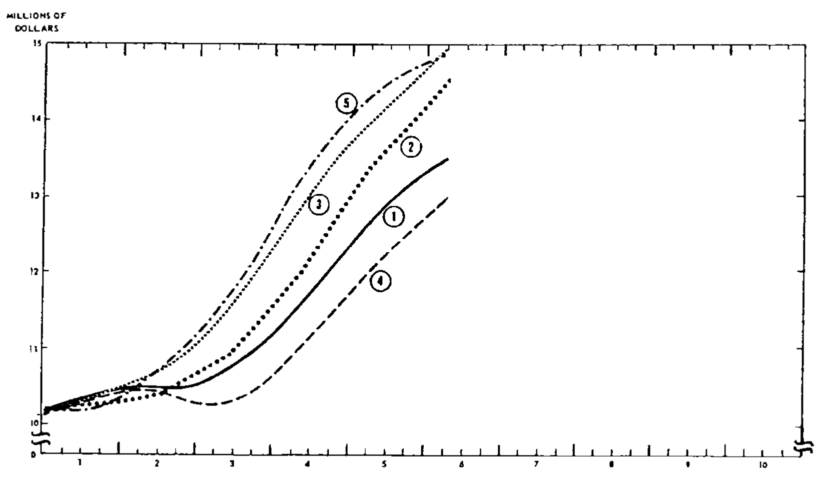


Fig. 9. Total assets, Game IIIA, New York, March 6-7, 1957.

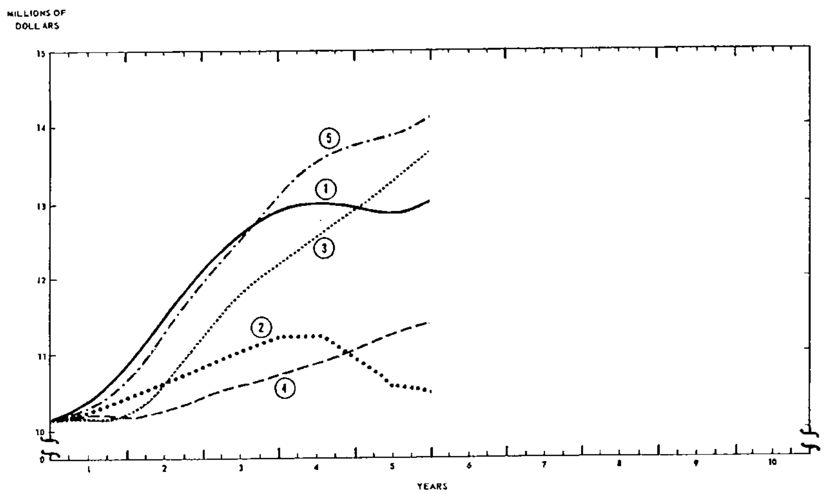


Fig. 10. Total assets, Game IIIB, New York, March 6-7, 1957.

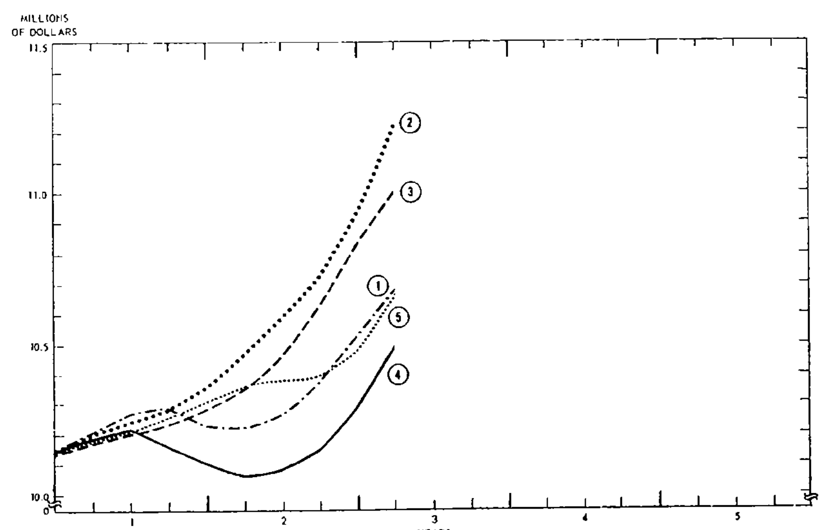


Fig. 11. Total assets, Game IV, New York, May 2, 1957.

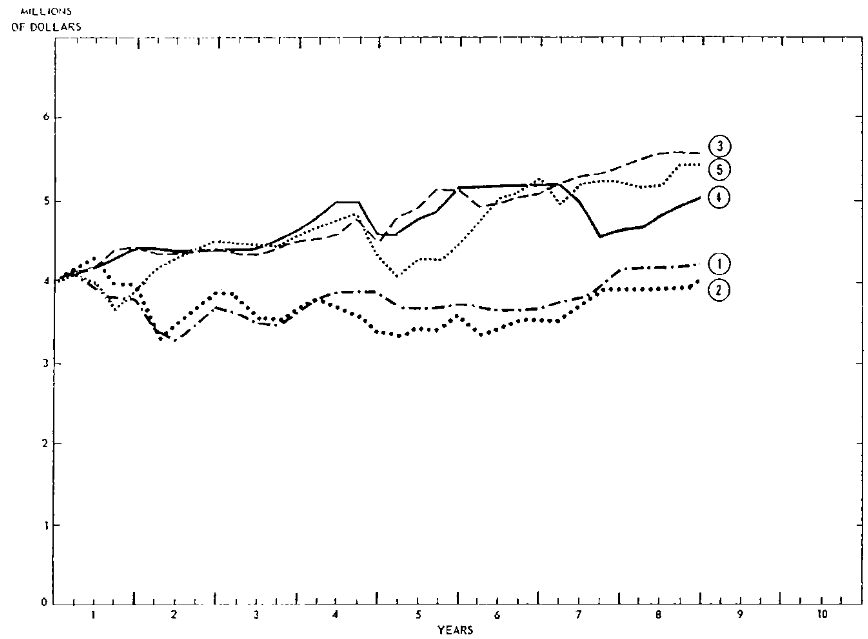


Fig. 12. Control chart, costs of production, Game IIB, Los Angeles, Dec. 3–4, 1956.

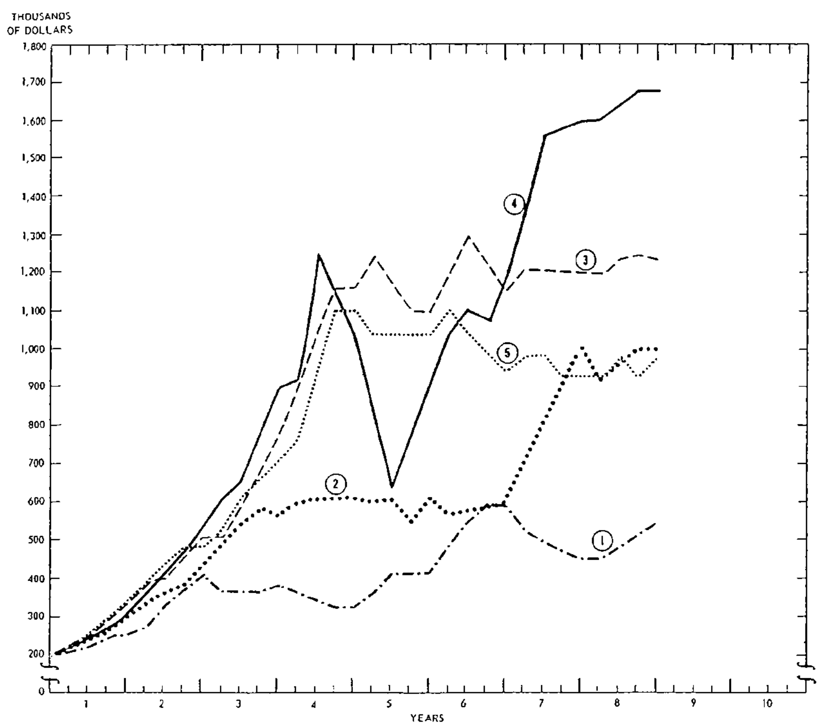


Fig. 13. Control chart, marketing expenditures, Game IIB, Los Angeles, Dec. 3–4, 1956.

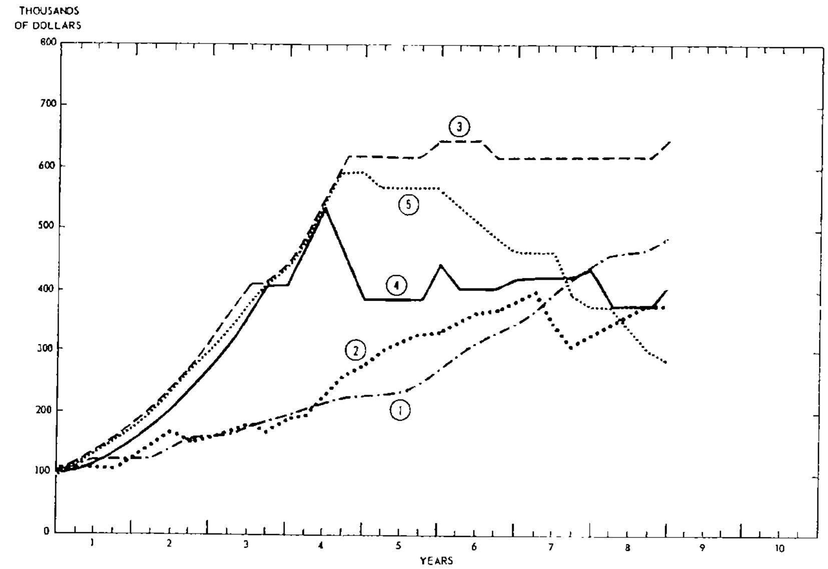


Fig. 14. Control chart, research & development expenditures, Game IIB, Los Angeles, Dec. 3-4, 1956.

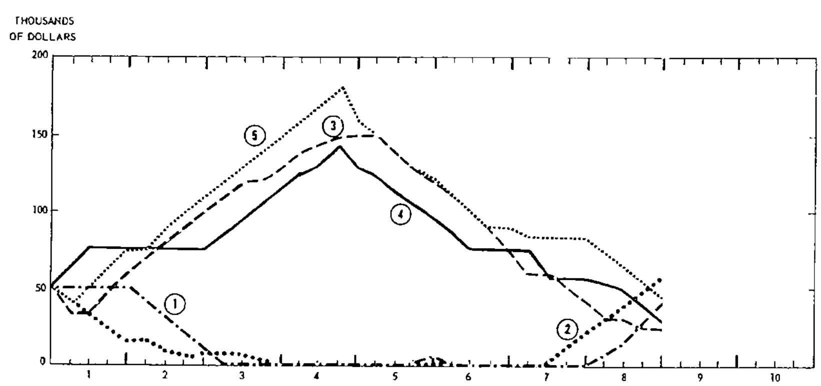


Fig. 15. Control chart, additional plant investment expenditures, Game IIB, Los Angeles, Dec. 3-4, 1956.

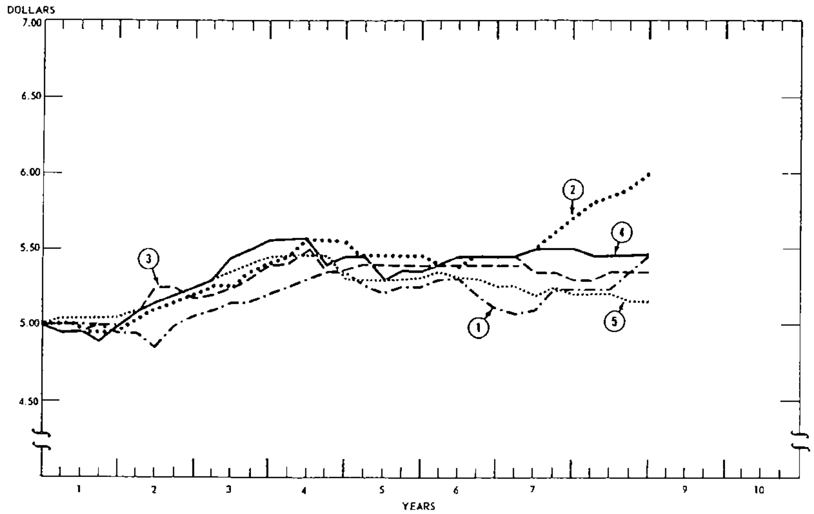


Fig. 16. Control chart, unit prices, Game IIB, Los Angeles, Dec. 3-4, 1956.

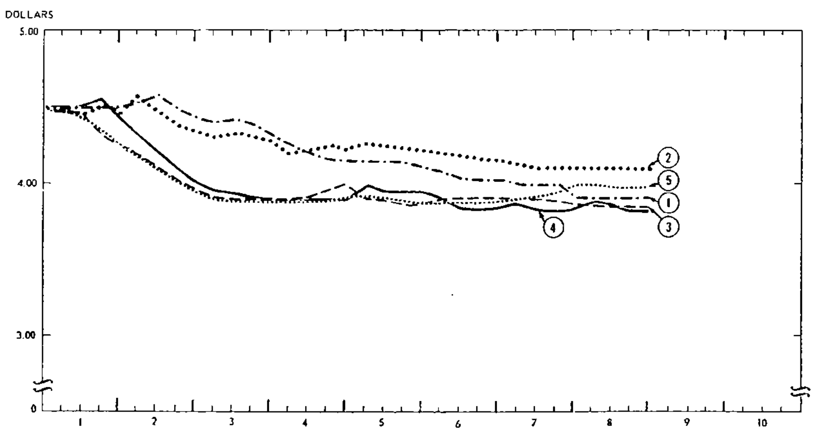


Fig. 17. Control chart, unit costs, Game IIB, Los Angeles Dec. 3-4, 1956.

REFERENCES

1. Bellman, R., On the Application of Dynamic Programming to the Study of Control Processes, Symposium on Nonlinear Control Processes, 1956, (to appear).

2. __________, Dynamic Programming and Its Application to Variational Problems in Mathematical Economics, Symposium on the Calculus of Variations and Applications, Amer. Math. Soc., Chicago, 1956, (to appear).

3. __________, Mathematical Aspects of Scheduling Theory, J. Soc. Industrial Applied Math., vol 4 (1956), pp. 168-205.

4. Bush, R. and R. Mosteller, Stochastic Models for Learning, J. Wiley and Sons, 1955.

5. Dantzig, G., Application of the Simplex Technique to a Transportation Problem, Activity Analysis of Production and Allocation, J. Wiley and Sons, 1953.

6. Mc Kinsey, J. C. C., Some Notions and Problems of Game Theory, Bull. Amer. Math. Soc., Vol 58 (1952), pp. 591-611.

7. Nash, J., Equilibrium Points in n-person Games, Proc. Nat. Acad. Sci., Vol 36 (1950), pp. 48-9.

8. Robbins, H., Some Aspects of the Sequential Design of Experiments, Bull. Amer. Math. Soc., Vol 58 (1952), pp. 527-36.

9. Shapley, L., A Value for n-person Games, Contributions to the Theory of Games, Vol 2 (1953), pp. 307-317, Annals of Math. Studies, Princeton U. Press.

10. von Neumann, J. and O. Morgenstern, Theory of Games and Economic Behavior, Princeton U. Press, 1950.

11. Wald, A., Statistical Decision Functions, J. Wiley and Sons, 1950.

12. Wiener, N., The Theory of Prediction, Modern Mathematics for the Engineer, McGraw-Hill, 1956.

13. Williams, J. D., The Compleat Strategyst, McGraw-Hill, 1954.

SUGGESTIONS FOR FURTHER READING

An enormous amount of work has been done in the field of simulation and gaming, and related disciplines. For those entering this field and wishing to obtain an overall perspective, we would like to suggest the following books and articles. Further references are contained in these.

I. Simulation and Operational Gaming.

1. Thomas, C.J., and W. L. Deemer, Jr., The Role of Operational Gaming in Operations Research, Journal Oper. Res. Soc., Vol 5 (1957), pp. 1-27.

2. Mood, A.M. and R. D. Specht, Gaming as a Technique of Analysis, P-579, The RAND Corporation, 1954.

II. The Theory of Games.

1. von Neumann, J. and O. Morgenstern, Theory of Games and Economic Behavior, Princeton U. Press, 1950.

2. Williams, J. D., The Compleat Strategyst, McGraw-Hill, 1954.

III. Dynamic Programming.

1. Bellman, R., Dynamic Programming, Princeton U. Press, 1957.

IV. Scheduling Theory.

1. Activity Analysis of Production and Allocation, J. Wiley and Sons, 1953.

V. Learning Theory.

1. Bush, R. and F. Mosteller, Stochastic Models for Learning J. Wiley and Sons, 1955.

2. Robbins, H., Some Aspects of the Sequential Design of Experiments, Bull. Amer. Math. Soc., Vol 58 (1952), pp. 527-36.