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On the Control of Nonlinear Systems with Unknown Prandtl-Ishlinskii Hysteresis

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***Abstract*— Control of nonlinear systems preceded by unknown hysteresis nonlinearities is a challenging task and has received great attention recently due to increasing industrial demands. In the literature, many mathematical models have been proposed to describe the hysteresis. The challenge addressed here is how to fuse those hysteresis models with available robust control techniques to have the basic requirement of stability of the system. The purpose of the paper is to show such a possibility by using the Prandtl-Ishlinskii (PI) hysteresis model. An adaptive variable structure control approach, serving as an illustration, is fused with the PI model without necessarily constructing a hysteresis inverse. The global stability of the system and tracking a desired trajectory to a certain precision are achieved. Simulations performed on a nonlinear system illustrate and further validate the effectiveness of the proposed approach.**

I. INTRODUCTION

The hysteresis phenomenon occurs in all the smart material-based actuators, such as piezoceramics and shape memory alloys [1]. When a nonlinear plant is preceded by the hysteresis nonlinearity, the system usually exhibits undesirable inaccuracies or oscillations and even instability [15] due to the nondifferentiable and nonmemoryless character of the hysteresis. The development of control techniques to mitigate the effects of hystereses has been studied for decades and has recently re-attracted significant attention, as can be seen in [10] and the references therein. Much of this renewed interest is a direct consequence of the importance of hysteresis in current applications. Interest in studying dynamic systems with actuator hysteresis is also motivated by the fact that they are nonlinear systems with *nonsmooth* nonlinearities for which traditional control methods are insufficient and so requiring development of new approaches [16]. Development of a general frame for control of a system in the presence of unknown hysteresis nonlinearities is a quite challenging task.

To address such a challenge, it necessitates to characterize these nonlinearities. Hysteresis models can be roughly classified into physics based models and purely phenomenological models. Physics-based models are built on first principles of physics. Phenomenological models, on the other hand, are used to produce behaviors similar to those physical systems without necessarily providing physical insight into the problems. The basic idea consists of the modeling of the real complex hysteresis nonlinearities by the weighted aggregate effect of all possible so-called elementary hysteresis operators. Elementary hysteresis operators are non-complex hysteretic nonlinearities with a simple mathematical structure. Models set up by the composition of operators of play and stop type are referred to as Prandtl-Ishlinskii models in the literature (see, e.g., [6], [18]). The reader may refer to [9] for a recent review for the hysteresis models.

With all the developed hysteresis models, it is by nature to seek the way to fuse those hysteresis models with available robust control techniques to mitigate the effects of hysteresis, especially when the hystereis is unknown, which is a typical case in the practical applications. However, the discussions on the fusion of the available hysteresis models with the available control techniques is surprisingly spare [14] in the literature. The most common approach is to construct an inverse operator, which was pioneered by Tao and Kokotovic [15], and the reader may refer to, for instance, [4], [3], [7] and references therein.

The challenge addressed here is to fuse those hysteresis models with available control techniques to have the basic requirement of stability of the system. As an illustration, in this paper we show such a possibility by fusing the Prandtl-Ishlinskii models with the adaptive variable structure control approach to mitigate the effects of the hysteresis without constructing inverse hysteresis nonlinearity. The proposed control law ensures the global stability of the adaptive system and achieves both stabilization and strict tracking precision. Simulations performed on a nonlinear system illustrate and further validate the effectiveness of the proposed approach. The proposed method can be observed as an initial step to to fuse the available hysteresis models with available control techniques.

II. PROBLEM STATEMENT

Consider a controlled system consisting of a nonlinear plant preceded by an actuator with hysteresis nonlinearity, that is, the hysteresis is presented as an input of the nonlinear plant. The hysteresis is denoted as an operator

$$w(t) = P[v](t) \tag{1}$$

with $v(t)$ as the input and $w(t)$ as the output. The operator $P[v]$ will be discussed in detail in the forthcoming section. The nonlinear dynamic system being preceded by the above

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hysteresis is described in the canonical form as,

$$x^{(n)}(t)+\sum_{i=1}^{k}a_iY_i(x(t),\dot{x}(t),...,x^{(n-1)}(t))=bw(t) \quad (2)$$

where Y_i are known continuous, linear or nonlinear functions. Parameters a_i and control gain b are constants. It is a common assumption that the sign of b is known. Without losing generality, we assume $b>0$. It should be noted that more general classes of nonlinear systems can be transformed into this structure [5].

The control objective is to design a control law for $v(t)$ in (1), to force the plant state $x(t)$ to follow a specified desired trajectory, $x_d(t)$, i.e., $x(t)\to x_d(t)$ as $t\to\infty$.

Through the paper the following assumption is made:

Assumption: The desired trajectory $\mathbf{x}_d = [x_d,\dot{x}_d,...,x_d^{(n-1)}]^T$ is continuous. Furthermore, $[\mathbf{x}_d^T,x_d^{(n)}]^T \in \Omega_d \subset R^{n+1}$ with Ω_d a compact set and is available.

III. Hysteresis Models

As mentioned in the introduction, in this paper we will focus on the Prandtl-Ishlinskii model to illustrate the fusion of the Prandtl-Ishlinskii models with the adaptive variable structure control approach to mitigate the effects of the hysteresis.

A. Stop and Play Operators

Before giving Prandtl-Ishlinskii model, we list below some basic well-known hysteresis operators. A detailed discussion on this subject can be found in the monographs [2], [6], [18]. One of the basic elements of the theory of hysteresis operators is expressed by a stop operator, $w(t)=E_r[v](t)$, with threshold r.

Analytically, suppose $C_m[0,t_E]$ is the space of piecewise monotone continuous functions, for any input $v(t)\in C_m[0,t_E]$, let $e_r: R\mapsto R$ be defined by

$$e_r(v)=\min(r,\max(-r,v)). \quad (3)$$

Then, for any initial value [1] $w_{-1}\in R$ and $r\geq 0$, the stop operator $E_r[\cdot;w_{-1}]$ is defined as [2]

$$\begin{aligned} E_r\quad &[v;w_{-1}](0)=e_r(v(0)-w_{-1}),\\ E_r\quad &[v;w_{-1}](t)=e_r(v(t)-v(t_i)+E_r[v;w_{-1}](t_i)), \end{aligned} \quad (4)$$

for $t_i<t\leq t_{i+1}$ and $0\leq i\leq N-1$, where $0=t_0<t_1<\cdots<t_N=t_E$ is a partition of $[0,t_E]$ such that the function v is monotone on each of the sub-intervals $[t_i,t_{i+1}]$. The argument of the operator is written in square brackets to indicate the functional dependence, since it maps a function to a function. The stop operator however is mainly characterized by its threshold parameter r which determines the hight of the hysteresis region in the (v,w) plane.,

There is another basic hysteresis nonlinearity operator, called the play operator [2]. For $r\geq 0$, the play operator $F_r[\cdot;w_{-1}]: C_m[0,t_E]\times w_{-1}\mapsto C_m[0,t_E]$ for a general initial value[2] $w_{-1}\in R$, is defined by

$$\begin{aligned} F_r\quad &[v;w_{-1}](0)=f_r(v(0),w_{-1}),\\ F_r\quad &[v;w_{-1}](t)=f_r(v(t),F_r[v;w_{-1}](t_i)), \end{aligned} \quad (5)$$

for $t_i<t\leq t_{i+1}$ and $0\leq i\leq N-1$, with

$$f_r(v,w)=\max(v-r,\min(v+r,w)). \quad (6)$$

where the partition $0=t_0<t_1<\cdots<t_N=t_E$ is the same as defined for the stop operator.

From the definitions given in (4) and (5), it can be proved [2] that the operator F_r is the complement of E_r, i.e., they are closely related through the equation

$$E_r[v](t)+F_r[v](t)=v(t), \quad (7)$$

for any piece-wise monotone input function v and $r\geq 0$.

In the sequel, we will simply write $E_r[v]$ or $F_r[v]$ to denote $E_r[v;w_{-1}]$ or $F_r[v;w_{-1}]$ so long as doing so does not affect the proof. Due to the natural of the play and stop operators, above discussions are defined on the space $C_m[0,t_E]$ of continuous and piecewise monotone functions; however, they can also be extended to the space $C[0,t_E]$ of continuous functions.

B. Prandtl-Ishlinskii Model

We are ready to introduce the Prandtl-Ishlinskii model defined by the stop or play hysteresis operators. The Prandtl-Ishlinskii model [12] was introduced to formulate the elastic-plastic behavior through a weighted superposition of basic elastic-plastic elements $E_r[v]$, or stop as follows:

$$w(t)=\int_0^R p(r)E_r[v](t)dr, \quad (8)$$

where $p(r)$ is a given density function, satisfying $p(r)\geq 0$ with $\int^{\infty} rp(r)dr<\infty$, and is expected to be identified from experimental data. With the defined density function, this operator maps $C[t_0,\infty)$ into $C[t_0,\infty)$,i.e., Lipschitz continuous inputs will yield Lipschitz continuous outputs [6]. Since the density function $p(r)$ vanishes for large values of r, the choice of $R=\infty$ as the upper limit of integration in the literature is just a matter of convenience [2].

It can be seen that the stop operator E_r serves as the building element in the Prandtl-Ishlinskii model (8). We should mention that the stop and play operators are rate-independent and the Prandtl-Ishlinskii model (8) is also rate-independent. As an illustration, Fig.1 shows $w(t)$ generated by model given in (8), with $p(r)=e^{-0.067(r-1)^2}$, $r\in[0,10]$, and input $v(t)=7sin(3t)/(1+t)$, $t\in[0,2\pi]$ with $w_{-1}=0$. This numerical result shows that the Prandtl-Ishlinskii model (8) indeed generates the hysteresis curves and is well-suited to model the rate-independent hysteretic behavior.

Since the operator F_r is the complement of E_r, the Prandtl-Ishlinskii model can also be expressed through the

[1] w_{-1} represent the value of $v-w$ before $v(0)$ is applied at time t=0.

[2] w_{-1} represent the initial state before $v(0)$ is applied at time t=0.

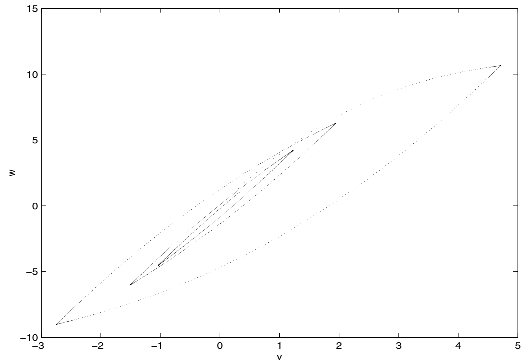

Fig. 1. Hysteresis curves given by (8)

play operator. Using Equation (7) and substituting E_r in (8) by F_r, the Prandtl-Ishlinskii model defined by the play hysteresis operator is expressed as follows:

$$w(t) = p_0 v(t) - \int_0^R p(r) F_r[v](t) dr, \quad (9)$$

where $p_0 = \int_0^R p(r)dr$ is constant which depends on the density function. It should be noted that Equation (9) decomposes the hysteresis behavior into two terms. The first term describes the linear reversible part and the second term describes the nonlinear hysteretic part. This decomposition is crucial since it facilitates the utilization of the currently available control techniques for the controller design.

IV. Controller Design

In this section, instead of constructing the inverse of the hysteresis model to mimic the hysteresis effects as frequently done in the literature, we shall propose, as an illustration, an adaptive variable structure controller for plants of the form described by (2) preceded by hysteresis described by the Prandtl-Ishlinskii model. The proposed controller will lead to global stability and yield tracking within a desired precision.

Consider the Prandtl-Ishlinskii model expressed by the play operator given in (9), the hysteresis output $w(t)$ can be expressed as

$$w(t) = p_0 v(t) - d[v](t), \quad (10)$$

where

$$d[v](t) = \int_0^R p(r) F_r[v](t) dr, \quad (11)$$

with $p_0 = \int_0^R p(r)dr$.

Using the hysteresis model of (10), the system (2) becomes,

$$x^{(n)}(t) + \sum_{i=1}^{k} a_i Y_i(x(t), \dot{x}(t), ..., x^{(n-1)}(t)) = b\{p_0 v(t) - d[v](t)\}, \quad (12)$$

which yields a linear relation to the input signal $v(t)$ plus a shifting term $bd[v]$.

Remark: It is clear that the first term on the right-hand side of (12) is expressed as a linear function of the control signal $v(t)$. In this case, it is possible to fuse the currently available controller design techniques with the hysteresis model for the controller design. It will become clear later, it is in fact this structure that makes it possible to design the adaptive variable structure control algorithm. This was also our primary motivation behind using the Prandtl-Ishlinskii model.

In the following development, we shall propose an adaptive variable structure controller for (12).

Equation (12) can be re-expressed as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ &\vdots \\ \dot{x}_{n-1} &= x_n \\ \dot{x}_n &= -\sum_{i=1}^{k} a_i Y_i(x_1(t), x_2(t), ..., x_{n-1}(t)) \\ &\quad + b\{p_0 v(t) - d[v](t)\} \\ &= \mathbf{a}^T Y + b_p v(t) - d_b[v](t) \end{aligned} \quad (13)$$

where $x_1(t) = x(t), x_2(t) = \dot{x}(t), \cdots, x_n(t) = x^{(n-1)}(t)$, $\mathbf{a} = [-a_1, -a_2, \cdots, -a_k]^T$, $Y = [Y_1, Y_2, \cdots, Y_k]^T$, $b_p = bp_0$, and $d_b[v](t) = \int_0^R p_b(r) F_r[v](t) dr$, with $p_b(r) = bp(r)$.

In presenting the developed adaptive variable structure control law, the following definitions are required:

$$\tilde{\mathbf{a}}(t) = \mathbf{a} - \hat{\mathbf{a}}(t), \quad (14)$$

$$\tilde{\phi}(t) = \phi - \hat{\phi}(t), \quad (15)$$

$$\tilde{p}_b(t, r) = p_b(r) - \hat{p}_b(t, r), \quad for\ all\ r \in [0, R], \quad (16)$$

$\hat{\mathbf{a}}$ is an estimate of $\mathbf{a}$, $\hat{\phi}$ is an estimate of ϕ, which is defined as $\phi \triangleq (b_p)^{-1}$, $\hat{p}_b(t, r)$ is an estimate of the density function $p_b(r)$. Let

$$B(v(t)) \triangleq \int_0^R p_b(r) |F_r[v](t)| dr, \quad (17)$$

and the estimation $\hat{B}(t)$ is given by $\int_0^R \hat{p}_b(t, r) |F_r[v](t)| dr$, which leads to

$$\tilde{B}(t) = \int_0^R (\hat{p}_b(t, r) - p_b(r)) |F_r[v](t)| dr. \quad (18)$$

Given the plant and hysteresis model subject to the assumptions described above, we propose the following control law:

$$v(t) = \hat{\phi}(t) v_1(t) \quad (19)$$

with

$$v_1(t) = -c_n z_n - z_{n-1} - \hat{\mathbf{a}}^T Y - sgn(z_n)\hat{B} + x_d^{(n)} + \dot{\alpha}_{n-1} \quad (20)$$

where

$$\begin{aligned} z_1(t) &= x_1(t) - x_d(t) \\ z_i &= x_i(t) - x_d^{(i-1)} - \alpha_{i-1},\ i = 2, 3, \cdots, n \quad (21) \\ \alpha_1(t) &= -c_1 z_1(t) \\ \alpha_i(t) &= -c_i z_i(t) - z_{i-1}(t) + \dot{\alpha}_{i-1}, \\ & \quad i = 2, 3, \cdots, n-1 \quad (22) \end{aligned}$$

where c_i, $i = 1, 2, \cdots, n-1$, are positive design parameters. The parameters $\hat{\phi}$, $\hat{\mathbf{a}}$, and function $\hat{p}_b(t,r)$ will be updated by the following adaptation laws

$$\dot{\hat{\mathbf{a}}} = \gamma Y z_n \quad (23)$$

$$\dot{\hat{\phi}} = -\eta v_1 z_n \quad (24)$$

$$\frac{\partial}{\partial t}\hat{p}_b(t,r) = q|F_r[v](t)||z_n|, \quad for\ r \in [0, R], \quad (25)$$

where parameters γ, η and q are positive constants determining the rates of the adaptations.

Remarks:

1) The term $u_N(t)$ represents the compensation component for the function $d[v](t)$. Unlike the traditional adaptive variable structure controller designs, where $d[v](t)$ is either assumed to be bounded by a constant or a known function [17], $d[v](t)$ is presented as an integral equation, and there is no assumption on its boundedness. Thanks to the density function $p(r)$, which is not a time function, we can thus treat this term as a parameter of the hysteresis model and develop an estimated law for it. This is crucial for the success of the adaptive law design.

2) For the calculation of $\hat{B}(t) = \int_0^R \hat{p}(r,t)\frac{|F_r[v](t)|}{p_{0min}}dr$ in the implementation, using numerical technique, we can simply replace the integration with the sum by dividing R into small intervals, i.e, $\hat{B}(t) = \sum_{l=0}^{N-1} \hat{p}(l\Delta r, t)\frac{|F_{l\Delta r}[v](t)|}{p_{0min}}\Delta r$, where N determines the size of the intervals as $\Delta r = R/N$. The selection of the size of the intervals depends on the accuracy requirement. As will be shown in the simulation example, the size of the small intervals may not necessarily be very small.

The stability of the closed-loop system described in (12), (19) and (23)-(25) is established in the following theorem:

Theorem: For the plant given in Equation (2) with the hysteresis (9), subject to Assumption 1, the adaptive variable structure controller specified by (19) and (23)-(25) ensures that all the closed-loop signals are bounded and $x(t) \to x_d(t)$ as $t \to \infty$.

Proof: Using the expression (13) and the definition of z_n in (21), noticing that $b_p v(t) = b_p\hat{\phi}v_1(t) = v_1(t) - b_p\tilde{\phi}v_1(t)$, one can obtain:

$$\begin{aligned} z_1\dot{z}_1 &= -c_1 z_1^2 + z_1 z_2 \\ z_i\dot{z}_i &= -z_{i-1}z_i - c_i z_i^2 + z_i z_{i+1}, \quad i = 2, 3, \cdots, n-1 \\ \dot{z}_n &= -c_n z_n - z_{n-1} + \tilde{\mathbf{a}}^T Y - sgn(z_n)\hat{B} \\ & \quad -d_b[v](t) - b_p\tilde{\phi}v_1(t) \quad (26) \end{aligned}$$

To establish global boundedness, we define the following Lyapunov function candidate

$$V(t) = \sum_{i=1}^n \frac{1}{2}z_i^2 + \frac{1}{2\gamma}\tilde{\mathbf{a}}^T\tilde{\mathbf{a}} + \frac{b_p}{2\eta}\tilde{\phi}^2 + \frac{1}{2q}\int_0^R \tilde{p}_b^2(t,r)dr. \quad (27)$$

The derivative $\dot{V}$ is given by

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^n z_i\dot{z}_i + \frac{1}{\gamma}\tilde{\mathbf{a}}^T\dot{\tilde{\mathbf{a}}} + \frac{b_p}{\eta}\tilde{\phi}\dot{\tilde{\phi}} \\ & \quad + \frac{1}{q}\int_0^R \tilde{p}_b(t,r)\frac{\partial}{\partial t}\tilde{p}_b(t,r)dr \\ &\leq -\sum_{i=1}^n c_i z_i^2 + \frac{1}{\gamma}\tilde{\mathbf{a}}^T(\dot{\tilde{\mathbf{a}}} + \gamma Y z_n) \\ & \quad + \frac{b_p}{\eta}\tilde{\phi}(\dot{\tilde{\phi}} - \eta v_1 z_n) \\ & \quad -|z_n|\hat{B} + |d_b[v](t)||z_n| \\ & \quad + \int_0^R \tilde{p}_b(t,r)|F_r[v](t)||z_n|dr \\ &\leq -\sum_{i=1}^n c_i z_i^2 + \frac{1}{\gamma}\tilde{\mathbf{a}}^T(\dot{\tilde{\mathbf{a}}} + \gamma Y z_n) \\ & \quad + \frac{b_p}{\eta}\tilde{\phi}(\dot{\tilde{\phi}} - \eta v_1 z_n) \\ & \quad -|z_n|\tilde{B} + \int_0^R \tilde{p}_b(t,r)|F_r[v](t)||z_n|dr \\ &= -\sum_{i=1}^n c_i z_i^2 \quad (28) \end{aligned}$$

Equations (27) and (28) imply that V is no increasing. Hence, z_i, i=1,..,n, $\hat{\mathbf{a}}$, $\hat{\phi}$, and $\frac{\partial}{\partial t}\hat{p}_b(t,r)$ are bounded. By applying the Lasalle-Yoshizawa theorem in [8] to (28), it further follws that $z_i \to 0$, i=1,..,n as $t \to \infty$, which implies that $\lim_{t\to\infty}[x(t) - x_d(t)] = 0$.

Remark: It is now clear that the developed control strategy to deal with the hysteresis nonlinearities can be applied to many systems and may not necessarily be limited to the system described by (2). However, we should emphasize that our goal in this paper is to illustrate the fusion of the hysteresis models with available control techniques in a simpler setting that reveals its essential features.

V. SIMULATION STUDIES

In this section, we illustrate the methodology presented in the previous sections using a simple nonlinear system described by

$$\dot{x} = a\frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} + bw(t) \quad (29)$$

where $w(t)$ represents the output of the hysteresis. The actual parameter values are $b = 1$ and $a = 1$. Without control, i.e., $v(t) = 0$, so $w(t) = 0$, the system in (29) is unstable since $\dot{x} = \frac{1-e^{-x(t)}}{1+e^{-x(t)}} > 0$ for $x > 0$, and $\dot{x} = \frac{1-e^{-x(t)}}{1+e^{-x(t)}} < 0$ for $x < 0$. The objective is to control the system state x to follow the desired trajectory $x_d = 5sin(2t) + cos(3.2t)$. The hysteresis is described by

$$w(t) = p_0 v(t) - \int_0^R p(r) F_r[v](t) dr, \qquad (30)$$

where $p(r) = \alpha e^{-\beta(r-\sigma)^2}$ for $r \in [0, 100]$, with parameters $\alpha = 0.5, \beta = 0.0014$, and $\sigma = 1$.

In the simulation, the adaptive variable structure control law (12) and (23)-(25) were used, taking $c_1 = 0.9368$. In the adaptation laws, we choose $\gamma = 0.13$, $\eta = 0.05$, $q = 0.437$, and the initial parameters $\hat{a}(0) = 0.13$, $\hat{\phi}(0) = 0.431$, and $\hat{p}_b(0, r) = 0$. The initial state is chosen as $x(0) = 2.05$, sample time is 0.002. We also assume that the hysteresis internal state was $w_{-1} = 0.07$ for $r \in [0, R]$ before $v(0)$ was applied. For the calculation of $\hat{B}(t)$, we replace the integration by the sum $\sum_0^N$. In the simulation, we choose $N = 6000$.

To illustrate the effectiveness of the proposed control scheme, the simulation has also been conducted without controlling the effects of hysteresis, which is implemented by setting $u_N(t) = 0$ in the controller $v(t)$. This implies that the control compensation for the hysteresis nonlinearity is ignored. Simulation results are shown in Figs. 2-5 for the system (12) to track the desired trajectory $x_d(t) = 5sin(2t) + cos(3.2t)$. Figs. 2 and 3 show the state trajectories and tracking errors for the desired trajectory with and without considering the effects of hysteresis, where the solid line is the results with $u_N(t) \neq 0$ and the dotted line is with $u_N(t) = 0$. Fig. 4 shows the role of signal $u_N(t)$ and Figs. 5 and 6 show the input control signal $v(t)$ and the hysteresis output $w(t)$. From Figs. 2 and 3 it illustrates that the proposed robust controller clearly demonstrates excellent tracking performance and the developed control algorithm can overcome the effects of the hysteresis.

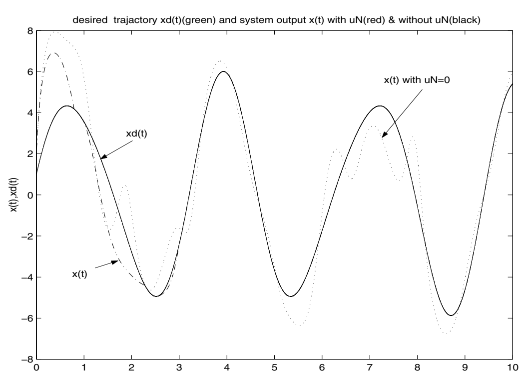

Fig. 2. Desired trajectory $x_d(t) = 5sin(2t) + cos(3.2t)$, system outputs $x(t)$ with control term u_N (-.) and $u_N = 0$ (dotted line)

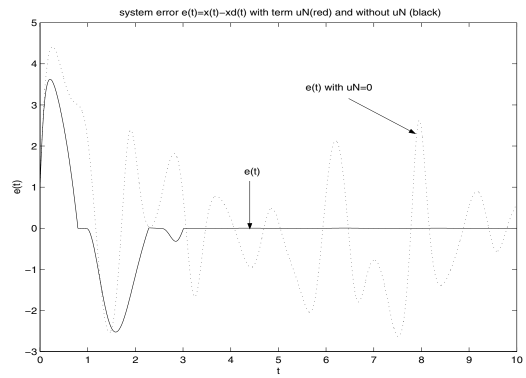

Fig. 3. Tracking errors of the state with control term u_N and $u_N = 0$ (dotted line)

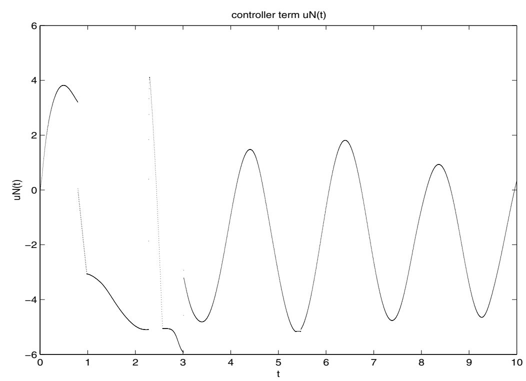

Fig. 4. Signal u_N designed to reduce the tracking error caused by the hysteresis

VI. Conclusion

In practical control systems, hysteresis nonlinearity with unknown parameters in physical components may severely limit the performance of control. In this paper, an adaptive variable structure control architecture is proposed for a class of continuous-time nonlinear dynamic systems preceded by a hysteresis nonlinearity with the Prandtl-Ishlinskii model presentation. The control law ensures global stability of the entire system and achieves both stabilization and tracking within a desired precision. Simulations performed on an unstable nonlinear system illustrate and further validate the effectiveness of the proposed approach. The primary purpose of exploring new avenues to fuse the model of hysteresis nonlinearities with the available adaptive controller design methodology without constructing a hysteresis inverse is achieved with highly promising results. The results presented in this paper can be considered as a stepping stone to be used towards the development of a general control framework for the systems with hysteretic behavior.

Acknowledgments

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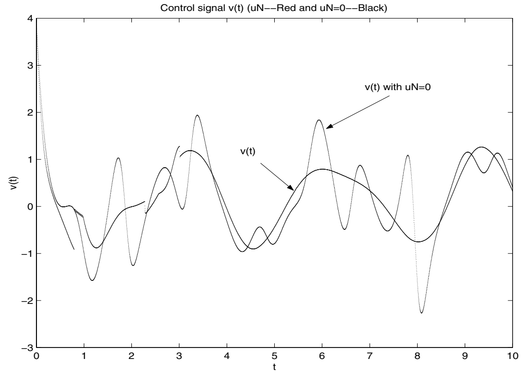

Fig. 5. Control signals $v(t)$ with control term u_N and without u_N (dotted line)

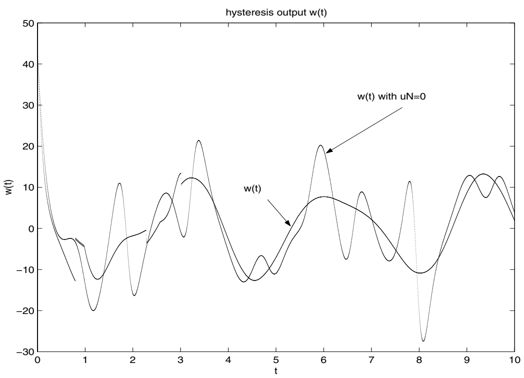

Fig. 6. Hysteresis outputs $w(t)$ with control term u_N and without u_N (dotted line)

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