XII. *On the Convection of Heat from Small Cylinders in a Stream of Fluid: Determination of the Convection Constants of Small Platinum Wires with Applications to Hot-Wire Anemometry.*

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[Plate 8.]

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PART I.

Mathematical Theory of the Convection of Heat from a Cylinder of any Form of Cross-section in a Stream of Fluid.

Section 1. *Introduction.*

The general problem of the convection of heat from bodies immersed in moving media has recently received considerable attention both from the theoretical and experimental point of view. The equation of the conduction of heat in a moving fluid was stated by Fourier as long ago as 1820,(1) and a few years later was expressed by Poisson(2) and Ostrogradsky(3) in the familiar form

$$c\frac{\mathrm{D}\theta}{\mathrm{D}t} = \frac{\partial}{\partial x}\left(\kappa\frac{\partial\theta}{\partial x}\right) + \frac{\partial}{\partial y}\left(\kappa\frac{\partial\theta}{\partial y}\right) + \frac{\partial}{\partial z}\left(\kappa\frac{\partial\theta}{\partial z}\right), \quad \ldots \ldots \quad (1)$$

where θ is the temperature of the fluid at any point (x, y, z), c the heat capacity of the fluid per unit volume, κ its thermal conductivity, and $\mathrm{D}/\mathrm{D}t$ the "mobile operator" $\mathrm{D}/\mathrm{D}t = \partial/\partial t + u\,\partial/\partial x + v\,\partial/\partial y + w\,\partial/\partial z$ of the hydrodynamical equations.

In 1901 the problem was taken up by Boussinesq,(4) whose memoir on the subject in 1905 contains a great number of successful calculations of heat losses from bodies of various shapes immersed in a stream of fluid.

A full account of the theoretical development of the subject is given by Russell,(5) and extensive references are given to papers and memoirs relating to the convection of heat.

Section 2. Boussinesq's *Transformation.*

Under certain assumptions Boussinesq, by an extremely elegant transformation, was able to reduce (1) to a differential equation capable of solution. Assuming a frictionless, incompressible fluid, the flow of liquid past an obstacle maps out the field in the neighbourhood of an obstacle by stream lines and equipotential surfaces which in some cases may constitute a set of orthogonal co-ordinates. If in such cases the general equation of heat conduction (1) be expressed in these co-ordinates, it takes a

(1) Fourier, 'Mémoires de l'Académie,' t. 12, p. 507, 1820.

(2) Poisson, 'Théorie Mathématique de la Chaleur,' 1835.

(3) Ostrogradsky, 'St. Pét. Ac. Sc. Bll.,' t. 1, p. 25, 1836.

(4) Boussinesq, 'Comptes Rendus,' vol. 133, p. 257; also 'Journal de Mathématiques,' vol. 1, pp. 285–332, 1905.

(5) Russell, 'Phil. Mag.,' vol. 20, pp. 591–610, October, 1910.

greatly simplified form, reducing in many cases to one or other of the known partial differential equations of mathematical physics. The case of two-dimensional flow lends itself especially well to such an investigation as well as to a comparison with the results of experiment. If $\alpha = constant$ represent the stream lines and $\beta = constant$ the equipotentials obtained from the solution of the hydrodynamical problem of flow past a cylinder of any form of cross-section (fig. 1), BOUSSINESQ first showed that the general equation could be transformed to the linear form

$$\frac{\partial^2\theta}{\partial\alpha^2} + \frac{\partial^2\theta}{\partial\beta^2} = 2n\frac{\partial\theta}{\partial\beta}, \quad \ldots \ldots \ldots \ldots \quad (2)$$

where θ is the temperature at any point of the liquid, κ its heat conductivity supposed to be independent of the temperature and therefore constant throughout the liquid,

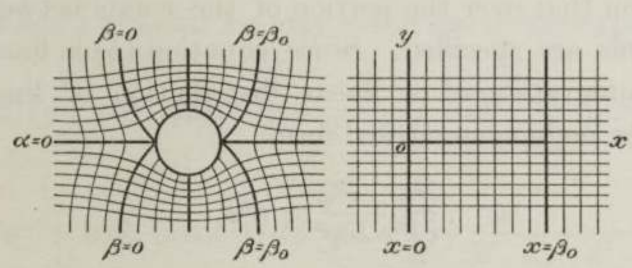


Fig. 1. BOUSSINESQ's transformation.

c is the specific heat per unit volume, and V the velocity of the stream at a great distance from the cylinder. The constant n is defined by the relation

$$2n = c\mathrm{V}/\kappa = s\sigma\mathrm{V}/\kappa, \quad \ldots \ldots \ldots \ldots \quad (3)$$

where s is the specific heat per unit mass of the fluid and σ its density.

The complete solution of (2) requires a knowledge of the conditions of heat-transfer over the interface between solid and liquid, a point which can only be settled by referring to the results of experiment. If the surface of the cylinder be the particular stream-line $\alpha = 0$, and the critical equipotentials be the curves $\beta = 0$ and $\beta = \beta_0$, the heat-flux per unit length of the cylinder is given by

$$\mathrm{H} = -\int_0^{\beta_0} \kappa\,(\partial\theta/\partial\alpha)_0\,d\beta, \quad \ldots \ldots \ldots \ldots \quad (4)$$

where the integral is taken to include both branches of the stream-line $\alpha = 0$.

It will be noticed that BOUSSINESQ's transformation reduces the problem to the simple case of calculating the temperature distribution in a uniform stream flowing parallel to the axis of x, ($\alpha = 0$), when the distribution of temperature or heat-flux is prescribed over the interval $x = 0$ to $x = \beta_0$. A solution of this simple *transformed* *problem* leads immediately to the corresponding solution in the case of a cylinder for which the hydrodynamical stream-lines can be calculated.([6])

Section 3. *Solution of* BOUSSINESQ'S *Transformed Problem.*

The complete statement of the problem just formulated may be made in terms of an *integral equation*, and, although the general solution is not yet forthcoming, an expression for the heat-loss per unit length may be obtained under special assumptions justified by a good agreement with the results of observation. The solution of the transformed problem requires us to solve the equation

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}=2n\frac{\partial\theta}{\partial x}, \quad \ldots \quad (5)$$

subject to the condition that over the portion of the x-axis between $x = 0$ and $x = \beta_0$ the boundary conditions are specified. Since equation (5) is linear, we may build up a solution by the integration of a linear distribution of line sources. Writing $\phi = \theta e^{-nx}$, (5) takes the more symmetrical form

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=n^2\phi, \quad \ldots \quad (6)$$

([6]) BOUSSINESQ, *loc. cit.* ('J. de Math.'), p. 295. The approximate solution given by BOUSSINESQ proceeds as follows: assuming the heat conductivity κ small the temperature varies very slowly with β, while it varies rapidly with α. If, in addition, the velocity is great enough so that the coefficient $c\mathrm{V}/\kappa$ is small, we may neglect the term $\partial^2\theta/\partial\beta^2$ in the differential equation (2), which then reduces to the simple Fourier form

$$\partial^2\theta/\partial\alpha^2 = 2n\,\partial\theta/\partial\beta, \quad \ldots \quad \text{(i.)}$$

of which the appropriate solution is

$$\theta = \sqrt{(2/\pi)}\int_0^\infty f(\beta - n\alpha^2/\nu^2)\,e^{-\frac{1}{2}\nu^2}\,d\nu, \quad \ldots \quad \text{(ii.)}$$

which prescribes the temperature over the boundary $\alpha = 0$ by the relation $\theta = f(\beta)$. Writing $\omega = n\alpha^2/\nu^2$, we have $(\partial\theta/\partial\alpha)_0 = -2\sqrt{(2n/\pi)}\int_0^\infty f'(\beta-\omega^2)\,d\omega$, from which we derive from (4) the result

$$\mathrm{H} = 4\kappa\sqrt{(2n/\pi)}\int_0^\infty [f(\beta_0-\omega^2)-f(-\omega^2)]\,d\omega. \quad \ldots \quad \text{(iii.)}$$

Making the further assumption that approximately $\theta = 0$ from $\beta = -\infty$ to $\beta = 0$, $\theta = \theta_0$ from $\beta = 0$ to $\beta = \beta_0$, and $\theta = 0$ from $\beta = \beta_0$ to $\beta = \infty$, (iii.) becomes

$$\mathrm{H} = 4\kappa\sqrt{(2n/\pi)}\,\beta_0^{\frac{1}{2}}\theta_0. \quad \ldots \quad \text{(iv.)}$$

Applying the results to the case of a circular cylinder, the hydrodynamical solution gives $\beta_0 = 4a$, where a is the radius of the cylinder. Also writing $c = s\sigma$, where s is the specific heat (at constant volume if the fluid is a gas) per unit mass and σ is the density, we obtain finally

$$\mathrm{H} = 8\,(s\sigma\kappa\mathrm{V}a/\pi)^{\frac{1}{2}}\,\theta_0, \quad \ldots \quad \text{(v.)}$$

which will be referred to as BOUSSINESQ'S formula. Here θ_0 is the temperature of the fluid cylinder above that of the surrounding fluid at a great distance. It must be kept in mind that the above formula cannot be expected to represent reality unless κ is small and the term $2s\sigma\mathrm{V}a/\kappa$ large.

of which a particular solution appropriate to a line source at the origin is seen, on transforming (6) to cylindrical co-ordinates, to be [7]

$$\phi = \mathrm{AK}_0(nr), \quad \text{where} \quad r = |\sqrt{(x^2+y^2)}|, \quad x = r\cos\theta, \quad y = r\sin\theta, \quad . \quad (7)$$

A is a constant of integration and $\mathrm{K}_0(z)$ that solution of BESSEL's equation most conveniently defined by the definite integral

$$\mathrm{K}_0(z) = \int_0^\infty e^{-z\cosh\phi}\,d\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

We write down for future reference the expansions

$$\mathrm{K}_0(z) = e^{-z}\sqrt{(\pi/2z)}\left[1-\frac{1}{8z}+\frac{1^2.3^2}{(8z)^2.2!}-\dots\right], \quad . \quad . \quad . \quad . \quad . \quad (9)$$

when z is large. When z is small we may make use of the expansions

$$\mathrm{K}_0(z) = -\mathrm{I}_0(z)[\gamma+\log(z/2)]+\frac{z^2}{2^2}+\frac{z^4}{2^2.4^2}(1+\tfrac{1}{2})+\frac{z^6}{2^2.4^2.6^2}(1+\tfrac{1}{2}+\tfrac{1}{3})+\dots, \quad (10)$$

where

$$\mathrm{I}_0(z) = \pi^{-1}\int_0^\pi \cosh(z\cos\phi)\,d\phi = 1+\frac{z^2}{2^2}+\frac{z^4}{2^2.4^2}+\frac{z^6}{2^2.4^2.6^2}+\dots, \quad . \quad . \quad (11)$$

and γ is EULER's constant, $\gamma = 0{\cdot}57721$.

If we denote by Q the rate at which heat is being supplied to the line source per unit length, Q should also be equal to the total flux calculated by integrating around any closed circuit enclosing the source; that is,

$$\mathrm{Q} = \int_c [-\kappa(\partial\theta/\partial\nu)+c\mathrm{V}\cos\epsilon\,.\,\theta]\,ds, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where the integral is taken around any closed circuit c enclosing the source, and $\mathrm{V}\cos\epsilon$ is the component of velocity along the outward drawn normal ν. This remarkable property of the solution $\theta = \mathrm{A}e^{nx}\mathrm{K}_0(nr)$ is easily verified by carrying out the integration around a circle of radius r, the result serving to determine the constant A. Remembering that

$$\frac{d\mathrm{K}_0(z)}{dz} = \mathrm{K}_1(z) \quad \text{and} \quad \frac{d\mathrm{I}_0(z)}{dz} = \mathrm{I}_1(z), \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

we have $d\theta/dr = \mathrm{A}ne^{nr\cos\theta}[\mathrm{K}_1(nr)+\cos\theta\,\mathrm{K}_0(nr)]$, and thus from (12)

$$\mathrm{Q} = 2r\int_0^\pi[2n\kappa\theta\cos\epsilon-\kappa(d\theta/dr)]\,d\epsilon = 2\mathrm{A}\kappa nr\left[-\mathrm{K}_1(nr)\int_0^\pi e^{nr\cos\epsilon}d\epsilon+\mathrm{K}_0(nr)\int_0^\pi\cos\epsilon\,e^{nr\cos\epsilon}d\epsilon\right].$$

[7] GRAY and MATTHEWS, 'Treatise on Bessel Functions,' 1895, pp. 77 and 90. The notation employed throughout is that of the above treatise.

It is not difficult to prove from (11) and (13) that

$$\int_0^\pi e^{z\cos\epsilon}\, d\epsilon = \pi \mathrm{I}_0(z) \quad \text{and} \quad \int_0^\pi \cos\epsilon\, e^{z\cos\epsilon}\, d\epsilon = \pi \mathrm{I}_1(z). \quad . \quad . \quad . \quad . \quad (14)$$

Making use of the relation(8) $\mathrm{I}_{n+1}\mathrm{K}_n - \mathrm{I}_n\mathrm{K}_{n+1} = (1/z)\cos n\pi$ for $n = 0$, we obtain finally $\mathrm{Q} = 2\mathrm{A}\kappa\pi$, and thus for the temperature at any point the expression

$$\theta = (\mathrm{Q}/2\pi\kappa)\, e^{nx}\mathrm{K}_0(nr), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

which is H. A. WILSON'S(9) solution for a line source.

Let $u(\xi)\, d\xi$ represent the total flux of heat from an elementary portion $d\xi$ of the x-axis between $x = 0$ and $x = \beta_0$. The contribution of this element to the temperature at any point is given by (15) in the form

$$d\theta = (1/2\pi\kappa)\, u(\xi)\, d\xi\, e^{n(x-\xi)}\mathrm{K}_0 | n\sqrt{[y^2+(x-\xi)^2]} | .$$

Since equation (5) is linear the temperature due to a distribution of line-sources along the x-axis between $x = 0$ and $x = \beta_0$ is obtained by integration,

$$\theta(x, y) = (1/2\pi\kappa)\int_0^{\beta_0} u(\xi)\, e^{n(x-\xi)}\mathrm{K}_0 | n\sqrt{[y^2+(x-\xi)^2]} |\, d\xi. \quad . \quad . \quad . \quad (16)$$

The boundary condition over $y = 0$ is expressed by the relation

$$2\pi\kappa\theta(x) = \int_0^{\beta_0} u(\xi)\, e^{n(x-\xi)}\mathrm{K}_0 | n(x-\xi) |\, d\xi. \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

If the temperature is prescribed over the boundary, equation (17) constitutes an *integral equation* for the determination of $u(\xi)$; if the flux of heat $u(\xi)$ is prescribed over the boundary from $x = 0$ to $x = \beta_0$ the same equation gives the temperature of the stream in contact with the boundary. In either case the total heat-loss of the cylinder per unit length is given by

$$\mathrm{H} = \int_0^{\beta_0} u(\xi)\, d\xi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

and the temperature at any point by (16).

Section 4. Calculation of Heat-Loss under the Hypothesis of Constant Flux over the Boundary.

The solution of the problem in hand which gives results in best agreement with experiment for the case of convection of heat from small cylinders is that obtained by

(8) GRAY and MATTHEWS, *loc. cit.*, p. 68.

(9) H. A. WILSON, "On Convection of Heat," 'Proc. Camb. Phil. Soc.,' 12, p. 413, 1904.

assuming the flux $u(\xi)$ to be constant over the boundary. From (18) we have, writing $u(\xi) = u_0$,

$$\mathrm{H} = u_0 \beta_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

As a result of the high heat-conductivity of the cylinder in the experiments carried out the temperature θ_0 of the cylinder may be considered constant over its entire boundary, an assumption justified by a calculation carried out in Section 13. There will therefore be a discontinuity in the temperature over the boundary; we assume that the temperature of the stream in contact with the cylinder becomes finally equal to that of the cylinder at the point β_0 where it leaves the boundary. The integral in (17) must be divided into two parts in order to make the argument $n(x-\xi)$ in the function $\mathrm{K}_0 |n(x-\xi)|$ positive; we then obtain

$$2\pi\kappa\theta(x) = u_0 \left[\int_0^x e^{n(x-\xi)} \mathrm{K}_0 \{n(x-\xi)\}\, d\xi + \int_x^{\beta_0} e^{n(\xi-x)} \mathrm{K}_0 \{n(\xi-x)\}\, d\xi \right].$$

Making the substitution $u = n(x-\xi)$ in the first integral and $v = n(\xi - x)$ in the second, we may write the above equation in the form

$$2\pi\kappa\theta(x) = (u_0/n) \left[\int_0^{nx} e^u \mathrm{K}_0(u)\, du + \int_0^{n(\beta_0 - x)} e^{-v} \mathrm{K}_0(v)\, dv \right]. \quad . \quad . \quad . \quad (20)$$

Writing $\theta = \theta_0$ when $x = \beta_0$ in the above equation and making use of (19) we find for the heat-loss of the cylinder per unit length the expression

$$\mathrm{H} = 2\pi\kappa\theta_0 n\beta_0 \Big/ \left[\int_0^{n\beta_0} e^u \mathrm{K}_0(u)\, du \right], \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

where θ_0 denotes the temperature of the cylinder above that of the stream at a great distance. The discontinuity of temperature occurring over the boundary may easily be calculated at any point from (20); the maximum discontinuity occurring at $x = 0$ is easily seen to be given by

$$\theta_1/\theta_0 = \left[\int_0^{n\beta_0} e^{-u} \mathrm{K}_0(u)\, du \right] \Big/ \left[\int_0^{n\beta_0} e^u \mathrm{K}_0(u)\, du \right]. \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Section 5. *Numerical Evaluation of the Functions Employed in the Preceding Section.*

Before we can proceed to evaluate the functions occurring in the preceding section we proceed to derive the convergent and asymptotic expansions of the function $\mathrm{F}(x) = \int_0^x e^u \mathrm{K}_0(u)\, du$. Expanding e^u in powers of u, and making use of the expansion (10), we obtain on integrating term by term the convergent series applicable for small values of the variable x,

$$(1/x) \int_0^x e^u \mathrm{K}_0(u)\, du = (1 + \tfrac{1}{4}x + \tfrac{1}{6}x^2 + \tfrac{17}{168}x^3 + \ldots) - (\gamma + \log \tfrac{1}{2}x)(1 + \tfrac{1}{2}x + \tfrac{1}{4}x^2 + \tfrac{5}{48}x^3 + \ldots). \quad (23)$$

Integrating term by term the asymptotic expansion (9), we obtain

$$\int_0^x e^u \mathrm{K}_0(u)\,du+1 = \sqrt{(2\pi x)}\left[1+\frac{1}{8x}-\frac{1^2.3}{(8x)^2.2!}+\frac{1^2.3^2.5}{(8x)^3.3!}-\ldots\right]. \quad . \quad . \quad (24)$$

The term 1 on the left-hand side represents a term of integration arising from the lower limit of the integral, which is proved by actual numerical calculation to be unity.

In order to evaluate $\mathrm{F}(x)$ numerically over the interval where neither (23) or (24) are convergent, use is made of EULER's formula for the quadrature of the function $y=f(x)$ ([10])

$$\int_{x_0}^{x_n} y\,dx = h\left(\tfrac{1}{2}y_0+y_1+y_2+\ldots\tfrac{1}{2}y_n\right)-\tfrac{1}{12}h^2\left[f'(x_n)-f'(x_0)\right]$$
$$+\tfrac{1}{720}h^4\left[f'''(x_n)-f'''(x_0)\right]-\ldots, \quad . \quad . \quad . \quad . \quad (25)$$

where h is the interval between the successive values of x, that is, $h=(x_n-x_0)/n$. From a table ([11]) of the BESSEL's functions $\mathrm{K}_0(x)$ and $\mathrm{K}_1(x)$, the values of $e^x\mathrm{K}_0(x)$ and $e^x\mathrm{K}_1(x)$ were tabulated over various ranges of equidistant intervals from $x=0{\cdot}1$ to $x=6{\cdot}0$. Numerical values of $\mathrm{F}(x)$ for values of $x<0{\cdot}1$ were easily calculated from the convergent formula (23). Beyond this point the integral between the limits $x=0{\cdot}1$ and $x=x_n$ was calculated by the use of EULER's formula (25), making use of a calculating machine for the purpose in such a way that the various entries were recorded as successive values of a single series of operations. Beyond $x=6{\cdot}0$ the function was evaluated from (24), the constant of integration proving to be unity. The functions $e^x\mathrm{K}_0(x)$, $e^x\mathrm{K}_1(x)$, $\int_0^x e^x\mathrm{K}_0(x)\,dx$ and $x\Big/\left[\int_0^x e^x\mathrm{K}_0(x)\,dx\right]$ are tabulated in Table I. together with a more detailed description of the method of computation; a graph of the last function is also given.

Section 6. *Approximate Formulæ for the Heat-Loss.*

When the variable x is small, equation (23) enables us to write

$$y = x\Big/\left[\int_0^x e^x\mathrm{K}_0(x)\,dx\right] \quad \text{in the form} \quad y = 1/[(1-\gamma)-\log\tfrac{1}{2}x]. \quad . \quad . \quad (26)$$

Hence when the variable $n\beta_0$ is so small that it may be neglected in comparison with $\log(1/n\beta_0)$ the expression (21) for the heat-loss may be written in the form

$$\mathrm{H} = 2\pi\kappa\theta_0/[\log(4b/\beta_0)], \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

where

$$c = s\sigma \text{ and } b = e^{1-\gamma}/(2n) = \kappa e^{1-\gamma}/(s\sigma\mathrm{V}). \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

([10]) EULER, 'Comm. Acad. Sci. Imp. Petrop.,' vi. (1732–33).

([11]) JAHNKE and EMDE, 'Funktionentafeln' (Teubner's, 1909), p. 135. The functions $(\pi i/2)\,\mathrm{H}_0{}^1(ix)$ and $-(\pi/2)\,\mathrm{H}_1{}^1(ix)$ of the above tables are here denoted by $\mathrm{K}_0(x)$ and $\mathrm{K}_1(x)$ respectively, and their computation is due to W. S. ALDIS ('Roy. Soc. Proc.,' vol. 64, p. 219, 1898).

When the variable x is large, equation (24) enables us to write approximately

$$y = 1/(2\pi)+\sqrt{x/(2\pi)}. \qquad (29)$$

Thus when the term $n\beta_0$ is sufficiently large and the term $1/(8n\beta_0)$ is small in comparison with unity, equation (21) takes the form

$$\mathrm{H} = \kappa\theta_0+\kappa\theta_0\sqrt{2\pi n\beta_0} = \kappa\theta_0+\sqrt{\pi\kappa s\sigma\beta_0}\,\mathrm{V}^{\frac{1}{2}}\theta_0. \qquad (30)$$

Expressions (27) and (30) hold for cylinders of any shape for which the constant β_0 can be calculated from the hydrodynamical problem. It is easily proved in the case of an elliptic cylinder of semi-axes (a, b) that

$$\beta_0 = 2\,(a+b). \qquad (31)$$

This result holds good independently of the direction of the axes of the cross-section with that of the stream and may be utilized in the special cases of calculating the heat-loss from a circular cylinder or a strip of breadth $2a$. The case of a circular cylinder of radius a represents the conditions of the experiments described in Part II. of the present paper; writing $\beta_0 = 4a$ we obtain finally the approximate formulæ [12] for the heat-loss

Small velocities . . . $\mathrm{H} = 2\pi\kappa\theta_0/[\log(b/a)]$, ... (32)

Large velocities . . . $\mathrm{H} = \kappa\theta_0+2\sqrt{\pi\kappa s\sigma a}\,\mathrm{V}^{\frac{1}{2}}\theta_0$. . . . . (33)

The limits within which these approximate formulæ represent the values of the heat-loss given by the exact expression (21) is examined in the description of Diagram I.

[12] It is interesting to compare (32) with LANGMUIR'S formula for free convection,

$$\mathrm{H} = 2\pi\,(\phi_2-\phi_1)/[\log b/a],$$

where ϕ denotes a function of the thermal conductivity and the temperature given by the relation $\phi = \int_0^\theta \kappa\, d\theta$. This result has been shown by LANGMUIR to represent with fair accuracy the results of his experiments on the free convection from small platinum wires ('Phys. Rev.,' 34, p. 401, 1912). Interpreted in the light of equation (32) the term b of LANGMUIR'S equation represents a term depending on the "effective" velocity $\bar{\mathrm{V}}$ of the free convection current set up by the heated wire.

Equation (33) may be compared with that derived by KENNELLY ('Trans. A.I.E.E.,' 26, p. 969, 1907, and 'Trans. A.I.E.E.,' 28, p. 368, 1909) as a result of his experiments on the forced convection of heat from small copper wires. In the notation of the present paper KENNELLY'S formula may be written in the form

$$\mathrm{H} = (\mathrm{C}+\mathrm{B}d)\,\theta_0\sqrt{\mathrm{V}+v_0},$$

where C and B are constants given by $\mathrm{C} = 300\times10^{-7}$, $\mathrm{B} = 5{\cdot}8\times10^{-8}$, the heat-loss H being measured in watts per unit length, the temperature difference θ_0 being expressed in degrees C. and the velocity V in cm./sec. v_0 is a constant whose value is $v_0 = 25$ cm./sec., and d is the diameter of the wire in cm. It must be noticed that in deriving the above formula no correction was made in the experiments for the "swirl" of the rotating arm in the determination of the velocity.

It is shown that when the cooling stream is air at ordinary pressure, there exists a value of the product Vd (V expressed in cm./sec. and the diameter d in cm.) given by $Vd = 0{\cdot}0187$ which discriminates between the two formulæ: when $Vd < 0{\cdot}0187$ (32) is appropriate to the problem, while for $Vd > 0{\cdot}0187$ (33) must be employed. In practically all cases except for very small wires and extremely low velocities the condition $Vd > 0{\cdot}0187$ is satisfied and equation (33) expresses with sufficient accuracy nearly all applications of the formula.

Section 7. Note on the Interpretation and Application of the Preceding Theory.

That the expression for the heat-loss from a cylinder cooled by a stream of fluid derived in the preceding sections and leading to equation (21) may be identified with reality involves many delicate considerations as to the nature of the boundary condition over the surface of the cylinder. A comparison with the results of experiment described in Section 13 gives strong support to the validity of this formula and seems to justify the boundary condition of constant flux by means of which it was derived. Looking at the matter from the point of view of the kinetic theory of gases, temperature conditions cannot strictly be defined in the immediate neighbourhood of the heated cylinder, and it is only at a distance of several free paths when equipartition is nearly complete that we may define temperature and that normal thermal conduction takes place. The boundary condition of constant flux is one which must look for its explanation in the light of the kinetic theory, making use of the precise knowledge which is now coming to hand regarding the nature of molecular impacts on solid boundaries from recent investigations on the properties of ultra-rarefied gases.([13]). This aspect of the question must, however, be left over for future discussion. That there should exist a discontinuity of temperature between the stream and the cylinder at ordinary pressures is not surprising; this discontinuity was considered possible by POISSON and is now known to exist in the case of rarefied gases.([14])

The solution of the present problem on the conduction of heat in moving media is only a particular case which may be applied to many other problems for which the same formal expression of the physical conditions holds; for instance, the results apply *mutatis mutandis* to the problem of diffusion or evaporation from liquid surfaces into streams of gases flowing over them. It must be kept in mind, however, that in each type of problem to which this analysis may be applied, the nature of the boundary condition must in each case be referred to a comparison with the results of experiment.

([13]) A summary of recent work on the subject is given by KNUDSEN ('La Théorie du Rayonnement et les Quanta,' Gauthier-Villars, Paris, 1912, p. 133), and more recently by DUNOYER ('Les Idées Modernes sur la Constitution de la Matière,' Gauthier-Villars, Paris, 1913, p. 215 *et seq.*).

([14]) KUNDT and WARBURG, 'Pogg. Ann.,' vol. 156, 1875, p. 177; also SMOLUCHOWSKI, 'Wied. Ann.,' 64, 1898, p. 101.

PART II.

Experimental Determination of the Convection Constants of Small Platinum Wires.

Section 8. *Introduction.*

The experimental study of thermal losses from heated bodies under various conditions dates back to the classical researches of Dulong and Petit[15] in 1817. Since that date numerous experiments have been carried out on the radiation and convection of heat, most of which fall under two categories:—

(i.) The study of the total heat losses from a heated body to an enclosure maintained at constant temperature and containing different gases under various conditions of pressure. In so far as these experiments refer to the heat losses from wires, we may cite the classical work of Ayrton and Kilgour[16] and that of Petavel.[17] More recently we may quote the experiments of Kennelly[18] on the forced convection of heat from small copper wires, and to a series of detailed papers recently published by Langmuir[19], where exhaustive references to experiments on convection problems are to be found.

(ii.) The study of radiation losses from heated solids to enclosures maintained at constant temperature, in which connection the earliest measurements carried out by an electrical method appear to have been due to Bottomley.[20] In high vacua this subject has in recent years been made the field of much research, especially in connection with the development of the metallic filament lamp. In the present work the heat-loss by radiation plays a very subordinate part and was not made the subject of special investigation.

The measurement of thermal losses in stagnant media, while simpler to carry out experimentally, and perhaps more important in practical applications, has thus far defied mathematical investigation on a rational physical basis. On the other hand, the mathematical interpretation of heat convection as a problem of heat conduction in moving media admits of fewer restrictions and leads to the results of Boussinesq[21]

(15) Dulong et Petit, 'Ann. de Chimie et de Physique,' t. 7, 1817.

(16) Ayrton and Kilgour, "The Thermal Emissivity of Thin Wires in Air," 'Phil. Trans.,' vol. 183, Part I., p. 371, 1892.

(17) Petavel, 'Phil. Trans.,' vol. 191, p. 501, 1898; also 'Phil. Trans.,' vol. 197, p. 229, 1901.

(18) Kennelly, Wright, and Van Bylevelt, 'Trans. A.I.E.E.,' vol. 26, p. 969, 1907; also 'Trans. A.I.E.E.,' vol. 28, pp. 363–396, 1909.

(19) Langmuir, 'Phys. Rev.,' vol. 34, p. 401, 1912; also 'Proc. A.I.E.E.,' June, 1912, p. 1011; 'Proc. A.I.E.E.,' April, 1913.

(20) Bottomley, 'Phil. Trans.,' A, vol. 178, p. 429, 1888; 'Roy. Soc. Proc.,' vol. 66, p. 269, 1900.

(21) Boussinesq, *loc. cit.*, p. 297.

and to the more general formulæ developed in the preceding sections. It is not difficult to understand from a physical point of view that the results of experiments on forced convection should be simpler in their interpretation in that the disturbance due to the free convection set up by the heated wire may be neglected in comparison with the impressed velocity provided the latter be sufficiently large. The first experimenter to have shown that the convection loss in a current of air is proportional to the temperature difference and to the square root of the velocity seems to have been SER(22), whose measurements on the variation of heat-loss with extent of surface indicate approximations to the theoretical formulæ. KENNELLY'S(23) observations led to an empirical formula closely resembling that finally derived in Section 6.

The experiments of COMPAN(24) on the cooling of spheres in air currents have verified BOUSSINESQ'S approximate equation between comparatively narrow limits of temperature and air-velocity. While the present investigation was in progress, the work of MORRIS(25) was published, verifying the application of a formula of the type obtained by BOUSSINESQ to the cooling of fine wires heated by an electric current to temperatures of about 70° C. above the surrounding air and for air-velocities as high as 40 miles an hour.

The present investigation was undertaken with two purposes in view:—

(i.) To study the laws of convection of heat from small platinum wires heated by an electric current over as wide a range as possible of temperature, air-velocity, and diameter in the light of the formulæ developed in the preceding sections and to obtain in absolute measure the convection constants of such wires.

(ii.) To make use of the constants thus obtained in the design of accurate and portable wind-measuring apparatus to form the basis of a standard system of anemometry, as well as to serve for use in a great variety of engineering and aerotechnical problems.

Section 9. *Experimental Arrangements.*

(i.) *General.*

The general arrangement of apparatus necessary to carry out the requisite measurements of heat losses from a series of platinum wires, of diameters 1 to 6 mils, consisted of a rotating arm capable of adjustment to any speed as calculated from a

(22) SER, 'Traité de Physique industrielle,' t. I., pp. 142–162, 1888. The paper is briefly abstracted by BOUSSINESQ, *loc. cit.*, p. 290.

(23) KENNELLY, 'Trans. A.I.E.E.,' 28, pp. 363–397, June, 1909. The formula referred to is (28), p. 388.

(24) COMPAN, 'Ann. de Chimie et de Physique,' 26, p. 488, 1902.

(25) MORRIS, "The Electrical Measurement of Wind Velocity," 'Electrician,' October 4, 1912, p. 1056. Paper read at the British Association Meeting, Dundee, September, 1912. See also 'Electrician,' October 4, 1912, p. 1056; 'Engineer,' September 27, 1912.

chronograph record. At various lengths along this arm could be clamped a light fork designed to hold the specimens of wire under test. The latter formed part of a Kelvin double bridge, electrical connection being obtained through a central mercury connecting switch and overhead wires to the remainder of the bridge. By means of a rheostat it was possible at each speed to adjust a measured current through the wire so as to bring its resistance to a value corresponding to a predetermined temperature. In this way it was possible in the case of each wire to vary in any chosen way the various factors of temperature, air-velocity and heat-loss. The general arrangement of apparatus and details are drawn in Diagram II. and shown photographically in Plate 8.

(ii.) *Details of Resistance Bridge.*

In order to measure the temperature of a length of platinum wire heated by an electric current under given conditions of wind-velocity, it is necessary to design a form of resistance-bridge suitable for an accurate determination under these conditions. The well-known connections of the Kelvin double bridge at once recommend themselves for the purpose and are shown diagrammatically in fig. 2. A represents the platinum wire whose resistance it is required to measure. B is a ten-metre bridge wire of No. 23 S.W.G. manganin wire. Connecting A and B is a resistance δ including that of the ammeter and leads to the fork on the rotating arm. The ratio-resistances a, b, α, β were in the neighbourhood of 10^5 ohms, so that the current in the wire A is to a very close approximation that indicated by the ammeter. A and B were connected through an adjustable rheostat to the 110-volt mains. The following formulæ are given for future reference. When the galvanometer is balanced it is not difficult to prove that

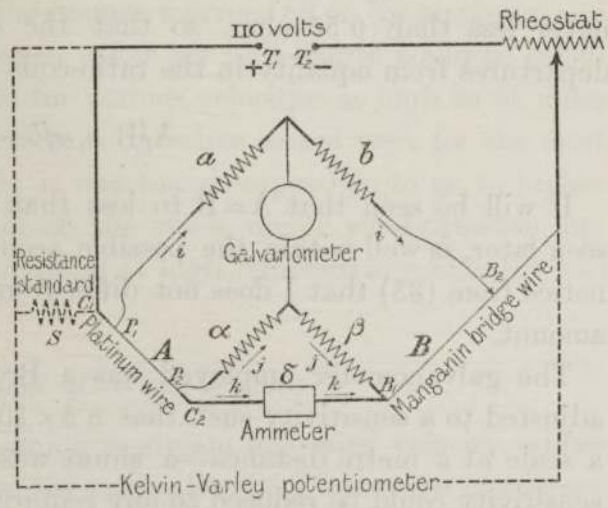


Fig. 2. Kelvin bridge connections.

$$\frac{\mathrm{A}}{\mathrm{B}} = \frac{a}{b} + \frac{\beta}{\mathrm{B}} \frac{\delta}{\alpha+\beta+\delta}\left(\frac{a}{b} - \frac{\alpha}{\beta}\right). \quad \quad (34)$$

Also if I be the current in the platinum wire, k that in the ammeter, i that through the coils (a, b), and j that through (α, β), we have

$$\mathrm{I} = k\left[1+\delta/(\alpha+\beta)\right], \qquad j = k\,.\,\delta/(\alpha+\beta), \qquad i = \mathrm{I}\left[\mathrm{A}/a + \alpha/a\,.\,\delta/(\alpha+\beta+\delta)\right]. \quad (35)$$

We notice from (34) that when the ratio coils are so adjusted that $a/b = \alpha/\beta$, then,

$A/B = a/b = \alpha/\beta$ independently of any connecting- or contact-resistance in the bridge. It is this characteristic property of the Kelvin bridge which makes it especially applicable to the present series of measurements.

In the actual apparatus the wire A to be tested is mounted in a specially constructed fork which is rotated at the extremity of a long revolving arm. The main circuit and the potential terminals P_1P_2 are carried through four mercury-contact slip-rings to the overhead wires and thence to the bridge.

In the actual apparatus the ratio coils had nearly equal resistances of the value 10^5 ohms; when compared by means of a Kelvin-Varley slide they were found to have the following ratios:—

α.	β.	a.	b.
24,984	24,986	25,020	25,010

making the correction factor $(a/b - \alpha/\beta) = 3/2500$.

The connecting resistance δ was found to be about 0·55 ohm; the resistance B was never less than 0·50 ohm, so that the correction in equation (34) due to small departures from equality in the ratio-coils is given by

$$A/B = a/b + \delta/B \,.\, 3/2500. \qquad (36)$$

It will be seen that A=B to less than one-tenth of 1 per cent., which, as will be seen later, is well within the possible accuracy of the present experiment. We also notice from (35) that I does not differ from the ammeter current k by an appreciable amount.

The galvanometer employed was a Broca instrument of resistance 97 ohms and adjusted to a sensitivity such that $8{\cdot}5 \times 10^{-10}$ amperes gave a deflection of 1 mm. on a scale at a metre distance—a shunt was used with the galvanometer so that the sensitivity could be reduced to any required fraction of this amount.

The ammeter was a direct-reading Weston instrument, consisting of milli-voltmeter and shunt, by means of which ranges 0·2, 2·0, 20 amperes were available; these ranges could be further subdivided by the insertion of a suitable resistance in series in the milli-voltmeter circuit. The instrument was calibrated against a Weston laboratory standard and was found to read correctly to within $\frac{1}{4}$ of 1 per cent., which represents the order of accuracy aimed at through the experiment.

The procedure carried out in order to obtain a set of readings was as follows: the wire under test was placed in the fork and potential terminals of fine platinum wire (about 1 mil diameter) fused on the wire at a measured distance apart. A 2-volt cell was inserted at T_1T_2 and a Kelvin-Varley slide placed in parallel with the circuit $SP_1P_2B_1B_2$, S being a standard resistance connected in series with the circuit A, B so that the resistance of the wire in ohms might be obtained. By using a very small measuring current (about 0·05 amperes) and the galvanometer at full sensitivity the resistance of the wire between potential terminals at standard room temperature

(17° C.) was obtained, care being taken to test for the absence of any appreciable heating effect of the measuring current since this may become quite noticeable in the case of the very fine wires. Points were then determined on the bridge-wire corresponding to a series of predetermined ratios R/R_{17}, each of these representing when balanced against the heated platinum wire a certain temperature as calculated from the constants of the wire. The series of temperatures employed was as far as possible kept the same for all the wires. In order to measure the heat-loss, the standard resistance and the Kelvin-Varley slide were cut out of the circuit, and the terminals T_1T_2 connected to the 110-volt mains through rheostats allowing of the continuous adjustment of current from zero to about 5 amperes. The rotating arm was then adjusted to a fixed speed, and the current set to such a value that a balance was obtained on the galvanometer (shunted to $\frac{1}{1000}$ of its sensitivity) when the adjustable contact B_2 was set on each of the points of the bridge-wire previously determined to represent certain temperatures. Readings were also taken with the apparatus at a standstill, and the wire in three positions, horizontal, vertical, and inclined at an angle of 45 degrees to the vertical. At each point the current was read off on the ammeter, and from the resistance of the wire the heat-loss in watts per unit length could be calculated. This series of readings was repeated for various velocities as high as 25 miles an hour. In order to eliminate the end correction the wires tested were for the most part of considerable length (about 23 cm.); it was found impossible to go to higher speeds owing to the sagging and vibration of the wires under wind-pressure and centrifugal force with consequent risk of breaking at high temperatures and the loss of a set of observations.

(iii.) *On the Measurement of Wind-velocity.*

The simplest laboratory method of obtaining a stream-line wind-velocity, whose value is known directly without reference to the calibration of Pitot-tubes or anemometers, is realized by the use of a whirling table to which is attached a light arm at the extremity of which the object to be experimented upon is attached. The advantage of this method is, however, more apparent than real. It is well-known that in such a disposition of apparatus a vortex is created in the neighbourhood of the rotating arm, so that the velocity of a point at any radius relative to the room does not represent its velocity relative to the air. In addition to the difficulty mentioned results obtained from air-velocities measured from motion in a circle cannot in some classes of work be applied with safety to linear motion. In fact so serious have these objections proved themselves to be that the method of obtaining velocity by means of a rotating arm has been abandoned in meteorological work and in aeronautical problems.([26])

([26]) These difficulties are described in detail by FRY and TYNDALL in a paper "On the Value of the Pitot Constant," 'Phil. Mag.,' 21, p. 352, 1911; also in the 'Report of the Advisory Committee for Aeronautics,' 1909–10, p. 15; 1910–11, 'Report,' No. 34, p. 50, by Messrs. BRAMWELL and SILLICK.

For most purposes a specially constructed wind-tunnel is employed in the production of wind-velocity. In order to eliminate the effect of the walls and to produce a uniform velocity over a considerable cross-section, a large size of tunnel is necessary, requiring at high velocities a very large delivery of air obtainable only from massive and expensive equipment. Moreover, it is necessary for the actual measurement of wind-velocity to depend on a Pitot-tube or other form of anemometer; in addition it is difficult to secure stream-line motion at high velocities.

(iv.) *On the Determination of the Correction Factor in the Measurement of Velocity.*

For the purposes of the present experiment the measurement of velocity by means of a rotating arm recommends itself as the simplest and the most direct. With the wire held parallel to the axis of rotation, the objection to circular motion does not hold, while the true velocity at any point on the arm relative to the air can be determined as follows:—

We denote by V_r the velocity of a point at radius r on the rotating arm relative to the room (apparent velocity), by V the true velocity relative to the air, and by v the velocity of the vortex at radius r relative to the room. Observation then shows that at the same radius the true velocity is proportional to the apparent velocity consistently with an expression of the form

$$V = V_r - v = (1-s)\,V_r. \qquad (37)$$

The constant s may conveniently be called the *swirl* and expressed in percentages of the apparent velocity. The quantity $v = sV_r$ represents the velocity at radius r of the vortex set up by the rotating arm. In order to measure this velocity one of the wires is placed in the fork of the rotating arm, and a series of currents required to heat up the wire to a determinate resistance for various velocities measured. The relation between the current and the apparent velocity is found to be of the form

$$i^2 = i_0^2 + \kappa_r V_r^{\frac{1}{2}}, \qquad (38)$$

and the constants i_0^2 and κ_r were determined by observation. The *same* wire was then mounted in another fork fixed relatively to the room and placed in such a position that the rotating fork passed within a centimetre or less from the fixed wire. The whirling table was then set into motion and the current required to bring the resistance to the same value for each of a series of velocities measured. In this way the apparent velocity of the vortex v_r was calculated from (38). It was found that the apparent velocity of the arm V_r was connected with v_r by a linear relation $v_r = s_r V_r$, s_r now denoting the *apparent swirl*. Making use of (37) we have the relations $v_r = v/(1-s) = sV_r/(1-s) = s_r V_r$ giving $s_r = s/(1-s)$ and finally $1-s = 1/(1+s_r)$ so that

$$V = V_r/(1+s_r), \qquad (39)$$

expressing the true velocity in terms of the apparent velocity and the apparent swirl, both easily measurable quantities. Details of measurements by means of which the swirl was obtained for the three positions on the rotating arm employed in the present experiment are given briefly under the description of Diagram II. We note in passing that the value of the swirl will depend on the construction and disposition of the apparatus as well as on the dimensions and shape of the room in which the rotating arm is set up; in the present case the swirl amounted to about 5 per cent. and could be determined to within 5 per cent., so that the true velocity of the wires relatively to the air of the room could be obtained to an accuracy well within $\frac{1}{4}$ of a per cent. In this manner the simplicity and ease of working of the rotating arm can be employed without loss of accuracy.(27)

(v.) *Note on the Construction of a Constant Low Resistance Set of Mercury Contact Slip-rings.*

In order that the resistance of the wire between potential terminals may be measured with accuracy at the extremity of the rotating arm, it is necessary to provide some form of slip-ring; since the contacts enter into the connections of the bridge, it is important that the contact resistances be as small as possible and remain constant while the apparatus is in rotation. A specially designed set of four slip-rings which fulfils these requirements is shown in (*b*) Diagram II., while the specifications are given under the accompanying description. The insertion of this set of slip-rings in the circuit $P_1P_2B_1B_2$ of fig. 2, increased the resistance by the amount 0·0077 ohm. only and was found to remain extremely constant. No change in the balance of the bridge when connected to measure the resistance of the wire at room temperature could be detected when the rotating arm was set into motion at various speeds.

(vi.) *Note on the Fork Carrying the Wire to be Tested and on the Whirling Table.*

The rotating arm consisted of a light strip of pine (6·3 × 1·3 cm.) about 2·6 m. long fastened to a whirling table in the manner shown in Diagram II. and illustrated by Plate 8. At three determinate positions on this arm could be clamped the fork for carrying the wires. This fork was designed to disturb as little as possible the flow of air past the wire, and is illustrated in detail in (*c*) Diagram II. In order to eliminate the cooling effect of the terminals and their disturbance on the flow of air past the ends of the wires, potential leads of extremely fine platinum wire (1 mil diameter) were fastened at the points P_1P_2 at distances of about 2 cm. from the ends C_1C_2. These potential leads were most easily fixed in position by heating the wire to

(27) The results here described and the method of correction for the swirl agree with the observations of Bramwell and Sillick (footnote (12)), who found nearly the same value for the ratio $s = v/V_r$. The correction is now regularly made in tests carried out on a rotating arm.

a bright red when the fine platinum wires could be easily fused in their proper places; this preliminary heating also served to anneal the wire before its resistance was determined. A small weight, attached by means of a silk thread to the lower current terminal C_2, served to keep the wire under slight tension while the distance between the potential terminals was accurately measured.

The whirling table was operated through suitable reducing gear by means of a $\frac{1}{4}$ h.p., 1200 r.p.m., 110-volt D.C. motor, with speed varied by means of an adjustable resistance in series with the armature, controlled by a rheostat conveniently situated near the bridge. The field of the motor was separately excited: in order to obtain constant speed it was found necessary to excite the field some time before commencing a series of observations in order that a steady temperature and therefore steady resistance in the field-coils shall have been attained. Another advantage is the elimination of disturbing effects on the galvanometer as the motor is stopped and started. A timing device attached to the driving shaft of the whirling table caused a contact to be made every tenth revolution of the rotating arm and to be registered on a chronograph simultaneously with the beats of a seconds pendulum. An examination of the chronograph sheet showed that with the above precautions the speed remained constant to about one-tenth of 1 per cent., well within the range of accuracy of the present observations.

Section 10.

(i.) *On the Constants of the Platinum Wires under Test.*

The most difficult measurement in the present study of heat convection and that which imposes a limit to the accuracy attainable is the calculation of the temperature of the wire from the change of resistance. In fact this point confines the present experiment to the use of platinum wires whose constants in the measurement of high temperatures are well known. It must be kept in mind, however, that the constants of a platinum wire which has been kept for some time at a high temperature for a considerable time as in the present experiments are liable to change. This source of uncertainty is aggravated in the case of very fine wires where the additional difficulty of a sensible change of resistance due to "*evaporation*" from the wire is to be met with.(28) Evidence on both these points will be noticed from an inspection of Table II., and the limit of accuracy due to these and other difficulties may be roughly set at $\frac{1}{4}$ of 1 per cent.

The wires employed in the present experiment were drawn through diamond dies(29) from a length of 6 mil pure platinum wire whose constants when used in platinum

(28) See BURGESS and LE CHATELIER, 'The Measurement of High Temperatures' (Wiley and Sons, New York, 1912), p. 232; also p. 196.

(29) The writer is indebted to the Imperial Wire and Cable Co., of Montreal, for the use of diamond dies employed in drawing down the wires.

thermometry were given (in the usual notation) as $\alpha = 0{\cdot}00388$, $\delta = 1{\cdot}54$. It was hoped that these constants would remain unchanged by drawing down the wire, so that by keeping the series of ratios in the bridge adjustment the same for all the specimens, the corresponding temperatures would only have to be calculated once. On testing some of the wires after they had been heated to high temperatures, it was found that the temperature coefficients varied appreciably with the size of the wires and that they differed materially from the rated value. The temperature coefficient was therefore separately determined for each wire by comparing its resistance at the ice- and steam-points with that of a manganin standard, making use of a potentiometer method for which the Kelvin-Varley slide was employed. In this way the resistance between potential terminals could be expressed in ohms, and the resistance at any temperature calculated. The results are given in Table II. and are explained in further detail in the accompanying description. It will be shown later that a considerable change in the value of δ does not affect the calculation of temperature beyond the limit of other experimental errors; the value of δ was therefore not re-determined, and the value given above was taken. A length of 3 mil wire drawn down from the original specimen was sent to a firm to be drawn down to 2·0, 1·5, and 1·0 mil. It will be noticed from Table II. that the temperature-coefficients of these extremely fine wires are so small that it is improbable that they are drawings from the specimen sent out. These fine wires were found extremely difficult to work with and could only be tested in comparatively short lengths of 3 to 4 cm.; they often broke while being heated to a dull redness for annealing so that it was not possible to measure the convection losses over as great a range of temperature as in the case of the larger wires. The friability of the wires under heating was probably due to their being drawn from a specimen of impure platinum.

(ii.) *On the Measurement of Diameters and Lengths.*

The measurement of the diameters of the wires to a fraction of a per cent. requires an estimation of length to about 10^{-5} cm. The direct measurement of the diameter by means of a high-power microscope was ultimately found to be the most satisfactory method.[30] The wires were mounted on a microscope slide and the diameters measured at centimetre intervals by means of a micrometer screw in the eye-piece. The readings of the micrometer screw were afterwards calibrated in centimetres by means of a Bausch and Lomb stage micrometer, care being taken to calibrate the eye-piece micrometer screw over the various portions of the field employed in measuring the

(30) The microscope employed was a Zeiss 1E, with an 8 mm. apochromatic objective fitted with a screw micrometer and × 6 compensating ocular; the total magnifying power was 187. The writer was indebted to Prof. J. C. Simpson, of the Department of Histology of McGill University, for the loan of this microscope.

wire in order to avoid errors due to optical distortion. It was found possible to reset the cross-hairs consistently to each division of the micrometer head, *i.e.*, to about 10^{-5} cm. The results are tabulated in Table II. and further details are given in the accompanying description.

The measurement of the lengths of wire between potential terminals offered no difficulties. In order to calculate the heat-loss per unit length at any temperature it was necessary to correct for expansion with temperature by means of the formula $l_\theta = l_0(1+\alpha\theta+\beta\theta^2)$, where l_θ is the length at θ° C., l_0 that at 0° C., and α and β the coefficients of linear expansion given by HOLBORN and DAY (31) as $\alpha = 0{\cdot}08868 \times 10^{-4}$, $\beta = 0{\cdot}001324 \times 10^{-6}$.

Section 11. *On the Calculation of Temperatures.*

From the mean temperature-coefficient of resistance c of platinum between 0° C. and 100° C. given by $c = (R_{100}-R_0)/100R_0$, the temperature of the wires were calculated from the formula $\theta_p = (R/R_0-1)/c$, R being the resistance at the temperature θ_p on the platinum scale and R_0 that at 0° C. In the present instance the resistances of the platinum wires were referred to the temperature $t = 17^\circ$ C., so that the ratio R/R_{17} are those from which the temperatures are to be calculated. We may take very approximately $R_t = R_0(1+ct)$ for $t = 17^\circ$ C., in which case it is not difficult to prove that $\theta_p - t = (R/R_t-1)(1/c+t)$ by means of which temperatures on the platinum scale were calculated. The temperatures of the wires were then obtained in terms of the true thermodynamic scale by means of CALLENDAR's formula,

$$\theta^\circ \text{ C.} = \theta_p + \delta(t/100-1) \, . \, t/100 \text{ with } \delta = 1{\cdot}54.$$

Actual calculation of temperatures was avoided by the use of the conversion tables given by BURGESS and LE CHATELIER (32) calculated for $\delta = 1{\cdot}50$. From an auxiliary table the temperature correction for a value of δ other than this value is given; it is seen that the correction $\Delta\theta$ for $\Delta\delta = 0{\cdot}01$ is $\Delta\theta = 0{\cdot}9^\circ$ C. at $\theta = 1000^\circ$ C., showing that the value of δ can vary considerably without altering the value of θ to an extent greater than the limit of accuracy of the other measurements.

No attempt was made to work with the wires at a higher temperature than 1200° C. owing to the fact that the use of CALLENDAR's formula beyond this point is subject to uncertainty (33); also at high temperatures the wires soften and are liable to break when driven through the air at the high velocities.

(31) 'Smithsonian Physical Tables,' 5th edition, 1910, p. 222.

(32) BURGESS and LE CHATELIER, 'The Measurement of High Temperature,' Table V., p. 493; also Table VII., p. 495.

(33) See LANGMUIR, 'Journal Amer. Chem. Soc.,' vol. 28, p. 1357, 1906.

Section 12. *On Possible Sources of Error in the Calculation of Temperatures.*

(i.) *Calculation of the Radial Temperature Gradient.*

The question arises as to whether under conditions of rapid rate of cooling there may not exist a temperature gradient in the wire of sufficient magnitude to cause the temperature as calculated from the apparent resistance of the wire to differ appreciably from the surface-temperature.(34) This point can be settled by the following analysis :—

Let ρ be the specific resistance of the wire at the temperature under consideration, e the voltage drop per unit length, constant at all points of the cross-section. The heat generated per unit volume is e^2/ρ and the equation for the radial conduction of heat is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\mathrm{K}r\frac{\partial\theta}{\partial r}\right) = \frac{e^2}{\rho} + c\frac{\partial\theta}{\partial t}, \qquad (40)$$

where K is the heat-conductivity and c the specific heat per unit volume. If H is the heat-loss per unit length at the boundary $r = a$, we have

$$\mathrm{H} = -2\pi a\mathrm{K}\left(\frac{\partial\theta}{\partial r}\right)_{r=a}. \qquad (41)$$

Also, if R is the apparent resistance of the wire at temperature θ we have $1/\mathrm{R} = \int_0^a (2\pi r/\rho)\,.\,dr$, giving from (40) and (41) when $\partial\theta/\partial t = 0$, the result $\mathrm{H} = -e^2/\mathrm{R}$ which holds independently of radial temperature gradients or variations of specific resistance and heat-conductivity with temperature. The radial temperature gradient may be calculated to a first approximation by assuming that the gradient is so small that K and ρ may be considered constant throughout the wire. We then have $1/\mathrm{R} = \pi a^2/\rho$ and therefore $\mathrm{H} = \pi a^2\,.\,e^2/\rho$. The integral of (40) consistent with (41) is $\mathrm{K}\,(\partial\theta/\partial r) = -\mathrm{H}r/2\pi a^2$, which integrates further to

$$\theta-\theta_a = \mathrm{H}/4\pi\mathrm{K}\,.\,(1-r^2/a^2), \qquad (42)$$

θ_a being the surface temperature at $r = a$. The maximum temperature difference is that between the centre of the wire and the surface and is given by $\theta-\theta_a = \mathrm{H}/4\pi\mathrm{K}$. For platinum the heat-conductivity is $\mathrm{K} = 0{\cdot}1664$ calories per cm.^{2} per sec., or $\mathrm{K} = 0{\cdot}698$ watts cm.^{2} sec. at 0° C.; in the case of the largest wire H does not exceed 5 watts per cm. at 1000° C., so that we have $\theta - \theta_a < 5/(4\pi \times 0{\cdot}698)$ or $\theta-\theta_a < 0{\cdot}6°$ C. We note that this small temperature gradient justifies the simplifying assumption regarding the constancy of K and ρ over the cross-section of the wire, and conclude, finally, that the error in the calculation of the surface temperature introduced by a radial temperature gradient is within the limits of accuracy of the other measurements.

(34) A similar problem is dealt with by KELVIN, 'Roy. Soc Proc.,' June 10, 1875; also 'Collected Works,' vol. 3, p. 245.

(ii.) *On the Error Introduced by the Cooling Effect of the Leads and Potential Terminals.*

It is of importance in the practical design of anemometers to determine the limits of error introduced by the leads and potential terminals. Practically all cases are covered by that illustrated in fig. 3, where the end-leads C_1C_2 may be considered so heavy that the heated wire is maintained at its extremities very nearly at air-temperature; in addition it is necessary to allow for the heat conducted away by the potential leads fused to the anemometer-wire at P_1P_2. The problem presents itself in the following way:—A wire whose constants have been determined from a very long specimen in which end-corrections are negligible is placed between two heavy terminals C_1C_2 at a distance $2(l+l_0)$ apart; a current i is passed through the wire until the resistance of length $2l$ between potential terminals P_1P_2 is $2Rl$; it is required to determine to what an extent the determination of air-velocity from the measurement of current and resistance is affected by the cooling effect of the leads and potential terminals. Let ω represent the cross-section of the anemometer-wire, ω' that of the potential leads, and let K be the heat-conductivity of platinum; further, let W and W′ represent the heat-losses per unit length of the two wires respectively when the air-velocity is V. The temperature θ at any point of the anemometer-wire is determined from the equation

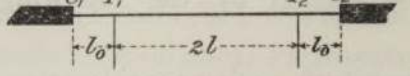


Fig. 3. Cooling effect of leads and potential terminals.

$$\omega \frac{\partial}{\partial x}\left(K\frac{\partial\theta}{\partial x}\right) + W = i^2R, \quad \ldots \quad (43)$$

where W and R are known functions of the temperature θ.

The complete expression for W is shown by the present experiment to be of the form

$$W = [\beta V^{\frac{1}{2}} + \gamma_0\{1+c(\theta-\theta_0)\}](\theta-\theta_0). \quad \ldots \quad (44)$$

In the case of platinum the variation of resistance with temperature can be represented by a formula of the type

$$R = R_0[1+a(\theta-\theta_0)+b(\theta-\theta_0)^2]. \quad \ldots \quad (45)$$

It will be noticed that, according to the experimental results expressed in (44) and (45), a solution of (43) can be completely determined in terms of elliptic integrals if we neglect the variation of the heat conductivity K with the temperature.[35] For

[35] Little seems to be known with regard to the variation of the heat conductivity of metals at high temperatures. The existence of definite experimental relations of the form (44) and (45) seems to indicate the possibility through an exact solution of (43) of carrying out an experiment for the determination of K at high temperatures along the lines of STRANEO's method (CARSLAW, 'Fourier Series and Integrals,' Macmillan and Co., 1906, p. 282).

the purpose of determining the small end-corrections it is sufficient to employ approximate expressions of the form

$$W = A(\theta-\theta_0) \qquad \text{and} \qquad R = R_0[1+\bar{a}(\theta-\theta_0)], \quad \ldots \quad (46)$$

where $A = \beta V^{\frac{1}{2}}+\bar{\gamma}$, and $\bar{a}$ and $\bar{\gamma}$ are average values of the coefficients over the range of temperature considered.

Under these conditions (43) becomes

$$K\omega\frac{d^2\theta}{dx^2} - A(\theta-\theta_0) = -i^2R_0[1+\bar{a}(\theta-\theta_0)],$$

or, writing $p^2 = (A-i^2R_0\bar{a})/K\omega$, the above equation takes the form

$$\frac{d^2\theta}{dx^2} - p^2(\theta-\theta_0) = i^2R_0/K\omega. \quad \ldots \quad (47)$$

Since we may neglect the current in the potential terminals (see equation (35)), the temperature θ' at a distance x' from the junction is determined from the equation

$$K\omega'\frac{d^2\theta'}{dx'^2} - A'(\theta'-\theta_0) = 0, \quad \ldots \quad (48)$$

where the accented symbols refer to the potential wire. The solution of (48), which makes $\theta' = \theta_0$ at $x' = \infty$ and $\theta' = \theta'_0$ at the junction $x' = 0$, may be written

$$\theta'_0-\theta' = (\theta'_0-\theta_0)\,e^{-x'\sqrt{(A'/K\omega')}}. \quad \ldots \quad (49)$$

The quantity of heat carried away by the potential lead from the point of junction is given by

$$H' = K\omega'\,d\theta'/dx' = (\theta'_0-\theta_0)\sqrt{(K\omega'A')} \quad \text{at} \quad x' = 0. \quad \ldots \quad (50)$$

Taking the origin at the centre of the anemometer-wire, the solution of (47), which makes $\theta = \theta'_0$ at $x = l$ is given by

$$\theta-\theta_0 = i^2R_0/K\omega p^2 - [i^2R_0/K\omega p^2 - H'/\sqrt{(K\omega'A')}]\,.\cosh px/\cosh pl. \quad \ldots \quad (51)$$

The flow of heat H_1 towards the junction at $x = l$ is given by

$$H_1 = -K\omega\,d\theta/dx = K\omega p\,[i^2R_0/K\omega p^2 - H'/\sqrt{(K\omega'A')}]\tanh pl. \quad \ldots \quad (52)$$

For the determination of the temperature in the portion of the anemometer-wire between the potential terminal P_1 and the terminal C_1, take the origin at the junction. The solution of (47), which makes $\theta = \theta'_0$ at $x = 0$ and $\theta = \theta_0$ at $x = l_0$, is given by

$$\theta-\theta_0 = \frac{i^2R_0}{K\omega p^2}\left\{1-\frac{\sinh px}{\sinh pl_0}\right\} + \left\{\frac{H'}{\sqrt{(K\omega'A')}} - \frac{i^2R_0}{K\omega p^2}\right\}\frac{\sinh p(l_0-x)}{\sinh pl_0}. \quad \ldots \quad (53)$$

The flow of heat away from the junction $x = 0$ is given by

$$H_2 = -K\omega\,d\theta/dx = (i^2R_0/p)\,.\operatorname{cosech} pl_0 + [H'/\sqrt{(K\omega'A')} - i^2R_0/K\omega p^2]\,.\,K\omega p\coth pl_0. \quad (54)$$

At the junction we have

$$H_1 = H_2 + H', \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

assuming perfect thermal contact between the anemometer-wire and the potential lead. From (55) we are now able to calculate H' and therefore from (51) the temperature of the anemometer-wire at any point; we find that

$$\theta - \theta_0 = (1 - Q \cosh px) \,.\, i^2 R_0 / K\omega p^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

where, writing $P = \sqrt{(K\omega' A')}/K\omega p$, Q is given by the equation

$$Q = \frac{1 + P \sinh pl_0}{\cosh p\,(l + l_0) + P \sinh pl_0 \cosh pl} . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

Let $\bar{\theta}$ and $\bar{R}$ represent the temperature and resistance per unit length of an infinitely long wire under the same conditions of wind velocity and heating by electric current. We then find the relations

$$\bar{\theta} - \theta_0 = i^2 R_0 / K\omega p^2 \,;\; p^2 = i^2 R_0 / K\omega\,(\bar{\theta} - \theta_0) = R_0 A / \bar{R} K\omega \,;\; P^2 = \omega' \bar{R} A' / \omega R_0 A. \quad (58)$$

The resistance Rl measured between potential leads is given by

$$Rl = \int_0^l R_0 \left[1 + \bar{a}\,(\theta - \theta_0)\right] dx = R_0 l + R_0 \bar{a}\,(\bar{\theta} - \theta_0) \int_0^l (1 + Q \cosh px)\, dx,$$

giving on integration and making use of the relation $\bar{R} = R_0 \left[1 + \bar{a}\,(\theta - \theta_0)\right]$ the final result

$$R/\bar{R} = \left[1 - (1 - R_0/\bar{R})\, Q\, (\sinh pl)\,/pl\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

This result enables us to estimate the effect of the potential leads and terminals on the measure of resistance. In the experiments under consideration the quantity measured is R; it is necessary in the determination of the convection constants to employ wires of such a length that R may not differ from $\bar{R}$ by more than a fraction of 1 per cent.

Since $A'/A = (\beta' V^{\frac{1}{2}} + \gamma')/(\beta V^{\frac{1}{2}} + \gamma)$ we may write as a rough approximation since from the experimental results the terms $\bar{\gamma}$ and $\bar{\gamma}'$ are small in comparison with the first terms of the expressions in which they occur, $A'/A = \beta'/\beta$. It also follows from the experimental results that $\beta'/\beta = (a'/a)^{\frac{1}{2}}$ so that since $\omega'/\omega = a'^2/a^2$ we may write for purposes of rough calculation

$$P = (\bar{R}/R_0)^{1/2}\,(a'/a)^{5/4}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

Writing from (46) $A = W/(\theta - \theta_0)$ we have for purposes of calculation

$$p = \left[R_0 W / \bar{R} K\omega\,(\theta - \theta_0)\right]^{1/2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

The quantities P and p are easily determined from the constants given in Table II. In the description of Diagram III. are set out the values of these quantities for various diameters in cases where the end corrections are liable to be large, *i.e.*, $\theta-\theta_0 = 1165°$ C., V = 81 cm./sec. for $\mathrm{R}_0/\overline{\mathrm{R}} = 1/4$. It will be noticed that p is always fairly large, and since in all the experimental arrangements we had $pl > 10$ and $pl_0 > 2$, the expression (57) may be written approximately in the form

$$\mathrm{Q} \sinh pl = (1+\mathrm{P} \sinh pl_0)/(e^{pl_0}+\mathrm{P} \sinh pl_0)$$
$$= (\mathrm{P}+\operatorname{cosech} pl_0)/(\mathrm{P}+1+\coth pl_0).$$

We have, finally, from (59) the convenient formula

$$\mathrm{R} = \mathrm{R}(1-\epsilon),$$

where

$$\epsilon = (1-\mathrm{R}_0/\overline{\mathrm{R}})(\mathrm{P}+2e^{-pl_0})/\{pl(\mathrm{P}+2)\}. \quad . \quad . \quad . \quad . \quad . \quad (62)$$

The values of ϵ are tabulated under the description of Diagram IV., from which it will be seen that the calculation of temperature from the resistance between the potential leads may be taken to refer to an infinitely long wire to an accuracy represented by these values of ϵ; in the case of the larger wires the experiment was carried out so that $l > 10$ cm. and $l_0 > 2$ cm., while the potential leads were of 1-mil wire. In this case the error introduced by the terminal conditions is well within the limits of accuracy imposed by the other measurements. In the case of the finest wires it was necessary to employ short specimens so that the possible error in the calculation of temperature due to terminal conditions is of the order of 2 or 3 per cent.

The preceding calculations are important in the design of hot-wire anemometers when the constant of the instrument is determined from the convection constants of platinum wire as determined in the present paper. Terminal errors do not need to be calculated if the anemometer is directly calibrated as long as the conditions of air velocity in the neighbourhood of the terminals are the same during actual service as during calibration.

(iii.) *On the Error due to the Vibration of the Anemometer Wire.*

A wire placed in a strong current of air tends to vibrate at right angles to the direction of the stream with a frequency depending both on the tension of the wire and the velocity of the air current. [36] Although the amplitude of vibration of the wire may be small, the high frequency may lead to an appreciable source of error the magnitude of which we now propose to investigate. We suppose the wire to perform simple harmonic vibrations of amplitude a and period T at right angles to the stream of velocity V. Representing the displacement from the equilibrium position by $x = a \sin \omega t$, the velocity at right angles to the stream is $v = a\omega \cos \omega t$. The velocity

[36] See RAYLEIGH, 'Sound,' vol. II., p. 412 (1896).

of the wire relative to the stream at any instant is given by $U = \sqrt{(V^2+v^2)}$. The heat-loss at any instant is given by the experimental relation $W = \beta(\theta-\theta_0)\sqrt{U}+\gamma$, so that the average heat-loss, $\overline{W} = Ri^2$, is given by

$$\overline{W} = 1/T\int_0^T W\,dt = \beta(\theta-\theta_0)\,T^{-1}\int_0^T (V^2+v^2)^{\frac{1}{4}}\,dt+\gamma.$$

Writing w $\omega a = 2\pi a/T$ and $\phi = 2\pi t/T$, we find

$$\overline{W} = \beta(\theta-\theta_0)\,V^{\frac{1}{2}}\,2\pi^{-1}\int_0^{\pi/2}[1+(w^2/V^2)\cos^2\phi]^{\frac{1}{4}}\,d\phi+\gamma. \quad . \quad . \quad . \quad . \quad (63)$$

If $\overline{V}$ be the apparent velocity corresponding to the observed heat-loss $\overline{W}$ we have, writing $\omega = 0$,

$$\overline{W} = \beta(\theta-\theta_0)\sqrt{\overline{V}}+\gamma. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

Comparing (63) and (64) we obtain finally

$$(V/\overline{V})^{\frac{1}{2}} = 1\Big/\left\{2\pi^{-1}\int_0^{\pi/2}[1+(w^2/V^2)\cos^2\phi]^{\frac{1}{4}}\,d\phi\right\} = \pi\sqrt{k'}\Big/\left\{2\int_0^K \mathrm{dn}^{3/2}(u,k)\,du\right\}, \quad (65)$$

where K is the complete elliptic integral of the first kind with modulus $k = w^2/(w^2+V^2)$. When w/V is small we have approximately,

$$(V/\overline{V})^{\frac{1}{2}} = 1-\tfrac{1}{8}w^2/V^2+\ldots \quad \text{or} \quad V/\overline{V} = 1-\tfrac{1}{4}w^2/\overline{V}^2. \quad . \quad . \quad . \quad . \quad . \quad (66)$$

As an illustration, take the amplitude $a = 0{\cdot}01$ cm., $T = 1/100$ sec., so that $w = 6{\cdot}2$ cm./sec. The error due to vibration will remain less than $\frac{1}{4}$ of a per cent. as long as V is less than 60 cm./sec. An increase of amplitude to $a = 0{\cdot}1$ cm. will introduce an appreciable error for velocities less than 600 cm./sec. It is thus important in the design of practical anemometers to construct the apparatus in such a way as to eliminate vibration or reduce it to so small an amplitude that the error contributed is negligible. In the present series of experiments the method adopted of keeping the wire under tension by means of a small weight in the manner shown in Diagram II. (*c*) proved to be effective in preventing the vibrations over the range of velocities employed. At very high velocities vibrations of considerable amplitude were at times set up in spite of the frictional constraint at the lower end of the wire.

Section 13. *On the Reduction and Interpretation of the Observations.*

Sample series of observations are given in Tables III. and IV. showing the method of entering the observations in the case of each wire. For each velocity (corrected for *swirl*) the current required to heat the wire to a predetermined series of resistances (from which the temperatures were calculated) were measured. The corresponding heat-loss in watts per unit length was then calculated and entered as the second entry in each compartment of the table. The theory of Part I. suggests that these results be examined in the light of the formula

$$W = B\sqrt{V} + C, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

where B and C are functions of the temperature and of the dimensions of the wire. If W is plotted against $\sqrt{V}$ for each temperature, we should, according to (67), obtain a family of straight lines, and figs. 4 and 5 show this conclusion to be justified within the limits of experimental error. By determining the line of closest fit through the observed points, we are enabled to calculate the constants B and C. The position of this line can be found by calculating for each system of observed points the position of the major axis of inertia of the corresponding system of material points of equal weight.(37) It was, however, found to be extremely tedious to make use of this method and it was deemed sufficiently accurate to adopt the simpler method of dividing the observations into two groups, finding the centre of gravity of each and taking the position of the line joining these two points as an approximation to the line of closest fit. The constants B and C were found in this way directly from the tabulated values of W and $\sqrt{V}$. The accuracy of the observations was tested by plotting for each wire the family of approximately straight lines corresponding to the various temperatures, and in this way a check was obtained on the determination of the constants B and C.

The constants B and C obtained in this manner are set out with the temperature and diameter of the wire in Tables V. and VI., and will now be discussed separately.

(i.) *Analysis of the Convection Constant* B.

The theoretical investigations of Sections 4, 5, and 6 suggest that the constant B be examined in the light of the formula $B = \beta(\theta - \theta_0)$. The third entry in each compartment of Table V. gives the value of $\beta = B/(\theta - \theta_0)$ for $\theta_0 = 17°$ C., from which it is seen in fact that β is very nearly independent of the temperature; the slight increase in its value with increasing temperature may be represented by a small temperature coefficient b such that $\beta = \beta_0[1 + b(\theta - \theta_0)]$ and which may be interpreted as due to the combined variation with temperature of the diameter of the wire, and the specific heat, density, and heat conductivity of air. The variation of β with temperature is very little greater than that due to the errors of experiment, and the coefficient b was roughly determined by plotting β against the temperature, drawing in the line of closest fit and hence determining β_0 and b, giving a mean value $b = 0{\cdot}000080$.

Theory further requires that β_0 be given by the expression $\beta_0 = 2\sqrt{\pi s_0 \kappa_0 \sigma_0 a_0}$, where the suffix $_0$ refers to the temperature 17° C. That the ratio $\beta_0/\sqrt{a_0}$ is constant is

(37) Formulæ for the determination of this straight line are given by KARL PEARSON ('Phil. Mag.,' 2, November, 1901, p. 559), and also by SNOW, E. C. ('Phil. Mag.,' 21, March, 1911, p. 367).

shown very satisfactorily in the last column of Table VI. and in fig. 7 where no systematic variation with the radius can be detected. The mean value of the ratio $\beta_0/\sqrt{a_0}$ as the final result of the entire series of observations is given by

$$\beta_0/\sqrt{a_0} = 1{\cdot}432 \times 10^{-3}, \quad \ldots \ldots \ldots \quad (68)$$

heat units being measured in watts.

In the numerical calculation of the theoretical factor $2\sqrt{\pi s_0 \sigma_0 \kappa_0}$ we take $\sigma_0 = 0{\cdot}001293$, $\kappa_0 = 5{\cdot}66 \times 10^{-5}$ calories(38) and $s_0 = 0{\cdot}171$ calories(39), the specific heat at constant volume being that appropriate to the problem since no external work is done by the expansion of the heated air. We find $2\sqrt{\pi s_0 \sigma_0 \kappa_0} = 3{\cdot}96 \times 10^{-4}$ expressed in calories; on multiplying by 4·18 to reduce to watts we have finally

$$2\sqrt{\pi s_0 \sigma_0 \kappa_0} = 1{\cdot}66 \times 10^{-3}. \quad \ldots \ldots \ldots \quad (69)$$

The agreement with the experimental factor in equation (68) must be considered fair in view of the uncertainty attached to the value of the heat conductivity, and also in that the theoretical investigation does not take into account the variation of this and the other factors with the temperature gradient in the neighbourhood of the wire.

(ii.) *Analysis of the Convection Constant* C.

The rational interpretation of the convection constant C in the light of the theory of Section 6 requires us to separate out the energy losses due to radiation. The characteristic form of the equation giving the total radiation from metallic surfaces is

$$e = \sigma\Theta^\beta, \quad \ldots \ldots \ldots \quad (70)$$

where Θ represents the absolute temperature and σ and β are constants depending on the nature of the metal forming the surface. From the observations of LUMMER and KURLBAUM (40) it can be shown that the radiation loss for polished platinum is given in watts per cm.² by the relation

$$e = 0{\cdot}514\,(\Theta/1000)^{5{\cdot}2}. \quad \ldots \ldots \ldots \quad (71)$$

Assuming the total radiation loss of a small wire to be proportional to its circumference, the loss per unit length is calculated for a wire of diameter 0·010 cm.,

(38) Recent determinations of the thermal conductivity of air are—

$4{\cdot}76 \times 10^{-5}$ at 24° C. (O. J. STAFFORD, 'Ztschr. f. Physik. Chemie,' 77, p. 67, 1911.)
$5{\cdot}22 \times 10^{-5}$ at 0° C. Mean of five observers (KAYE and LABY, 'Tables,' p. 52).
$5{\cdot}66 \times 10^{-5}$ at 0° C. (EUCKEN, A., 'Physik. Zeit.,' 12, 1101, 1911.)
$5{\cdot}69 \times 10^{-5}$. Mean value adopted by CHAPMAN, 'Phil. Trans.,' A, vol. 211, p. 465, 1912, where further references to recent determinations are given.

(39) There is little variation in the experimental determinations of the specific heat at constant volume. The above value is taken from KAYE and LABY's 'Tables,' p. 58.

(40) LUMMER and KURLBAUM, 'Verh. Phys. Ges.,' Berlin, 17, p. 106, 1898.

and is found to agree with the determinations of LANGMUIR ([41]) obtained by a somewhat different calculation. The value of the radiation loss $E = 2\pi ae$ per unit length is plotted against the temperature, and from this curve the radiation loss for any wire may be easily determined at any temperature. The values of the radiation losses thus obtained are set down in Table VI. under the constants C and the true convection loss $C_0 = C - E$ obtained by subtraction. For each wire the ratio $C_0/(\theta-\theta_0)$ is determined over the range of temperatures and is plotted against $\theta-\theta_0$. The result shows very approximately a straight line whose constants are determined from the position of the line of closest fit in the manner already described. In this way the constants in the formula

$$C_0 = \gamma_0(\theta-\theta_0)\left[1+c(\theta-\theta_0)\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

were obtained for each wire. The constant γ_0 is very nearly independent of the diameter of the wire as we should expect from the theoretical discussion of Part I. By plotting its value against the radius we find approximately

$$\gamma_0 = 2{\cdot}50\times10^{-4}(1+70a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

expressed in watts. According to the theoretical development we should expect the relation $C_0 = \kappa_0(\theta-\theta_0)$. Taking $\kappa_0 = 5{\cdot}66\times10^{-5}$ calories $= 2{\cdot}37\times10^{-4}$ watts, this result is in surprisingly good agreement with the value of γ_0 given in (73), when the uncertainty of the exact correction for radiation is taken into account.([42]) The slight dependence of γ_0 on the radius of the wire is probably due to the effect of viscosity. The coefficient c is seen to vary remarkably little from the mean value $c = 0{\cdot}00114$, and may be considered to represent in large measure the variation of the heat-conductivity with the temperature.([43])

([41]) LANGMUIR, "Convection and Conduction of Heat in Gases," 'Phys. Rev.,' 34, p. 415, 1912.

([42]) The success of the theoretical formula (33),

$$H = \kappa\theta_0 + 2\sqrt{\pi\kappa\sigma s a}\,V^{\frac{1}{2}}\theta_0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (i)$$

in representing experimental results leads to a possibility worth future investigation as to whether the convection method may not be used in the determination of the heat-conductivity and specific heat of gases as a method of continuous flow. By making the temperature difference θ in (i.) small and introducing refinements in the methods of measurement, the procedure of the present experiment is especially well adapted to the study of the variation of the conductivity and specific heat of gases over an extremely wide range of temperature and pressure. A slight modification of the disposition of the apparatus would enable the viscosity of the same specimen of gas to be measured under the same conditions, an experimental procedure of the greatest importance in the interpretation of these results according to the kinetic theory of gases (see CHAPMAN, "The Kinetic Theory of a Gas Constituted of Spherically Symmetrical Molecules," 'Phil. Trans.,' A 482, vol. 211, pp. 462–476).

([43]) It is interesting to note that WINKLEMANN gives a temperature coefficient $0{\cdot}00190$ for the variation of thermal conductivity with temperature (see the 'Smithsonian Physical Tables,' 1910, p. 200).

Section 14. *On the Variation of the Convection Constants with Inclination to the Direction of the Stream.*

In order to study the effect of the inclination of the wire on the convection constants, a series of observations was taken on Wire No. 1 (diameter 0·0153 cm.), mounted in the fork of the rotating arm and set at various angles to the vertical. The wire was arranged in such a manner as to eliminate the disturbing effect of the fork when set horizontally. For each inclination the relation $W = B\sqrt{V}+C$ was found to hold good, and the values of the constants B and C were determined in the manner already described. It was further found that the relation $B = \beta(\theta-\theta_0)$ held good, and that β increased slightly with the temperature according to the formula $\beta = \beta_0[1+b(\theta-\theta_0)]$, b having a value very nearly the same for each inclination and agreeing roughly with the value already given. The constant C was also determined and was found to vary very little with the inclination. These variations were very little greater than errors arising from uncertainties of determination, so that the term C was not studied in further detail. In Table VII. are given the variations of B and β with the inclination of the wire to the vertical. It will be noticed that the current required to maintain the wire at a given temperature is markedly greater when the wire is perpendicular to, than when its direction coincides with that of, the stream, a point which is utilized in the design of practical hot-wire anemometers. Since the study of the effect of inclination on convection is much more easily carried out in a wind-tunnel, experiments were not carried further than those described under Table VII.

Section 15. *Note on the Calculation of the Free Convection of Heat from Small Platinum Wires.*

Although the complete investigation of the problem of free convection from a heated cylinder is one which at present transcends mathematical representation, it is not without interest to examine to what an extent the problem may be dealt with by supposing that the wire is cooled by a current of "effective" velocity V imposed on the wire by the ascent of heated air from the surface of the cylinder. Since the velocity will be small, we may employ formula (32) on the supposition that κ is the value of the conductivity appropriate to the temperature difference $\theta-\theta_0$. From the examination of the experimental results of forced convection, described in Section 13, it would appear that the variation of the thermal conductivity with the temperature could be expressed by the relation

$$\kappa = \kappa_0[1+c(\theta-\theta_0)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

with $c = 0·00114$ and $\kappa_0 = 5·66\times10^{-5}$ calories per cm.^{2} per sec. $= 2·37\times10^{-4}$ watts.

It follows from (32) that the heat-loss by convection can be obtained from a formula of the form

$$H = 2\pi\kappa_0 (\theta-\theta_0) [1+c(\theta-\theta_0)]/[\log b/a], \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

to which must be added the radiation loss given by

$$E = 2\pi a \times 0{\cdot}514 (\theta/1000)^{5\cdot2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

E being expressed in watts per cm.[2] Formula (75) is tested in the light of LANGMUIR's observations on free convection([44]) which were carried out over a much wider range of temperature than those of the writer. The analysis of these observations is discussed under Table VIII., from which it appears that to a fair degree of accuracy the denominator $\log b/a$ is independent of the temperature, but depends on the radius of the wire in the manner shown in fig. 9, the use of which affords a simple method of calculating the actual convection loss in any actual case.

It is interesting from the relation (28), $b = \kappa e^{1-\gamma}/(s\sigma\mathrm{V})$ to determine the effective velocity $\overline{\mathrm{V}}$ of the convection current by means of which the wire is cooled. The variation of the term $\sqrt{\kappa s\sigma}$ with the temperature is small, and from the results of Section 13 may be expressed by means of the relation

$$\sqrt{\kappa s\sigma} = \sqrt{\kappa_0 s_0 \sigma_0}\,[1+b(\theta-\theta_0)], \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

where $b = 0{\cdot}000080$. Thus making use of (74) and writing

$$\overline{\mathrm{V}}_0 = \kappa_0 e^{1-\gamma}/(s_0\sigma_0 b), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (78)$$

we have

$$\overline{\mathrm{V}} = \overline{\mathrm{V}}_0 [1+c(\theta-\theta_0)]^2/[1+b(\theta-\theta_0)]^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

Values of the effective velocities $\overline{\mathrm{V}}_0$ and $\overline{\mathrm{V}}$ are given in Table VIII. It is probable that the values of $\overline{\mathrm{V}}$ present velocities of the same order as the actual average velocity of the convection current generated by the heated wire, estimates of which are useful in determining the limits of temperature and velocity within which a hot-wire anemometer can be employed with accuracy.

([44]) LANGMUIR, 'Phys. Rev.,' 34, p. 415, 1912.

PART III.

On the Design of Portable Hot-Wire Anemometers.

Section 16. *Introduction.*

That the measurement of the current required to keep a wire at a given temperature (measured by its resistance) might be employed as a method of measuring air-velocity seems to have first been suggested by KENNELLY(45) in 1909. This method was developed independently both by BORDINI,(46) and by MORRIS(47) about the same time, and actual measurements were carried out by these investigators. In the latter case a platinum wire was inserted in one arm of a Wheatstone bridge, the remaining resistances being constructed of manganin and unaffected by variation in temperature due to the heating of the measuring current; under these circumstances it was found that the square of the current required to keep a wire at a constant temperature of about 70° C. above the surrounding air was very nearly proportional to the square root of the air velocity for as high values as 40 miles per hour; it was found necessary to make a small correction corresponding to the term C of the formula $W = B\sqrt{V}+C$.

A form of integrating anemometer, suitable for measuring the total flow across a given cross-section, has recently been described by GERDIEN.(48) All of the instruments described are more especially suited to the measurement of average velocities over a considerable area.(49)

The aim of the experiments described in Part II. of the present paper was to determine the convection constants of platinum wire in absolute measure, in order that it might be possible to construct from the easily determined electrical constants

(45) See footnote (23).

(46) BORDINI, U., "Un procedimento per la misura della velocità dei gas," read before the Società Italiana per il Progresso delle Scienze, October 13, 1911; published in the 'Nuovo Cimento,' Series VI., vol. III., pp. 241–283, April, 1912; see also the 'Electrician,' 70, p. 278, November 22, 1912.

(47) MORRIS, J. T., "The Electrical Measurement of Wind Velocity," read at the British Association, Dundee, September, 1912; published in the 'Engineer,' September 27, 1912; the 'Electrician,' October 4, 1912, p. 1056; see also the 'Electrician,' 70, p. 278, November 22, 1912.

(48) GERDIEN, H., "Der Luftgeschwindigkeitsmesser der Siemens und Halske," 'Ber. der Deutschen Phys. Gell.,' Heft 20, 1913.

(49) An interesting application of electric heating to the measurement of gas flow is to be found in the Thomas Gas Meter (THOMAS, C. C., 'Jour. of the Franklin Inst.,' November, 1911, pp. 411–460; also 'Proc. of the American Gas Inst.,' 7, p. 339, October, 1912; and 'Trans. Amer. Soc. Mech. Eng.,' 31, p. 655, 1909). In this form of instrument, however, the entire cross-section of gas is heated by means of a special heating grid, and the electric power required to maintain a constant difference of temperature, as measured by means of a pair of differential electric thermometers on each side of the heater, is recorded.

of the wire an anemometer capable of giving an accurate measure of air-velocity without reference to a calibration in terms of some other form of wind-measuring instrument. The problem is analogous to the determination of the constants of platinum wire for the purposes of thermometry; in fact standard thermometer wire is the most suitable for the purpose of hot-wire anemometry in that its electrical constants are usually determined with great accuracy.

The specifications described in the next section were carried out independently of any previous work, and as they offer several distinct advantages with regard to the construction of a practical standard of anemometry, it has been thought by the writer worth while to describe them in some detail. The special type of instrument developed may be called a *linear anemometer* consisting only of a single wire, and is especially suited for the study of turbulent flow and the analysis of sharp gradients of velocity such as are to be found in the neighbourhood of obstacles in streams, in jets, &c. The advantage to be derived from the use of a single wire lies in the fact that it is possible to measure with considerable accuracy the velocity in the neighbourhood of the wire with no appreciable disturbance of the flow at a distance of a few diameters away. This statement is easily verified from the classical solution of the flow of a stream past a cylindrical obstacle. In the case of a circular cylinder of radius a in a stream of a perfect fluid of undisturbed velocity V, the velocity potential of the point (r, θ) is given by $\phi = \mathrm{V}(r+a^2/r)\cos\theta$, θ being measured from the downstream direction of the stream. It is easily proved that the maximum velocity at a distance r, $(r > a)$, occurs for $\theta = \pi/2$ and is given by $q = \mathrm{V}(1+a^2/r^2)$. Thus at a distance $r = 10a$, the velocity of the stream is disturbed by only 1 per cent. of its value; hence, in the case of a 3 mil anemometer wire, the theoretical resolving power in measuring a sharp gradient may roughly be set at 15 mils (about 0·04 cm.), the accuracy of velocity measurement being set at 1 per cent.

Section 17. *General Considerations on the Design of Hot-Wire Anemometers.*

The Kelvin bridge connections employed in the determination of the convection constants, as described in Part II., offered so many advantages that this system of connections was at once taken as the starting point in the design of an instrument for the measurement of wind-velocity.

The experiments showed that over the entire range of diameters employed, the power required to maintain the wire at the same resistance, corresponding to a temperature θ° C., is given by the formula

$$\mathrm{W} = \mathrm{B}\sqrt{\mathrm{V}}+\mathrm{C}, \qquad \text{where } \mathrm{B} = \beta(\theta-\theta_0), \qquad \beta = \beta_0[1+0{\cdot}00008\,(\theta-\theta_0)],$$

and $\beta_0 = 1{\cdot}432\times10^{-3}\sqrt{a}$; furthermore, $\mathrm{C} = \mathrm{C}_0+\mathrm{E}$ where the radiation is given by $\mathrm{E} = 2\pi a\times0{\cdot}514\,(\Theta/1000)^{5\cdot2}$, $\mathrm{C}_0 = \gamma_0(\theta-\theta_0)[1+0{\cdot}00114\,(\theta-\theta_0)]$, and finally

$\gamma_0 = 2{\cdot}50 \times 10^{-4}(1+70a)$. From this series of formulæ and constants it is possible to calibrate in absolute measure an anemometer wire provided its thermometric constants and its diameter are accurately known. It is simpler, however, to make use of a wire whose constants have been directly determined by calibration either by reference to a standard instrument or preferably in absolute measure by means of a rotating arm. The advantage of the Kelvin bridge connections enables a number of wires with potential terminals fused in place to be independently calibrated and inserted at will in a suitable anemometer fork, the readings of the instrument when balanced being independent of the connecting or contact resistances. It will be noticed from the above formulæ that by maintaining the anemometer wire at a sufficiently high temperature, the variations of the factor $\theta-\theta_0$ with changes of room temperature can be made small enough to be neglected. This result is important from the point of view of a practical instrument in that the calculation of wind velocity from the current required to balance the resistance of the anemometer wire against a fixed resistance is independent of the room temperature for such variations as may be met with under ordinary conditions of use; in any case, a small correction for the variation of room temperature may be applied if necessary. The most suitable temperature was found to be in the neighbourhood of 1000° C. The most convenient method of obtaining this adjustment is to adjust the manganin resistance B (fig. 2) so that its resistance is four times that of the platinum wire between potential terminals at room temperature. At this temperature variations of as much as 5° C. in room temperature only affect the velocity determination by about 1 per cent.—about the limit of accuracy in most measurements of turbulent flow. At the high temperatures employed the wires were found to remain in a very constant state, and no variations of electrical constants sufficient to affect the calibration constants for velocity measurements could be detected after extended use. It was found of great convenience in practice to be able to make rough adjustments of current necessary to balance the resistances by judging the degree of brightness of the anemometer wire which is dull red at a temperature for which the resistance is four times the resistance at room temperature. It will be noticed from Table VIII. that the effective velocity of the free convection current increases comparatively slowly with increased temperature to 1000° C., and in most applications does not interfere seriously with the measurement of velocity.

Although it is desirable to operate the instrument with as small an expenditure of power as possible by making use of a very fine anemometer wire, it was found by experience to be impracticable to employ wires of diameter smaller than 3 mils; for a wire of this diameter, maintained in the neighbourhood of 1000° C., velocities from zero to 25 miles per hour are represented by currents between 1 and 2 amperes. For wires smaller than 3 mils, the electrical constants are subject to change on drawing down the wire, and errors due to the cooling effect of the potential leads become relatively larger, although by special calibration an instrument with a wire as small as 1 mil, or less, could be successfully constructed to meet special requirements.

For the same wire the current i required to bring the wire to the same temperature is given by a relation of the form

$$i^2 = i_0^2 + k\sqrt{V}, \quad \ldots \ldots \ldots \quad (80)$$

where i_0^2 and k are the constants of the instrument determined either by calculation or by direct calibration; in practice there is some advantage in making use of a dynamometer form of ammeter in which the scale reading is proportional to i^2, provided it is sufficiently accurate; the range of readings can be much increased by arranging the instrument so that the scale has a suppressed zero.

Another point connected with hot-wire anemometry may be mentioned here. The constants of the instrument or its calibration-curve determined by means of a rotating arm represent determinations of velocity appropriate to stream-line motion; when employed to measure turbulent flow the velocity measured by the hot-wire instrument may be referred to as the *effective velocity*. The question then arises as to whether this measure is a consistent one and is independent of the diameter of the wire. This point was settled in the course of experiments carried out by Prof. A. M. GRAY[50] and the writer on the distribution of air-velocities in the neighbourhood of rapidly rotating cylinders. The velocity gradient in the neighbourhood of a rapidly revolving wooden pulley was measured by the use of an anemometer fitted with a 3-mil wire and also under the same conditions with a 5-mil wire. It was found that the velocity gradients agreed within the limits of experimental error. It would thus seem possible to obtain a consistent measure of turbulent flow under conditions where other methods of measurement would be inadequate.

Section 18. *Note on the Applications of the Hot-Wire Anemometer.*

The advantages of the anemometer designs and connections described under Diagram III. in the establishment of a standard system of anemometry may be briefly stated as follows :—

(i.) The use of the Kelvin bridge connections makes it possible to standardize and calibrate anemometer wires at a central laboratory independently of the remainder of the apparatus with which the wire is to be employed.

(ii.) These connections also enable the wire to be heated to a high temperature and so makes the determinations of velocity practically independent of ordinary variations of room temperatures.

(iii.) The use of the linear anemometer makes it possible to establish a consistent measure of turbulent flow.

(iv.) The linear anemometer makes it possible to analyse sharp gradients of velocities without disturbing the flow at the point of measurement.

[50] Of the Department of Electrical Engineering, McGill University.

(v.) The properties of the Kelvin bridge connections make it possible to connect an anemometer through low resistance slip-rings and provide a means of attacking such problems as the analysis of velocities in the neighbourhood of rapidly revolving areoplane propeller blades, or between the blades of centrifugal fans.

In addition to the applications already mentioned incidentally, it will be noticed that the use of the linear anemometer makes it easily possible to analyse the distribution of air-velocities in the neighbourhood of obstacles of various dimensions and shapes. An important application of this method is to be found in aerotechnical problems, such as the analysis of propeller wakes, distribution of velocities over planes of various dimensions and camber, &c. A compilation of such data would prove to be of the greatest service in developing a rational theory of turbulent flow. An instrument of the type under consideration could easily be constructed to give a continuous graphical record, thereby greatly extending its usefulness in analyses of complicated velocity distributions.

The application of the theoretical part of the present paper to the measurement of the thermal conductivity of gases has already been mentioned; in the case of liquids the chief application of the method lies in the investigation of a possible instrument for the convenient and rapid measurement of thermal conductivity. Experiments by ROGOVSKIJ[51] seem to indicate the existence of a law of heat-loss of the form already studied.

In a somewhat different field of application, the laws of heat-loss already studied probably determine the temperature which a small solid object is able to acquire in a current of highly heated gas such as a Bunsen or blow-pipe flame. In fact the hot-wire method seems capable of affording a convenient means of analysing the velocities and true temperatures of flames or of heated gases in boiler-flues, &c.

It is hoped at a latter date to undertake in detail the study of some of the problems suggested in the present section.

Before closing the writer wishes to express his sincere thanks to Prof. H. T. BARNES, F.R.S , Director of the Macdonald Physics Laboratory, for the kind way in which he has facilitated the present work by every means in his power.

([51]) ROGOVSKIJ, E. A., 'Comptes Rendus Acad. Sci., Paris,' vol. 136, pp. 1391–1393, 1903; also vol. 141, pp. 622–624, 1905.

NUMERICAL TABLES AND DIAGRAMS: COMPARISON OF THEORY WITH OBSERVATION.

Discussion of Results.

Table of the Functions $e^x K_0(x)$, $e^x K_1(x)$, $\int_0^x e^x K_0(x)\,dx$, $x\Big/\Big[\int_0^x e^x K_0(x)\,dx\Big]$.

The functions tabulated in the first two columns of the table opposite are based on JAHNKE and EMDE'S tables([1]) of the functions $K_0(x)$ and $K_1(x)$ for values of x between $x = 0\cdot1$ and $x = 12\cdot0$. The process of multiplication by the factor e^x was easily carried out by means of a "Brunsviga" calculating machine.([2]) For values of the argument $x < 0\cdot1$ the functions were calculated directly from the convergent expansions

$$K_0(x) = -I_0(x)[\gamma+\log\tfrac{1}{2}x]+\frac{x^2}{2^2}+\frac{x^4}{2^2.4^2}(1+\tfrac{1}{2})+\frac{x^6}{2^2.4^2.6^2}(1+\tfrac{1}{2}+\tfrac{1}{3})+\dots \quad \text{(i.)}$$

$$-K_1(x) = I_1(x)[\gamma+\log\tfrac{1}{2}x]+\frac{1}{2}\cdot\frac{x}{2}\left(\frac{2}{x}\right)^2-\frac{1}{2}\cdot\frac{x}{2}\left[\frac{1}{1!}+\frac{1+1+\frac{1}{2}}{1!.2!}\left(\frac{x}{2}\right)^2+\frac{1+\frac{1}{2}+1+\frac{1}{2}+\frac{1}{3}}{2!.3!}\frac{(x)^2}{(2)}+\dots\right]. \quad \text{(ii.)}$$

For large values of the argument numerical values were obtained from the asymptotic expansions

$$e^x K_0(x) = \left(\frac{\pi}{2x}\right)^{1/2}\left[1-\frac{1}{8x}+\frac{1^2.3^2}{2!(8x)^2}-\dots\right], \quad \text{(iii.)}$$

$$-e^x K_1(x) = \left(\frac{\pi}{2x}\right)^{1/2}\left[1+\frac{3}{8x}-\frac{3.5}{2!(8x)^2}+\frac{3.5.7}{3!(8x)^3}-\dots\right]. \quad \text{(iv.)}$$

For values of $x < 0\cdot1$, the function $\int_0^x e^x K_0(x)$ is most easily calculated from the convergent series obtained by integrating (i.) term by term after multiplying by the factor e^x:

$$x^{-1}\int_0^x e^x K_0(x)\,dx = 1+\tfrac{1}{4}x+\tfrac{1}{6}x^2+\tfrac{17}{168}x^3+\dots-(\gamma+\log\tfrac{1}{2}x)(1+\tfrac{1}{2}x+\tfrac{1}{4}x^2+\tfrac{5}{48}x^3+\dots). \quad \text{(v.)}$$

From the point $x = 0\cdot1$ onwards it is convenient to make use of EULER'S ([3]) formula for quadratures in the form

$$2h^{-1}\int_{x_0}^{x_n} y\,dx = [(y_0+y_1)+(y_1+y_2)+\dots+(y_{n-1}+y_n)]-\tfrac{1}{6}h[f'(x_n)-f'(x_0)]+\tfrac{1}{360}h^3[f'''(x_n)-f'''(x_0)]-\dots \quad \text{(vi.)}$$

In the case under consideration we have

$$f'(x) = e^x[K_0(x)+K_1(x)], \qquad f'''(x) = (4-3x^{-1}+2x^{-2})e^x K_1(x)+(4-x^{-1})e^x K_0(x). \quad \text{(vii.)}$$

The first term on the right-hand side of (vi.) in square brackets [ ] is especially well suited for computation on an adding machine; the totals (y_0+y_1), $(y_0+y_1)+(y_1+y_2)$, &c., are obtained as successive results of a single series of operations and are thus easily tabulated. Opposite these entries are written down the values of $\frac{1}{6}hf'(x_n)$, obtained by using equation (vii.) from the first two columns of Table I. The third order correction term in (vi.) can be easily shown to be too small to affect the value of the series to 1 part in 1000, as seen from the numerical example given below. Starting with $\int_0^{0\cdot1} y\,dx = 0\cdot3572$ calculated from the convergent series, and taking $h = 0\cdot1$, the work is tabulated as follows:—

x.	y.	$\frac{1}{6}hf'(x)$.	$\frac{1}{360}h^3f'''(x)$.	$2h^{-1}\int_0^{0\cdot1} y\,dx+S_n$.	C'_n.	C'''_n.	$\int_0^{x_n} y\,dx$.
0·1	2·6824	−0·1368	−0·0052	—	—	—	0·3572
0·2	2·1407	−0·0615	−0·0006	11·9681	0·0753	0·0046	0·5946

([1]) See footnote ([11]).

([2]) For a description of this instrument see D'OCAGNE, 'Le Calcul Simplifié' (Gauthier Villars, Paris, 1905), pp. 62–65.

([3]) See footnote ([10])

where

$$S_n = (y_0 + y_1) + (y_1 + y_2) + (y_2 + y_3) + \ldots + (y_{n-1} + y_n),$$

$$C'_n = \tfrac{1}{6}h[f'(x_n) - f'(x_0)], \qquad \text{and} \qquad C'''_n = \tfrac{1}{360}h^3[f'''(x_n) - f'''(x_0)].$$

By carrying out this process the function $\int_0^x e^x K_0(x)\,dx$ was tabulated as far as $x = 6 \cdot 0$, beyond which point the calculation by means of the asymptotic series involves less labour. We obtained, in equation (24), an expansion of the form

$$\int_0^x e^x K_0(x)\,dx + C = \sqrt{(2\pi x)}\left[1 + \frac{1}{8x} - \frac{1^2 . 3}{2!\,(8x)^2} + \frac{1^2 . 3^2 . 5}{3!\,(8x)^3} - \ldots\right] \quad \text{(viii.)}$$

That the constant C is unity is very probably true although an analytical proof of the result would be very laborious; for the purposes of the present work the case is sufficiently justified by the following numerical calculations of the sum of the series on the right-hand side of (viii.). These calculations, at the same time, furnish a check on the results obtained by quadratures :—

	$x = 1.$	$x = 2.$	$x = 3.$	$x = 6.$
Series (v.)	1·779	—	—	—
Series (viii.)	[2·778] (4)	3·7519	4·5165	6·2643
Quadratures	1·7804	2·7499	3·5127	5·2640
C	[1·002]	1·0020	1·0038	1·0003

The values of the function $x \Big/ \left[\int_0^x e^x K_0(x)\,dx\right]$ are easily obtained from the functions previously tabulated. This function when plotted against $\sqrt{x}$, as shown in Diagram I., gives very approximately a straight-line relation over the range dealt with in the present experiments. Simple approximate formulæ for the function in question are derived and tested in the description of Diagram I.

(4) The series (viii.) fails to give accurate values for so small a value of the argument as $x = 1$. The sum of four terms is 2·798, and of five 2·758, the value given above being the mean of these two determinations.

TABLE I.

x.	$e^xK_0(x)$.	$-e^xK_1(x)$.	$\int_0^x e^xK_0(x)\,dx$.	$x\Big/\Big[\int_0^x e^xK_0(x)\,dx\Big]$.	$\sqrt{x}$.
·0001	9·3272	10001·000	·001033	·09683	·01
·0004	7·9412	2500·998	·003576	·1119	·02
·0009	7·1355	1112·108	·007319	·1230	·03
·0016	6·5642	625·994	·012095	·1323	·04
·0025	6·1227	400·992	·017788	·1405	·05
·0036	5·7630	278·767	·024313	·1481	·06
·0049	5·4611	205·068	·031601	·1551	·07
·0064	5·2005	157·233	·039578	·1617	·08
·0081	4·9718	124·435	·048224	·1680	·09
·0100	4·7638	100·974	·057472	·1740	·10
·0200	4·1139	50·966	·1015	·1971	·1414
·0300	3·7338	34·282	·1406	·2134	·1732
·0400	3·4726	25·940	·1765	·2266	·2000
·0500	3·2949	20·931	·2102	·2378	·2236
·0600	3·1142	17·588	·2421	·2478	·2449
·0700	2·9814	15·199	·2726	·2568	·2646
·0800	2·8680	13·406	·3018	·2650	·2828
·0900	2·7693	12·009	·3300	·2727	·3000
·1000	2·6823	10·891	·3572	·2799	·3162
·200	2·1407	5·8334	·5946	·3363	·4472
·300	1·8527	4·1253	·7936	·3780	·5477
·400	1·6626	3·2587	·9683	·4131	·6325
·500	1·5241	2·7309	1·1273	·4435	·7071
·600	1·4167	2·3738	1·2742	·4709	·7746
·700	1·3301	2·1151	1·4064	·4977	·8367
·800	1·2581	1·9179	1·5407	·5192	·8944
·900	1·1971	1·7623	1·6634	·5411	·9487
1·000	1·1444	1·6361	1·7804	·5616	1·0000
1·10	1·0983	1·5315	1·8925	·5812	1·049
1·20	1·0574	1·4429	2·0002	·6000	1·095
1·30	1·0208	1·3668	2·1041	·6178	1·140
1·40	·9883	1·3009	2·2045	·6351	1·183
1·50	·9582	1·2432	2·3018	·6516	1·225
1·60	·9311	1·1917	2·3963	·6677	1·265
1·70	·9059	1·1462	2·4881	·6832	1·304
1·80	·8826	1·1046	2·5775	·6983	1·342
1·90	·8611	1·0677	2·6646	·7121	1·378
2·00	·8416	1·0337	2·7498	·7273	1·414
2·1	·8232	1·0020	2·8330	·7412	1·449
2·2	·8057	·9738	2·9145	·7548	1·483
2·3	·7893	·9475	2·9942	·7681	1·517
2·4	·7740	·9229	3·0724	·7811	1·549
2·5	·7596	·9002	3·1490	·7938	1·581
2·6	·7459	·8789	3·2243	·8064	1·612
2·7	·7328	·8592	3·2982	·8186	1·643
2·8	·7206	·8405	3·3709	·8306	1·673
2·9	·7090	·8231	3·4424	·8424	1·703
3·0	·6978	·8066	3·5127	·8540	1·732

TABLE I. (continued).

x.	$e^xK_0(x)$.	$-e^xK_1(x)$.	$\int_0^x e^xK_0(x)\,dx$.	$x\Big/\left[\int_0^x e^xK_0(x)\,dx\right]$.	$\sqrt{x}$.
3·2	·6768	·7762	3·6499	·8768	1·789
3·4	·6580	·7491	3·7836	·8986	1·844
3·6	·6405	·7243	3·9135	·9200	1·897
3·8	·6245	·7023	4·0399	·9406	1·949
4·0	·6093	·6814	4·1632	·9607	2·000
4·2	·5953	·6627	4·2837	·9804	2·049
4·4	·5823	·6453	4·4014	·9997	2·098
4·6	·5700	·6292	4·5169	1·0184	2·145
4·8	·5586	·6142	4·6300	1·0367	2·191
5·0	·5478	·6003	4·7406	1·0547	2·236
5·2	·5377	·5871	4·8491	1·0724	2·280
5·4	·5281	·5750	4·9557	1·0897	2·324
5·6	·5187	·5633	5·0604	1·1066	2·366
5·8	·5100	·5526	5·1633	1·1233	2·408
6·0	·5019	·5422	5·2640	1·1398	2·449
6·5	·4828	·5187	5·5114	1·1794	2·550
7·0	·4658	·4981	5·7482	1·2178	2·646
7·5	·4506	·4797	5·9775	1·2547	2·739
8·0	·4367	·4632	6·1972	1·2909	2·828
8·5	·4240	·4482	6·4120	1·3255	2·915
9·0	·4123	·4347	6·6221	1·3591	3·000
9·5	·4016	·4222	6·8328	1·3903	3·082
10·0	·3916	·4108	7·0234	1·4239	3·162
11·0	·3738	·3904	7·4076	1·4850	3·317
12·0	·3582	·3729	7·7723	1·5431	3·464
13·0	·3442	·3574	8·1248	1·6000	3·606
14·0	·3321	·3439	8·4583	1·6551	3·742
15·0	·3210	·3317	8·7887	1·7067	3·873
16·0	·3110	·3207	9·1036	1·7575	4·000
17·0	·3016	·3105	9·4102	1·8066	4·123
18·0	·2934	·3014	9·7090	1·8540	4·243
19·0	·2857	·2932	9·9974	1·9005	4·359
20·0	·2787	·2856	10·2790	1·9457	4·472
25·0	·2492	·2544	11·596	2·1559	5·000
36·0	·2082	·2111	14·090	2·5550	6·000
49·0	·1786	·1804	16·591	2·9535	7·000
64·0	·1564	·1576	19·092	3·3522	8·000
81·0	·1390	·1399	21·594	3·7509	9·000
100·0	·1252	·1258	24·097	4·1500	10·000

Table of the Functions $e^xK_0(x)$, $e^xK_1(x)$, $\int_0^x e^xK_0(x)\,dx$, $x\Big/\left[\int_0^x e^xK_0(x)\,dx\right]$, and $\sqrt{x}$.

TABLE II.

Constants of Wires Used in the Determination of Convection Constants.

Specimen.	r_{17}.	r_0.	r'_0.	α.	d cm.	$\rho_0 \times 10^{-6}$.
1	·0582	·0547	·0546([1])	$\cdot 00375_5$([4])	·01530	10·02
2	·0706	[·0662]	—	[·00375]	[·01387]([7])	[10·00]
3	·0881	·0828	·0834([2])	$\cdot 00374_5$([5])	·01230	9·98
4	·1030	·0968	·0953	$\cdot 00374_5$	·01165	10·19
5	·1340	·1262	·1225	$\cdot 00371_6$	·01013	9·91
6	·1672	·1577	·1534	$\cdot 00370_2$	·00930	10·38
7	·2220	·2092	·2192([3])	$\cdot 00370_9$([6])	·00775	10·38
8	1·570	1·519	1·590	$\cdot 00188_5$	·00383	18·3
9	1·599	1·550	1·606	$\cdot 00183_9$	·00390	19·15
10	1·852	1·795	1·847	$\cdot 00332_5$	·00283	11·60
10_1	1·865	1·765	—	·00328	·00278	10·70
$10'_1$	1·771	1·692	—	·00341	·00278	10·25

([1]) Mean of two determinations, 0·0545 and 0·0547.

([2]) Mean of two determinations each, 0·0834.

([3]) Mean of two determinations, 0·2193 and 0·2191.

([4]) Mean of three determinations, 0·003761, 0·003752, and 0·003752.

([5]) Mean of 0·003746 and 0·003745.

([6]) Mean of 0·003703 and 0·003715.

([7]) Wire No. 2 burned out during the experiment and the constants given in square brackets are the means of those of wires No. 1 and No. 2.

Description of Table II.—Constants of Wires Used in the Determination of Convection Constants.

The resistance per unit length at 17° C., denoted in Table II. by r_{17}, was obtained by the method described in Section 9 with the wire placed in the fork and annealed for several minutes before the convection measurements were made; care was taken to use a very small measuring current to avoid heating. These values of resistance were reduced to 0° C., making use of the temperature coefficients given in the fifth column. After the test had been carried out the temperature coefficient was determined for each wire by coiling it loosely in the grooves of a double screw thread cut in an ebonite cylinder. Potential terminals were fused to the wire at a measured distance apart and these were with the current terminals brought out to the bridge connections by means of fine copper leads from the glass test-tube in which the ebonite cylinder was inserted. The resistance between potential terminals was compared with that of a manganin standard at the ice and steam points. In this way both the resistance per unit length r'_0 and the temperature coefficient α were determined. It will be noticed that in the case of the large wires the prolonged annealing is more effective in diminishing the resistance than the reduction of diameter by "evaporation" in increasing it, while the contrary may be noticed in the finer wires. Wires No. 1 to No. 7 show a marked diminution in the temperature coefficient with diameter, a fact well worth a special investigation. Wires No. 8 to No. 10 were probably drawn from a specimen of impure platinum.

As described in Section 10, the diameters of the wires were measured directly by means of a microscope.([8]) It was found that the diameters of various portions of each wire varied about 1 per cent. The mean diameter was found from ten to fifteen observations. A typical set of readings for the finest wire (1 mil) is given below.

Wire No. 10. Readings of micrometer head at intervals of 3 mm. :—

0·405 0·395 0·412 0·413 0·395 0·395 0·400 0·400 0·395 0·041 Mean 0·401

Calibration of micrometer readings; micrometer readings corresponding to 0·030 cm. on stage micrometer situated in the same part of the field :—

0·432 0·434 0·431 Mean 0·432

Mean diameter of wire No. 10 . . . . . $d = 0{\cdot}00278$ cm.

([8]) An attempt was made to measure the diameter of the wire by rolling it between two narrow plates of smooth ground glass and counting the number of revolutions of the wire for a given displacement of the plates as measured by a micrometer screw—according to the method described by HORTON ('Phil. Trans.,' A, vol. 204, p. 407, 1905) in the measurement of the diameter of quartz fibres. In the present instance the platinum wire proved to be too soft for the employment of this method; the surface of the wire proved on microscopic examination to be badly roughened and the diameter sensibly changed by the action of the ground glass. The use of smooth glass plates was found to be unsuitable owing to the danger of slip.

TABLE III.

Specimen Observation Sheet, September 15, 1912.

Air temperature, 16°·8 C. Relative humidity, 62 per cent.
Wire No. 1. Diameter, 0·01530 cm. Length between potential terminals, 23·2 cm.
Total length, 27·2 cm. Resistance per unit length at 17° C., 0·0582 ohms.

R/R_{17}		1·727	2·37	2·965	3·51	4·00
Temperature, degrees C.		*227°*	*426°*	*627°*	*817°*	*1004°*
r/r_{17}		1·730	2·40	2·98	3·54	4·05
r ohms		*·1008*	*·1397*	*·1733*	*·2025*	*·2360*
T = 17·63 sec.	V_r = 19·11	1·40	1·76	2·02	2·22	2·44
$2\pi L$ = 337 cm.	V = 17·4					
$(1-s)$ = ·912	$\sqrt{V}$ = 4·17	*·198*	*·435*	*·703*	*1·000*	*1·405*
T = 6·84 sec.	V_r = 49·3	1·615	2·02	2·29	2·52	2·72
$2\pi L$ = 337 cm.	V = 45·0					
$(1-s)$ = ·912	$\sqrt{V}$ = 6·70	*·263*	*·572*	*·908*	*1·285*	*1·745*
T = 9·77 sec.	V_r = 86·4	1·81	2·27	2·50	2·74	2·92
$2\pi L$ = 844 cm.	V = 82·4					
$(1-s)$ = ·956	$\sqrt{V}$ = 9·07	*·331*	*·721*	*1·081*	*1·521*	*2·015*
T = 9·50 sec.	V_r = 174·8	2·05	2·54	2·82	3·08	3·28
$2\pi L$ = 1660 cm.	V = 165·0					
$(1-s)$ = ·947	$\sqrt{V}$ = 12·83	*·424*	*·905*	*1·375*	*1·925*	*2·54*
T = 5·55 sec.	V_r = 299	2·33	2·84	3·17	3·42	3·64
$2\pi L$ = 1660 cm.	V = 283					
$(1-s)$ = ·947	$\sqrt{V}$ = 16·8	*·549*	*1·127*	*1·740*	*2·370*	*3·140*
T = 3·425 sec.	V_r = 485	2·57	3·08	3·43	3·70	3·90
$2\pi L$ = 1660 cm.	V = 459					
$(1-s)$ = ·947	$\sqrt{V}$ = 21·4	*·667*	*1·328*	*2·030*	*2·77*	*3·60*
T = 2·60 sec.	V_r = 638	2·66	3·25	3·61	3·89	4·10
$2\pi L$ = 1660 cm.	V = 603					
$(1-s)$ = ·947	$\sqrt{V}$ = 24·55	*·712*	*1·480*	*2·260*	*3·07*	*3·97*
T = 1·954 sec.	V_r = 850	2·86	3·44	3·81	4·10	4·32
$2\pi L$ = 1660 cm.	V = 803					
$(1-s)$ = ·947	$\sqrt{V}$ = 28·3	*·825*	*1·660*	*2·525*	*3·42*	*4·41*
Constants B and C in the formula $W = B\sqrt{V}+C$ determined from the line of closest fit.		B = ·0264	·0508	·0772	·1012	·1272
		C = ·089	·241	·388	·602	·884

Description of Tables III. and IV.—Specimen Observation Sheets.

In Tables III. and IV. are set out the observations and calculations of the heat-losses in the case of the specimens Nos. 1 and 10 of the wires tested at different velocities. Although observations of relative humidity and of barometric pressures were made at the time of the experiment, no effect due to the variation of these factors could be detected in the final results. The ratio R/R_{17} refers to the entire length of wire between the potential terminals, while the ratio r/r_{17} refers to unit length at the temperature under consideration and is corrected for thermal expansion by the formula of Section 10. The time of a single revolution of the rotating arm, denoted by T, was obtained from the measurement of a chronograph sheet. The fork containing the wire was clamped in one of three positions so that the radius L was one of the values 53·7 cm., 134·0 cm., and 264·3 cm.; the apparent velocity of the wire through the air was given by $V_r = 2\pi L/T$. Owing to the "swirl" set up by the rotating arm, the true velocity of the wire through the air was obtained from the formula $V = (1-s) V_r$, the correction factor $(1-s)$ having been determined for each value of the radius in the manner described under Diagram II. The values printed in italics in the Tables III. and IV. are the heat-losses expressed in watts per unit length, as calculated from the current required to increase the resistance of the wire to the value R, and entered in the same compartment. These values when plotted against the square root of the velocity give rise to a family of straight lines, the temperature being the variable parameter. These are illustrated in figs. 4 and 5, and indicate how closely these curves correspond to the theoretical curve shown in Diagram I. It was found impossible to work at very low velocities owing to the disturbing effect of the free convection current set up by the heated wire. It was thus impossible to follow out the experimental curves to small velocities with a view to comparison with theory. It will be seen from the graph of the theoretical curve that the straight-line asymptote lies extremely close to the curve over the interval covered by experiment. In the reduction of the observations the line of closest fit through the experimentally determined points was taken to represent the equation of the asymptote given by formula (33).

TABLE IV.

Specimen Observation Sheet, October 28, 1912.

Air temperature, 15°·3 C. Relative humidity, 55 per cent.
Wire No. 10. Diameter, 0·00283 cm. Length between potential terminals, 3·64 cm.
Total length, 4·6 cm. Resistance per unit length at 17° C., 1·852 ohms.

R/R_{17}	1·398	1·727	2·00	2·37	2·60	2·965
Temperature, degrees C.	*162°*	*272°*	*366°*	*499°*	*584°*	*724°*
r/r_{17}	1·400	1·730	2·010	2·39	2·63	2·98
r ohms	*2·59*	*3·20*	*3·72*	*4·43*	*4·87*	*5·52*
T = 18·00 sec. V_r = 18·72	—	·200	·226	·260	·280	·296
2πL = 337 cm. V = 17·1						
$(1-s)$ = ·912 $\sqrt{V}$ = 4·13	—	*·128*	*·190*	*·300*	*·382*	*·486*
T = 6·64 sec. V_r = 50·8	—	·228	·256	·288	·306	·328
2πL = 337 cm. V = 46·3						
$(1-s)$ = ·912 $\sqrt{V}$ = 6·80	—	*·166*	*·244*	*·394*	*·456*	*·597*
T = 9·097 sec. V_r = 182·7	·171	·284	·314	·349	·370	·392
2πL = 1660 cm. V = 173·0						
$(1-s)$ = ·947 $\sqrt{V}$ = 13·13	*·0751*	*·258*	*·367*	*·540*	*·668*	*·851*
T = 4·83 sec. V_r = 342	·190	·314	·348	·388	·410	·432
2πL = 1660 cm. V = 324						
$(1-s)$ = ·947 $\sqrt{V}$ = 18·0	*·0939*	*·316*	*·451*	*·668*	*·819*	*1·035*
T = 3·77 sec. V_r = 441	·201	·328	·362	·408	·427	·451
2πL = 1660 cm. V = 417						
$(1-s)$ = ·947 $\sqrt{V}$ = 20·4	*·1051*	*·345*	*·488*	*·740*	*·890*	*1·126*
T = 2·41 sec. V_r = 688	·222	·358	·396	·440	·460	·485
2πL = 1660 cm. V = 650						
$(1-s)$ = ·947 $\sqrt{V}$ = 25·47	*·128*	*·411*	*·583*	*·859*	*1·030*	*1·305*
T = 1·80 sec. V_r = 922	(·234)(1)	·380	·420	·462	·489	·515
2πL = 1660 cm. V = 871						
$(1-s)$ = ·947 $\sqrt{V}$ = 29·5	(*·1422*)	*·463*	*·657*	*·944*	*1·163*	*1·470*
Constants B and C in the formula	**B = ·00425**	**·01292**	**·0181**	**·0256**	**·0308**	**·0384**
$W = B\sqrt{V}+C$ determined from the line of closest fit.	**C = ·0182**	**·081**	**·123**	**·205**	**·257**	**·338**

(1) Owing to an interruption in the series of readings the velocity corresponding to this observation was V=856 cm./sec.

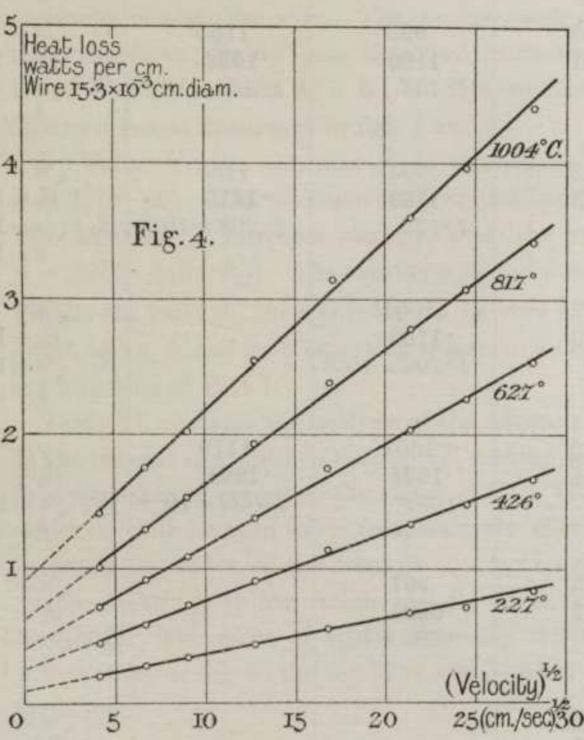


Fig. 4. *Heat-Loss Velocity Curves. Wire No. 1.*

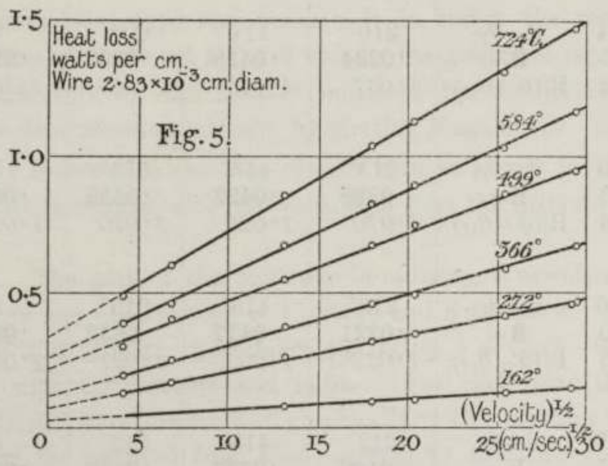


Fig. 5. *Heat-Loss Velocity Curves. Wire No. 10.*

TABLE V.

Wire No. 1	$\theta-\theta_{17}$	210°	409°	610°	800°	987°		$\sqrt{a}=\cdot0875$ $\beta_0=1\cdot257\times10^{-4}$ $\beta_0/\sqrt{a}=1\cdot437\times10^{-3}$ $b=4\cdot3\times10^{-5}$
$d=\cdot01530$	B =	**·0264**	**·0508**	**·0772**	**·1012**	**·1272**		
$a=\cdot00765$	$B/(\theta-\theta_{17})$	*1·255*	*1·241*	*1·265*	*1·265*	*1·290* $\times10^{-4}$		
Wire No. 2	$\theta-\theta_{17}$	210°	410°	609°	801°	990°	1161°	$\sqrt{a}=\cdot0832$ $\beta_0=1\cdot207\times10^{-4}$ $\beta_0/\sqrt{a}=1\cdot448\times10^{-3}$ $b=3\cdot7\times10^{-5}$
$d=\cdot01387$	B =	**·0255**	**·0515**	**·0728**	**·0985**	**·1233**	**·1481**	
$a=\cdot00693$	$B/(\theta-\theta_{17})$	*1·230*	*1·255*	*1·192*	*1·230*	*1·248*	*1·275* $\times10^{-4}$	
Wire No. 3	$\theta-\theta_{17}$	210°	411°	608°	803°	993°	1165°	$\sqrt{a}=\cdot0784$ $\beta_0=1\cdot130\times10^{-4}$ $\beta_0/\sqrt{a}=1\cdot441\times10^{-3}$ $b=8\cdot2\times10^{-5}$
$d=\cdot01230$	B =	**·0238**	**·0468**	**·0718**	**·0961**	**·1190**	**·1422**	
$a=\cdot00615$	$B/(\theta-\theta_{17})$	*1·133*	*1·142*	*1·180*	*1·196*	*1·198*	*1·222* $\times10^{-4}$	
Wire No. 4	$\theta-\theta_{17}$	210°	411°	608°	803°	993°	1165°	$\sqrt{a}=\cdot0763$ $\beta_0=1\cdot078\times10^{-4}$ $\beta_0/\sqrt{a}=1\cdot412\times10^{-3}$ $b=11\cdot4\times10^{-5}$
$d=\cdot01165$	B =	**·0224**	**·0458**	**·0696**	**·0932**	**·1168**	**·1410**	
$a=\cdot00582$	$B/(\theta-\theta_{17})$	*1·067*	*1·116*	*1·145*	*1·161*	*1·175*	*1·208* $\times10^{-4}$	
Wire No. 5	$\theta-\theta_{17}$	213°	413°	613°	807°	1001°		$\sqrt{a}=\cdot0710$ $\beta_0=1\cdot020\times10^{-4}$ $\beta_0/\sqrt{a}=1\cdot438\times10^{-3}$ $b=8\cdot1\times10^{-5}$
$d=\cdot01013$	B =	**·0228**	**·0422**	**·0655**	**·0880**	**·1102**		
$a=\cdot00506$	$B/(\theta-\theta_{17})$	*1·070*	*1·023*	*1·070*	*1·090*	*1·102* $\times10^{-4}$		
Wire No. 6	$\theta-\theta_{17}$	213°	416°	616°	812°	1001°	1179°	$\sqrt{a}=\cdot0682$ $\beta_0=\cdot988\times10^{-4}$ $\beta_0/\sqrt{a}=1\cdot448\times10^{-3}$ $b=8\cdot3\times10^{-5}$
$d=\cdot00930$	B =	**·0221**	**·0417**	**·0632**	**·0834**	**·1094**	**·1291**	
$a=\cdot00465$	$B/(\theta-\theta_{17})$	*1·038*	*1·002*	*1·027*	*1·027*	*1·092*	*1·092* $\times10^{-4}$	
Wire No. 7	$\theta-\theta_{17}$	213°	413°	612°	805°	997°		$\sqrt{a}=\cdot0622$ $\beta_0=\cdot872\times10^{-4}$ $\beta_0/\sqrt{a}=1\cdot405\times10^{-3}$ $b=12\cdot0\times10^{-5}$
$d=\cdot00775$	B =	**·0193**	**·0377**	**·0577**	**·0771**	**·0984**		
$a=\cdot00387$	$B/(\theta-\theta_{17})$	*·907*	*·913*	*·927*	*·958*	*·985* $\times10^{-4}$		
Wire No. 8	$\theta-\theta_{17}$	230°	434°	613°	877°			$\sqrt{a}=\cdot0438$ $\beta_0=\cdot640\times10^{-4}$ $\beta_0/\sqrt{a}=1\cdot460\times10^{-3}$
$d=\cdot00383$	B =	**·0140**	**·0272**	**·0372**	**·0540**			
$a=\cdot00192$	$B/(\theta-\theta_{17})$	*·657*	*·652*	*·625*	*·628* $\times10^{-4}$			
Wire No. 9	$\theta-\theta_{17}$		439°		885°			$\sqrt{a}=\cdot0441$ $\beta_0=\cdot630\times10^{-4}$ $\beta_0/\sqrt{a}=1\cdot430\times10^{-3}$
$d=\cdot00390$	B =		**·0261**		**·0542**			
$a=\cdot00195$	$B/(\theta-\theta_{17})$		*·634*		*·625* $\times10^{-4}$			
Wire No. 10	$\theta-\theta_{17}$	145°	255°	349°	482°	567°	707°	$\sqrt{a}=\cdot0380$ $\beta_0=\cdot531\times10^{-4}$ $\beta_0/\sqrt{a}=1\cdot396\times10^{-3}$ $b=8\cdot1\times10^{-5}$
$d=\cdot00283$	B =	**·00425**	**·01292**	**·0181**	**·0256**	**·0308**	**·0384**	
$a=\cdot00144$	$B/(\theta-\theta_{17})$	*·332*	*·544*	*·545*	*·551*	*·561*	*·557* $\times10^{-4}$	
$W=B\sqrt{V}+C,\quad B=\beta(\theta-\theta_{17})$ $\beta=\beta_0[1+b(\theta-\theta_{17})]$						*Mean values*		$\beta_0/\sqrt{a}=1\cdot432\times10^{-3}$ $b=8\cdot0\times10^{-5}$

Analysis of the Convection Constant B

Description of Tables V. and VI.—Analysis of the Convection Constants B *and* C.

In Tables V. and VI. are summarised the results of the entire series of observations on the ten specimens of platinum wire. The measurement of the diameters and the calculation of the temperature of the wires have already been described in Sections 10 and 11. In the case of each wire the constants B and C of the formula $W = B\sqrt{V} + C$ were determined from the position of the lines of closest fit to the observed points illustrated in figs. 4 and 5.

In Table V. the constant B is printed in heavy-faced type and beneath it in italics the ratio $\beta = B/(\theta - \theta_0)$. The variation of B with temperature is illustrated for each wire by the graphs of fig. 6. The constant β increases slightly with the temperature in the manner indicated by the formula $\beta = \beta_0[1 + b(\theta - \theta_0)]$. The constants β_0 and b were determined graphically by plotting β against $(\theta - \theta_0)$. Finally the ratio $\beta_0/\sqrt{a}$ was found to be very nearly constant in the case of each wire as shown in the graph of fig. 7, and its mean value is seen to be in fair agreement with that calculated from the theoretical investigation of Part I.

Table VI. contains the analysis of the constant C. The part of this term due to radiation is calculated in the manner described in Section 13, making use of the observations of LUMMER and KURLBAUM. In heavy-faced type are given the values of $C_0 = C - E$ and beneath it, in italics, the ratios $\gamma = C_0/(\theta - \theta_0)$, which it will be seen vary comparatively slowly with temperature and radius. For each wire the constants γ_0 and c of the formula $\gamma = \gamma_0[1 + c(\theta - \theta_0)]$ were determined graphically. The constant γ_0 varies slightly with the radius and its value is not far removed from that required by the theory of Section 6. The value of γ_0 for Wire No. 2 shows a discrepancy which was explained, on microscopic observation, as due to the fact that owing to the use of an imperfect die the wire was badly scored along its length.

TABLE VI.

Wire No. 1	$\theta-\theta_{17}$	210°	409°	610°	800°	987°		
	C	*·089*	*·241*	*·388*	*·602*	*·884*		$\gamma_0=3·85\times10^{-4}$
$a=$ ·00765	E	·002	·005	·014	·040	·087		
	$C_0=C-E=$	**·087**	**·237**	**·374**	**·562**	**·797**		
	$C_0/(\theta-\theta_{17})$	*4·15*	*5·79*	*6·13*	*7·03*	*8·07* $\times10^{-4}$		$c=$ ·001075
Wire No. 2	$\theta-\theta_{17}$	210°	410°	609°	801°	990°	1161°	
	C	*·085*	*·183*	*·392*	*·601*	*·850*	*1·220*	$\gamma_0=3·06\times10^{-4}$
$a=$ ·00693	E	·001	·004	·012	·036	·079	·154	
	$C_0=C-E=$	**·084**	**·179**	**·380**	**·565**	**·771**	**1·066**	
	$C_0/(\theta-\theta_{17})$	*4·00*	*4·37*	*6·23*	*7·05*	*7·78*	*9·17* $\times10^{-4}$	($c=$ ·001649)
Wire No. 3	$\theta-\theta_{17}$	210°	411°	608°	803°	993°	1165°	
	C	*·092*	*·228*	*·357*	*·574*	*·814*	*1·106*	$\gamma_0=3·62\times10^{-4}$
$a=$ ·00615	E	·001	·003	·010	·030	·069	·129	
	$C_0=C-E=$	**·091**	**·225**	**·347**	**·544**	**·745**	**·977**	
	$C_0/(\theta-\theta_{17})$	*4·34*	*5·48*	*5·62*	*6·65*	*7·51*	*8·38* $\times10^{-4}$	$c=$ ·001045
Wire No. 4	$\theta-\theta_{17}$	210°	411°	608°	803°	993°	1165°	
	C	*·096*	*·217*	*·382*	*·567*	*·823*	*1·085*	$\gamma_0=3·49\times10^{-4}$
$a=$ ·00582	E	·001	·004	·011	·032	·071	·138	
	$C_0=C-E=$	**·095**	**·213**	**·371**	**·535**	**·752**	**·947**	
	$C_0/(\theta-\theta_{17})$	*4·53*	*5·18*	*6·11*	*6·66*	*7·58*	*8·13* $\times10^{-4}$	$c=$ ·001159
Wire No. 5	$\theta-\theta_{17}$	213°	413°	613°	807°	1001°		
	C	*·070*	*·209*	*·369*	*·561*	*·800*		$\gamma_0=3·39\times10^{-4}$
$a=$ ·00506	E	·001	·003	·009	·026	·063		
	$C_0=C-E=$	**·069**	**·206**	**·360**	**·535**	**·737**		
	$C_0/(\theta-\theta_{17})$	*3·24*	*5·00*	*5·87*	*6·63*	*7·36* $\times10^{-4}$		$c=$ ·001181
Wire No. 6	$\theta-\theta_{17}$	213°	416°	616°	812°	1001°	1179°	
	C	*·088*	*·211*	*·354*	*·536*	*·764*	*1·057*	$\gamma_0=3·31\times10^{-4}$
$a=$ ·00465	E	·001	·003	·008	·025	·056	·110	
	$C_0=C-E=$	**·087**	**·208**	**·346**	**·511**	**·708**	**·947**	
	$C_0/(\theta-\theta_{17})$	*4·08*	*5·00*	*5·62*	*6·28*	*7·07*	*8·03* $\times10^{-4}$	$c=$ ·001155
Wire No. 7	$\theta-\theta_{17}$	213°	413°	612°	805°	997°		
	C	*·081*	*·213*	*·358*	*·546*	*·728*		$\gamma_0=3·29\times10^{-4}$
$a=$ ·00387	E	·001	·002	·007	·020	·046		
	$C_0=C-E=$	**·080**	**·211**	**·351**	**·526**	**·682**		
	$C_0/(\theta-\theta_{17})$	*3·76*	*5·10*	*5·74*	*6·53*	*6·83* $\times10^{-4}$		$c=$ ·001164
Wire No. 8	$\theta-\theta_{17}$	230°	434°	613°	877°			
	C	*·096*	*·204*	*·349*	*·584*			$\gamma_0=3·21\times10^{-4}$
$a=$ ·00192	E	·000	·001	·004	·009			
	$C_0=C-E=$	**·096**	**·203**	**·345**	**·575**			
	$C_0/(\theta-\theta_{17})$	*4·17*	*4·68*	*5·64*	*6·56* $\times10^{-4}$			$c=$ ·001191
Wire No. 9	$\theta-\theta_{17}$		439°		885°			
			—		—			
$a=$ ·00195			—		—			
Wire No. 10	$\theta-\theta_{17}$	145°	255°	349°	482°	567°	707°	
	C	*·0182*	*·081*	*·123*	*·205*	*·257*	*·338*	$\gamma_0=2·26\times10^{-4}$
$a=$ ·00144	E	·000	·000	·001	·001	·002	·005	
	$C_0=C-E=$	**·0182**	**·081**	**·122**	**·204**	**·255**	**·333**	
	$C_0/(\theta-\theta_{17})$	—	*3·18*	*3·50*	*4·23*	*4·50*	*4·72* $\times10^{-4}$	($c=$ ·001681)

$W = B\sqrt{V}+C$, $C_0 = C-E$, $E = 2\pi a\times ·514\,(\theta/1000)^{5·2}$, *Mean value of c*

$C_0=\gamma_0(\theta-\theta_{17})[1+c(\theta-\theta_{17})]$, $\gamma_0 = 2·50\times10^{-4}(1+70a)$, $c =$ ·00114

***Analysis of the Convection Constant* C.**

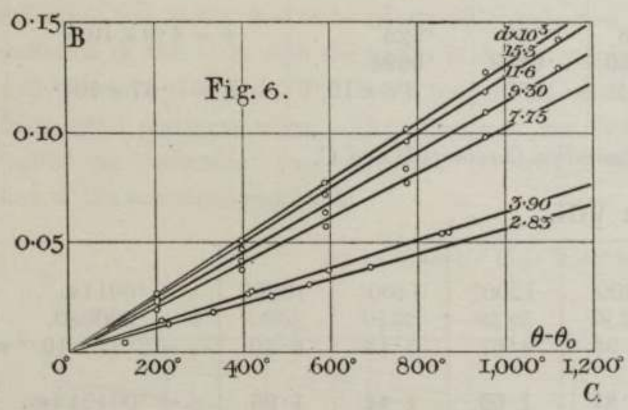


Fig. 6. *Variation of B with the Temperature.*

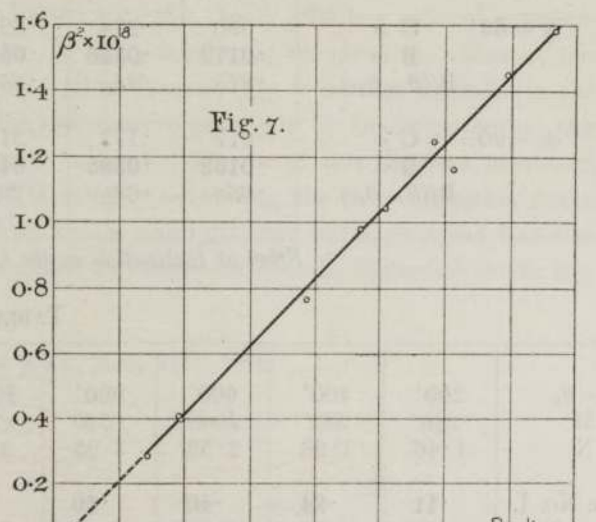


Fig. 7. *Variation of β_0 with the Radius of the Wires.*

TABLE VII.

Wire No. 1. Diameter, 0·0153 cm. Inclined at angle ϕ with the vertical : direction of stream horizontal.

	$\theta - \theta_{17}$	210°	409°	610°	800°	987°	
$\phi = 0°$	C =	·089	·241	·388	·602	·884	$b = 1{\cdot}5 \times 10^{-5}$
	B =	**·0264**	**·0508**	**·0772**	**·1012**	**·1272**	
	$B/(\theta - \theta_{17})$	*1·255*	*1·241*	*1·265*	*1·265*	*1·290* $\times 10^{-4}$	$\beta_0 = 1{\cdot}26 \times 10^{-4}$
$\phi = 31°$	C =	·109	·259	·362	·520	·860	$b = 8{\cdot}0 \times 10^{-5}$
	B =	**·0227**	**·0460**	**·0725**	**·0985**	**·1213**	
	$B/(\theta - \theta_{17})$	*1·081*	*1·122*	*1·188*	*1·231*	*1·230* $\times 10^{-4}$	$\beta_0 = 1{\cdot}15 \times 10^{-4}$
$\phi = 45°$	C =	·084	·206	·413	·639	1·000	$b = 5{\cdot}0 \times 10^{-5}$
	B =	**·0216**	**·0452**	**·0644**	**·0843**	**·1034**	
	$B/(\theta - \theta_{17})$	*1·030*	*1·103*	*1·056*	*1·054*	*1·049* $\times 10^{-4}$	$\beta_0 = 1{\cdot}02 \times 10^{-4}$
$\phi = 63°$	C =	·090	·224	·396	·562	·846	$b = 9{\cdot}0 \times 10^{-5}$
	B =	**·0172**	**·0346**	**·0522**	**·0704**	**·0885**	
	$B/(\theta - \theta_{17})$	*·818*	*·845*	*·856*	*·880*	*·897* $\times 10^{-4}$	$\beta_0 = {\cdot}81 \times 10^{-4}$
$\phi = 90°$	C =	·077	·177	·318	·551	·823	$b = 4{\cdot}0 \times 10^{-5}$
	B =	**·0138**	**·0285**	**·0430**	**·0549**	**·0694**	
	$B/(\theta - \theta_{17})$	*·658*	*·696*	*·704*	*·687*	*·703* $\times 10^{-4}$	$\beta_0 = {\cdot}67 \times 10^{-4}$

Effect of Inclination on the Convection Constants B *and* C.

TABLE VIII.

$\theta - \theta_0$ / M / N	200°	400°	600°	800°	1000°	1200°	1400°	1600°	
M	*246*	*582*	*1000*	*1530*	*2140*	*2840*	*3640*	*4520*	$c = {\cdot}00114.$
N	1·46	1·98	2·56	3·23	3·93	4·67	5·43	6·30	$b = {\cdot}000080.$ $\kappa_0 = 2{\cdot}37 \times 10^{-4}$ watts.
Wire No. I.	**·11**	**·24**	**·40**	**·60**	**·81**	**1·09**	**1·44**	**1·96**	$d = {\cdot}00404$ cm.
$\log b/a$	*3·33*	*3·60*	*3·73*	*3·80*	*3·93*	*3·88*	*3·77*	*3·43*	
V_0	6·9	5·3	4·6	4·3	3·8	4·0	4·5	6·2	*Mean* $\log b/a = 3{\cdot}68$
V cm./sec.	*10·0*	*10·5*	*11·8*	*13·9*	*15·0*	*18·7*	*24·4*	*39·0*	
Wire No. II.	**·12**	**·29**	**·47**	**·70**	**·95**	**1·24**	**1·62**	**2·19**	$d = {\cdot}00691$ cm.
$\log b/a$	*3·05*	*2·98*	*3·17*	*3·26*	*3·35*	*3·41*	*3·35*	*3·07*	
V_0	5·3	5·7	4·7	4·3	3·9	3·7	4·0	5·2	*Mean* $\log b/a = 3{\cdot}20.$
V cm./sec.	*7·7*	*11·3*	*12·0*	*13·9*	*15·3*	*17·2*	*21·7*	*32·8*	
Wire No. III.	**·13**	**·31**	**·52**	**·75**	**1·03**	**1·29**	**1·64**	**2·18**	$d = {\cdot}01262$ cm.
$\log b/a$	*2·81*	*2·79*	*2·87*	*3·04*	*3·09*	*3·28*	*3·31*	*3·08*	
V_0	3·7	3·8	3·5	3·0	2·8	2·3	2·3	2·8	*Mean* $\log b/a = 3{\cdot}03.$
V cm./sec.	*5·4*	*7·5*	*9·0*	*9·7*	*11·0*	*10·7*	*12·5*	*17·6*	
Wire No. IV.	**·17**	**·38**	**·66**	**·95**	**1·29**	**1·67**	**2·06**	**2·49**	$d = {\cdot}02508$ cm.
$\log b/a$	*2·15*	*2·28*	*2·26*	*2·40*	*2·46*	*2·53*	*2·63*	*2·70*	
V_0	3·6	3·2	3·2	2·8	2·7	2·5	2·2	2·1	*Mean* $\log b/a = 2{\cdot}43.$
V cm./sec.	*5·2*	*6·3*	*8·2*	*9·0*	*10·6*	*11·7*	*11·9*	*13·2*	
Wire No. V.	**·22**	**·51**	**·85**	**1·28**	**1·71**	**2·22**	**2·85**	**3·50**	$d = {\cdot}0510$ cm.
$\log b/a$	*1·66*	*1·70*	*1·75*	*1·78*	*1·86*	*1·91*	*1·91*	*1·92*	
V_0	2·8	2·8	2·6	2·6	2·4	2·3	2·3	2·2	*Mean* $\log b/a = 1{\cdot}81.$
V cm./sec.	*4·1*	*5·5*	*6·7*	*8·4*	*9·4*	*10·7*	*12·5*	*13·9*	

$$H = 2\pi\kappa_0(\theta-\theta_0)\left[1+c(\theta-\theta_0)\right]/\left[\log b/a\right], \qquad M = (\theta-\theta_0)\left[1+c(\theta-\theta_0)\right].$$
$$N = \left[1+c(\theta-\theta_0)\right]^2/\left[1+b(\theta-\theta_0)\right]^2, \qquad \mathbf{V} = N\mathbf{V}_0, \qquad \mathbf{V}_0 = \kappa_0 e^{1-\gamma}/(s_0\sigma_0 b).$$

Analysis of LANGMUIR'S *Observations on Free Convection from Small Platinum Wires.*

Description of Table VII.—Effect of Inclination on the Convection Constants of Small Wires.

In Table VII. is set out the analysis of observations on the effect of inclination on the convection constants of one of the wires. The wire was mounted in the fork in such a manner that the end connections offered no obstruction to the flow of air, illustrated in (*d*) Diagram II. As described in Section 14, the previously determined laws of convection were found to hold good; the values of the constants b and β_0 were determined in the manner already described, and the variation of the latter with the inclination is shown graphically in fig. 8.

Description of Table VIII.—Analysis of Observations on Free Convection.

In the accompanying table are set out LANGMUIR'S(1) observations on the free convection of heat from small platinum wires, discussed in terms of the theory of Part I. of the present paper. For the small velocities of the free convection current set up by the heated wire this theory affords a rational interpretation of the empirical formula obtained by LANGMUIR. In heavy-faced type are given the values of the heat-losses corrected for radiation; the denominator $\log(b/a)$ of formula (75) is set out in italics in the following line and is seen to be nearly independent of the temperature, especially for the larger wires; the variation of this term with the radius is shown graphically in fig. 9. The use of this diagram, combined with formulæ (75) and (76), offers a convenient method of roughly estimating the free convection losses from small platinum wires. The velocity of the air-flow, which would give rise to the observed heat-loss (called the "effective" free convection current), is calculated and is set out in italic characters in the last line of the accompanying table.

(1) LANGMUIR, 'Phys. Rev.,' 34, p. 415, Table VIII., 1912.

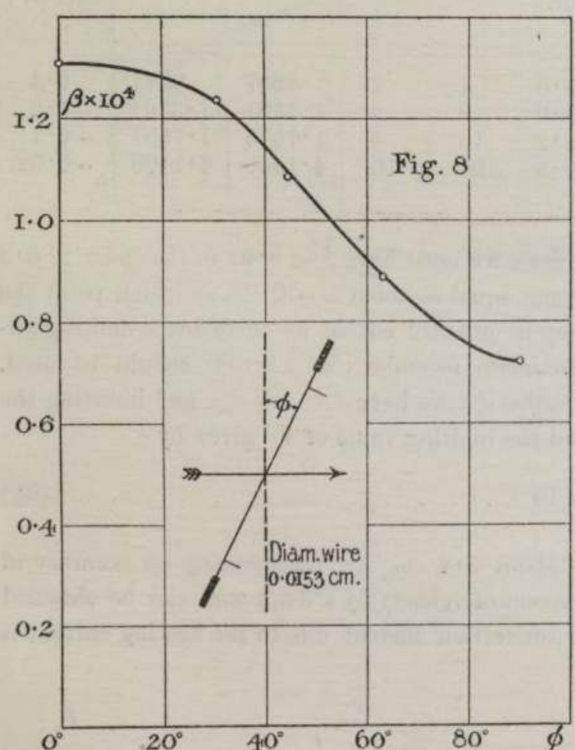


Fig. 8. *Effect of inclination on the convection constant* β_0.

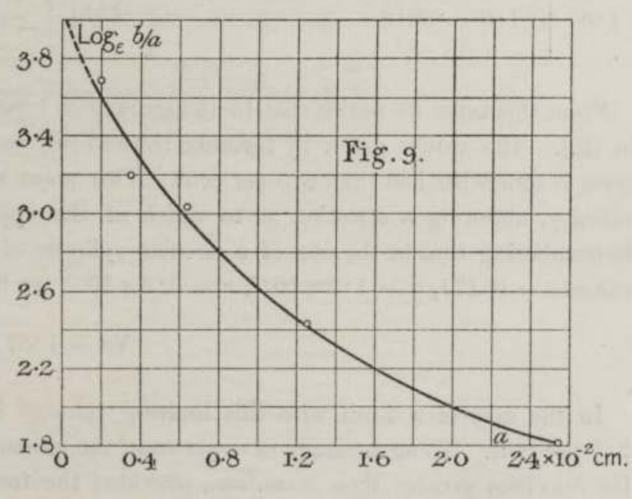


Fig. 9. *Variation of* $\log b/a$ *with diameter of the wire.*

Diagram I.—Graph of the Function $x \Big/ \left[\int_0^x e^x K_0(x)\,dx\right]$.

The values of the function $y = x \Big/ \left[\int_0^x e^x K_0(x)\,dx\right]$ are required for the purpose of making a comparison between theory and experiment in the convection losses of a cylindrical wire cooled by a stream of air. These values may be easily obtained from Table I. or from the graph of the function given in Diagram I. We notice that, for small values of the variable x, equation (23) enables us to write approximately

$$y = 1/[(1-\gamma) - \log(x/2)] = 1/[1\cdot11593 + \log(1/x)] \quad \text{(i.)}$$

(Error < 1 per cent. if $x < 0\cdot02$.)

When x is sufficiently large (24) gives approximately

$$y = 1/(2\pi) + \sqrt{x/2\pi} = 0\cdot15915 + 0\cdot39894\sqrt{x} \quad \text{(ii.)}$$

(Error < 1 per cent. if $x > 0\cdot3$.)

It is of importance in the application of these approximate equations to know for what values of x they may be employed with sufficient accuracy. The following table sets forward this comparison :—

x.	$\sqrt{x}$.	y.	Formula (i.)	Per cent. error.	Formula (ii.)	Per cent. error.	x.	$\sqrt{x}$.	y.	Formula (ii.)	Per cent. error.
·01	·1	·1740	·1748	+0·5	·1990	+15·0	4	2	·9607	·9571	−0·4
·04	·2	·2266	·2307	+2·0	·2389	+ 5·0	9	3	1·3591	1·3561	−0·2
·09	·3	·2727	·2809	+3·0	·2788	+ 2·2	16	4	1·7575	1·7551	−0·1
1·0	1·0	·5616	—	—	·5581	− 0·6	100	10	4·1500	4·1490	−0·02

From this table we notice that to an accuracy of 1 per cent. we must have $x < 0\cdot02$ in (i.) and $x > 0\cdot3$ in (ii.). The values given by formulæ (i.) and (ii.) become equal at about $x = 0\cdot08$, at which point the error is somewhat less than $2\cdot5$ per cent. This point may in practice enable us to define a limiting air-velocity, affording a criterion as to which of the approximate formulæ (32) or (33) should be used. Remembering that in the case of a circular cylinder of radius $d/2$ we have $x = s\sigma V\,d/\kappa$, and inserting the values $s = 0\cdot171$, $\sigma = 1\cdot3\times10^{-3}$, $\kappa = 5\cdot2\times10^{-5}$, we find the limiting value of Vd given by

$$Vd = 1\cdot87\times10^{-2} \quad \text{(iii.)}$$

In the case of a 3-mil wire this limiting velocity is about $2\cdot4$ cm./sec. representing an accuracy of $2\cdot5$ per cent. To an accuracy of 1 per cent. the measurement of velocity by a 3-mil wire can be obtained for velocities greater than 9 cm./sec., provided the free convection current due to the heating current is less than this amount.

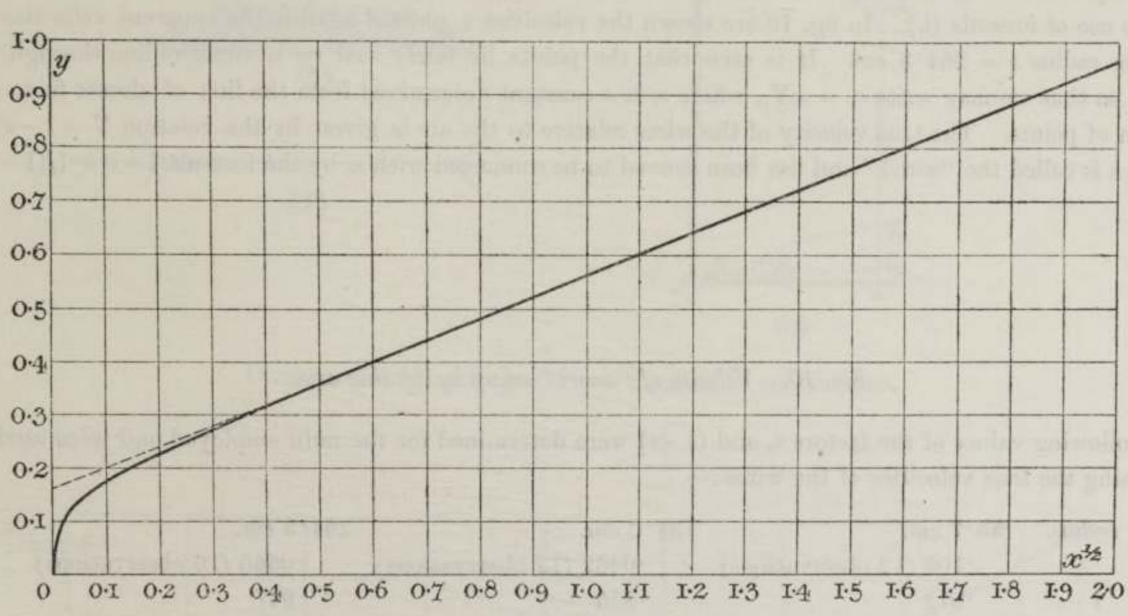


Diagram I. Graph of the Function $x \Big/ \left[\int_0^x e^x \mathrm{K}_0(x)\, dx\right]$.

Description of Diagram II.

In fig. (*a*) of Diagram II. is shown the general arrangement of the rotating arm and of the electrical connections by means of which the air-velocities in the determination of the convection constants of the small platinum wires in absolute measure were determined. In order to make the correction for the "swirl" described in Section 9, a wire was inserted in the fork which was then clamped to the rotating arm at the various radii employed during the tests. Representing by V_r the velocity of the wire relative to the room, and by i the current required to bring the wire to a given resistance, the constants i_o^2 and k_r of the formula

$$i^2 = i_o^2 + k_r \sqrt{V_r}, \quad \text{(i.)}$$

were determined. The wire was then removed and placed in a stationary fork as close to the original position C_1C_2 as possible. The arm was then set into motion and the velocity v_r at radius r of the vortex set up by the rotation measured in terms of the current required to bring the wire to the same resistance by the use of formula (i.). In fig. 10 are shown the velocities v_r plotted against the apparent velocities V_r for the radius $r = 264 \cdot 3$ cm. It is seen that the points lie fairly well on a straight line through the origin so that we may write $v_r = s_r V_r$, where s_r is a constant determined from the line of closest fit to the system of points. The true velocity of the wires relative to the air is given by the relation $V = 1 - s) V$ where s is called the "swirl" and has been proved to be connected with s_r by the formula $1 - s = 1/(1 + s_r)$.

Fig. 10. Velocity of "swirl" set up by rotating arm.

The following values of the factors s_r and $(1 - s)$ were determined for the radii employed and were used in obtaining the true velocities of the wires:—

radius.	53·7 cm.	134·0 cm.	264·3 cm.
s_r	·103 (12 observations)	·0462 (12 observations)	·0565 (16 observations)
$1 - s$	·912	·956	·947.

The mercury-contact slip-rings by means of which connections were made to the rotating arm are drawn in fig. (*b*). In the upper block of ebonite are fastened four concentric rings of copper strip 0·036 cm. thick connected to the four terminals $C_1C_2P_1P_2$ by means of heavy brass screws, three to each ring. These four rings fit into four concentric troughs cut into the lower block and filled with mercury. Coiled in the bottom of each trough are heavy copper wires of diameter 0·23 cm. passing out through the ebonite to four terminals fastened to the lower block. The two blocks were kept in position by a steel pin through the centre of each. The resistance of the two outer rings in series was found to be 0·0077 ohms and remained extremely constant while the rotating arm to which they were attached was set in motion. The two outer rings were found to be capable of carrying as much as 50 amperes and this capacity could easily be increased to 100 amperes by slight changes of detail.

Fig. (*c*) shows in greater detail the construction of the fork for holding the wires under test and illustrates the manner in which the potential terminals were attached to avoid end corrections and the way that the wire was kept under suitable tension with a frictional resistance to damp the vibrations of the wires.

Fig. (*d*) illustrates the manner of arranging the end-connections in the measurement of the effect of inclination on the convection constants.

In Plate 8 are reproduced photographs of the apparatus illustrated in figs. (*a*), (*b*), and (*c*).

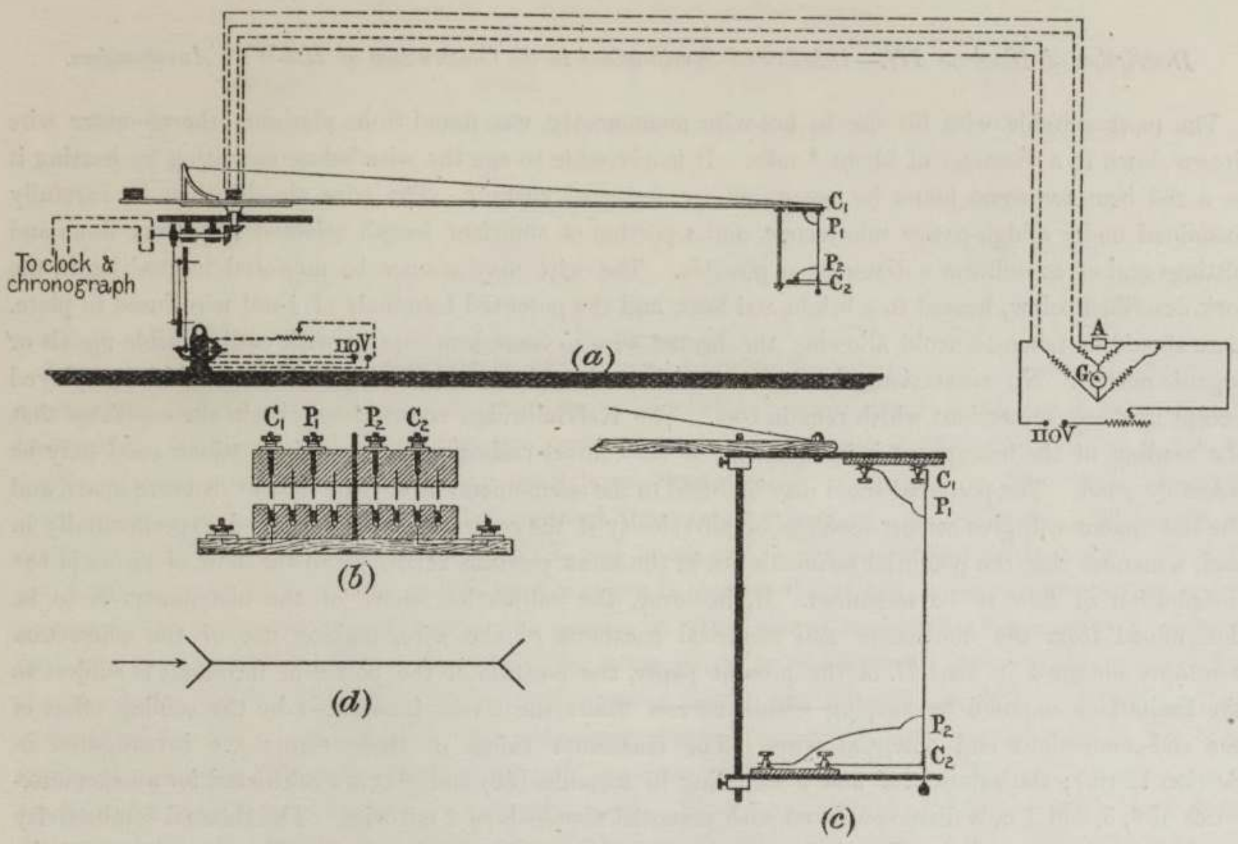


Diagram II. Rotating arm and diagram of connections.

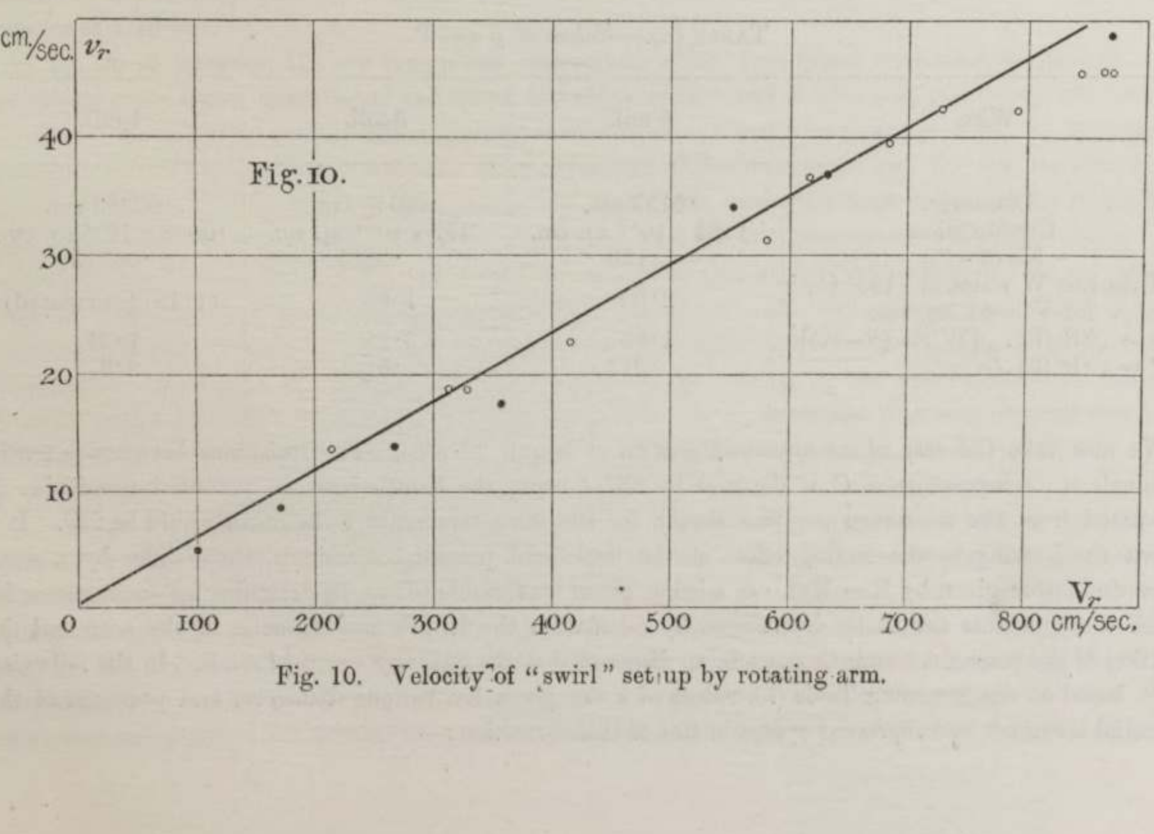


Fig. 10. Velocity of "swirl" set up by rotating arm.

Description of Diagram III.—Details and Specifications in the Construction of Hot-Wire Anemometers.

The most suitable wire for use in hot-wire anemometry was found to be platinum thermometer wire drawn down to a diameter of about 3 mils. It is advisable to age the wire before mounting by heating it to a red heat for some hours by means of an electrical current. The wire should then be carefully examined under a high-power microscope, and a portion of sufficient length selected free from flaws and pittings and of as uniform a diameter as possible. The wire may at once be mounted in position in the fork described below, heated to a bright red heat, and the potential terminals of 1-mil wire fused in place. Care should be taken to avoid allowing the heated wire to come into contact with easily fusible metals or organic matter. No connections should be fused in a blow-pipe flame, and no solder should be employed except in those connections which remain cool. The Kelvin-bridge connections obtain the condition that the reading of the instrument is independent of the contact resistances at C_1 and C_2, which need only be tolerably good. The potential leads may be fused to the anemometer at any convenient distance apart, and the instrument will give correct readings of air-velocity if the constants are determined experimentally in such a manner that the potential terminals are in the same position relatively to the flow of air as in the distribution of flow to be measured. If, however, the calibration curve of the instrument is to be determined from the dimensions and electrical constants of the wire, making use of the convection constants obtained in Part II. of the present paper, the position of the potential terminals is subject to the limitations imposed by keeping within narrow limits the errors introduced by the cooling effect of the end-connections and potential wires. The maximum values of these errors are investigated in Section 12 (ii.); the values of P and p occurring in formulæ (60) and (61) are calculated for anemometer-wires of 6, 3, and 1 mils diameter, fitted with potential terminals of 1-mil wire. The thermal conductivity of platinum is taken to be $K = 0\cdot7$ watts per sq. cm. per sec. per degree C. The temperature of the wire is that corresponding to the ratio $\overline{R}/R_0 = 4$, adopted as the most suitable for hot-wire anemometry and representing a value $\theta - \theta_0 = 1165°$ C. The heat-loss W employed in the table given below is that obtained from experiment at the lowest velocity $V = 81$ cm./sec. :—

TABLE (i.).—*Values of p and* P.

Wire.	6-mil.	3-mil.	1-mil.
Diameter	·0153 cm.	·00775 cm.	·00283 cm.
Cross-section	$1\cdot84 \times 10^{-4}$ sq. cm.	$\cdot472 \times 10^{-4}$ sq. cm.	$\cdot0628 \times 10^{-4}$ sq. cm.
$K\omega(\theta - \theta_0)$	·150	·0385	·00513
Heat-loss W watts at 1165° C., for V = 81 cm./sec.	2·00	1·60	(1·10) (calculated)
$p = \sqrt{(R_0/\overline{R})} \cdot \sqrt{[W/K\omega(\theta - \theta_0)]}$	1·83	3·22	7·31
$P = \sqrt{(\overline{R}/R_0)} \cdot (a'/a)^{3/4}$	·213	·51	2·0

We now take the case of an anemometer-wire of length $2(l + l_0)$. The resistance between potential terminals at a temperature $\theta°$ C. is denoted by $2Rl$, l being the length between potential terminals. If calculated from the resistance per unit length for the same temperature the result would be $2\overline{R}l$. It is shown that, owing to the cooling effect of the leads and potential terminals, these differ by a small correction-factor given by $R = \overline{R}(1 - \epsilon)$, ϵ being given by formula (62). In designing an anemometer for which the constants are to be determined by calculation the length and diameter of the wire and the position of the potential terminals must be so disposed that the quantity ϵ remain small. In the following table, based on the preceding Table (i.), values of ϵ are given for various diameters and positions of the potential terminals and represent overestimates of this correction :—

TABLE (ii.).—*Values of the Correction Factor* ϵ.

Wire.	6-mil.	3-mil.	1-mil.
cm. $l = 10 \cdot 0$, $l_0 = 2 \cdot 0$	·0050	·0047	·0051
$l = 2 \cdot 0$, $l_0 = 0 \cdot 5$	·092	·041	·026

A convenient form of fork suitable for holding in position the anemometer wires, and offering a minimum of disturbance to the flow of air in its neighbourhood, is illustrated in fig. (*a*) of Diagram III. Fastened to a block of ebonite are the two arms of the fork, consisting of steel strips about 5 mm. in width. At the end of each is soldered a small brass block, drilled to receive two fine needles fastened about 1 cm. apart. Threaded through the eyes of these two needles is a 3-mil platinum wire, having its extremities firmly clamped in the brass block just mentioned. The anemometer-wire is held by these two loops as indicated in the figure and by this means is kept under tension by the elastic support, and is protected from accidental damage by the two needles on either side of each end. The tension is adjusted by a fine thread carried down from each of the brass blocks to an adjustable screw in the centre of the ebonite block; this thread is also effective in preventing lateral vibrations of the fork. Carried up from each end of the ebonite block are two thin steel strips crossing each other to the opposite arm of the fork, insulated from each other and also from the fork by means of thin mica strip. These strips serve to brace the fork and at the same time serve as potential leads. At each extremity is soldered a small brass block drilled to hold a fine needle, at the extremity of which is soldered a thin copper wire. The 1-mil platinum potential terminals fused to the anemometer-wire are carried to these copper wires to which they are easily soldered. The complete apparatus is illustrated photographically in Plate 8 (*c*), mounted on a micrometer screw for measuring rapid gradients of turbulent flow. In the experiments previously referred to in Section 17 it was found possible to resolve a gradient in which the velocity changed by 5 cm./sec. over a distance of 1/10 mm.

In fig. (*b*) of Diagram III. are drawn the connections which were found convenient in practice. The resistances *a* and *b* were made equal and about 500 ohms while α and β were adjusted to equality at about 250 ohms. In order to protect the anemometer-wire from accidently burning out, a key K_1 was inserted by means of which it was automatically short-circuited; a double-contact key K_2 was inserted in the galvanometer circuit in such a way that contact was first made through a high resistance for preliminary adjustments; it was also found convenient to connect the galvanometer to an adjustable shunt. The resistance B was constructed of No. 23 B. and S. manganin wire wound non-inductively on an asbestos frame in such a manner as to dissipate a maximum amount of heat; its resistance measured between potential terminals soldered to the wire was adjusted to four times that of the anemometer-wire at room temperature. By means of a fine-adjustment rheostat R the current in the anemometer-wire could be adjusted until a balance was obtained on the galvanometer. It is important that the rheostat be always re-adjusted to the position of minimum current to avoid over-heating the wire should the velocity of the air-flow suddenly diminish; this may be easily accomplished by means of a spring control.

In taking a measurement of velocity, the key K_1 is pressed down and the current as read by the ammeter slowly increased until on pressing down the key K_2 a balance is obtained on the galvanometer. From the reading of the current the velocity may readily be obtained from a calibration curve corresponding to the formula (80), $i^2 = i_0^2 + k\sqrt{V}$. It has already been mentioned that a dynamometer form of instrument will, if at the same time sufficiently accurate, prove to be more suitable for the measurement of current in that the scale-readings will be more open at high velocities; the same result could also be achieved by suitably shaping the pole-pieces of the permanent magnet of a direct-current instrument and in the event of being able to employ standardized anemometer-wires, the scale of the ammeter could be graduated to read velocities directly.

The galvanometer employed was a Weston portable instrument with jewel bearings and capable of detecting a current of about 10^{-6} amperes. This degree of sensitivity is more than necessary; in fact a millivoltmeter was found to be sufficiently sensitive for most purposes. The constants of damping are very important in determining the rapidity with which observations can be made, and it was found that equally sensitive galvanometers varied within wide limits in this respect. By employing an alternating current and a telephone receiver instead of a galvanometer it was found that the same calibration curve was obtained as in the case of direct current measurements.

It was found possible to test for any suspected change in the constants of the anemometer-wire by measuring the current required to bring the wire to the standard temperature in a stagnant atmosphere, care being taken to protect the instrument from draughts.

In analysing a complicated distribution of air-flow, the direction of the current of air may be determined from the effect discussed in Section 14; if the anemometer is rotated about an axis perpendicular to the wire, it will be at right-angles to the stream when the current required to bring it to the standard

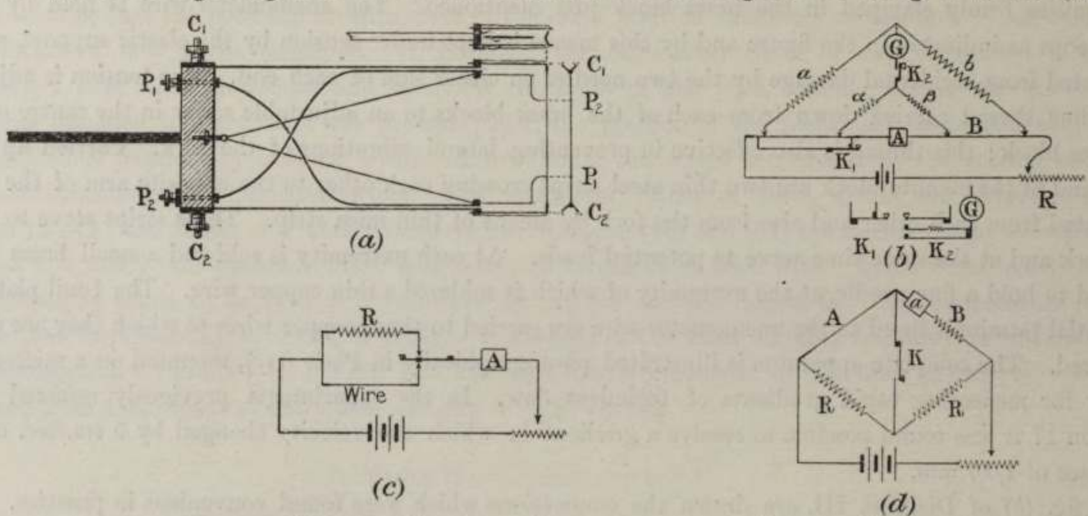
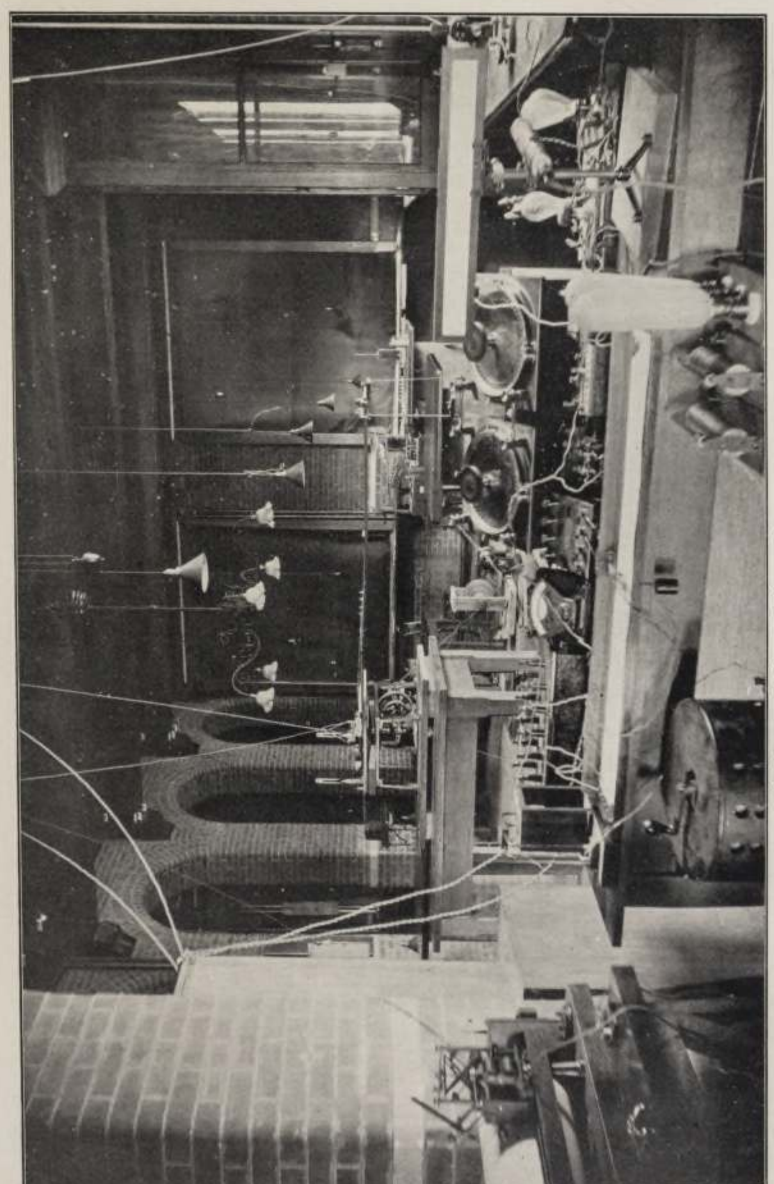


Diagram III. Details of hot-wire anemometers and connections.

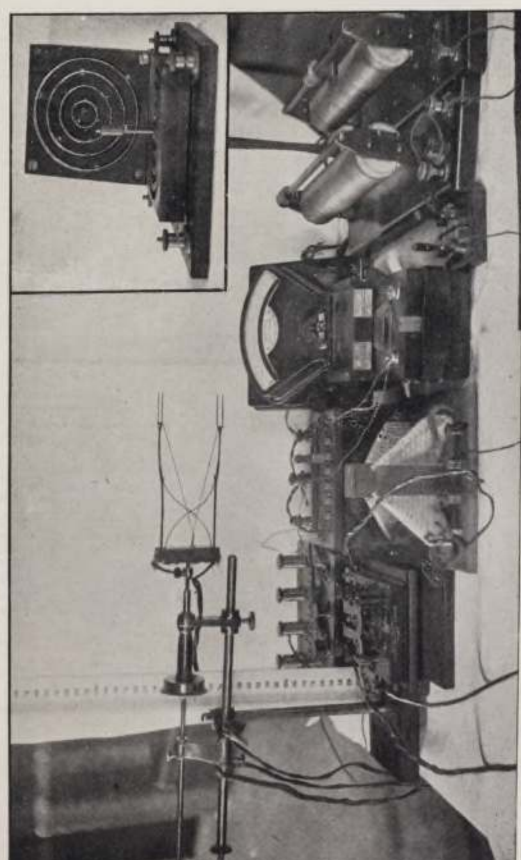
temperature is a maximum. This effect is sufficiently marked to enable the direction of a stream to be fixed with a fair degree of accuracy.

In figs. (*c*) and (*d*) of Diagram III. are illustrated simple methods of making the connections to an anemometer-wire so as to dispense with a galvanometer. In fig. (*c*) a low resistance storage-cell is connected by means of a three-way key in such a way that the anemometer-wire and a manganin resistance of four times the resistance may be successively included in the circuit; if the current be adjusted so that the ammeter reading is unchanged on successively including these two resistances, they must have the same resistance. Fig. (*d*) illustrates an adaptation of KELVIN's method for measuring a galvanometer resistance. The ammeter *a* is in one of the arms in series with a manganin resistance so that the total resistance is four times that of the anemometer-wire in the adjacent arm. The ratio resistances R are adjusted to equality and have approximately the same value as that of the remaining arms. A contact key K replaces the galvanometer. If the current in the bridge be adjusted so that on depressing the key there is no change in the ammeter reading, it is easily seen that the condition of balance of the Wheatstone-bridge is satisfied and the resistances A and B are equal.

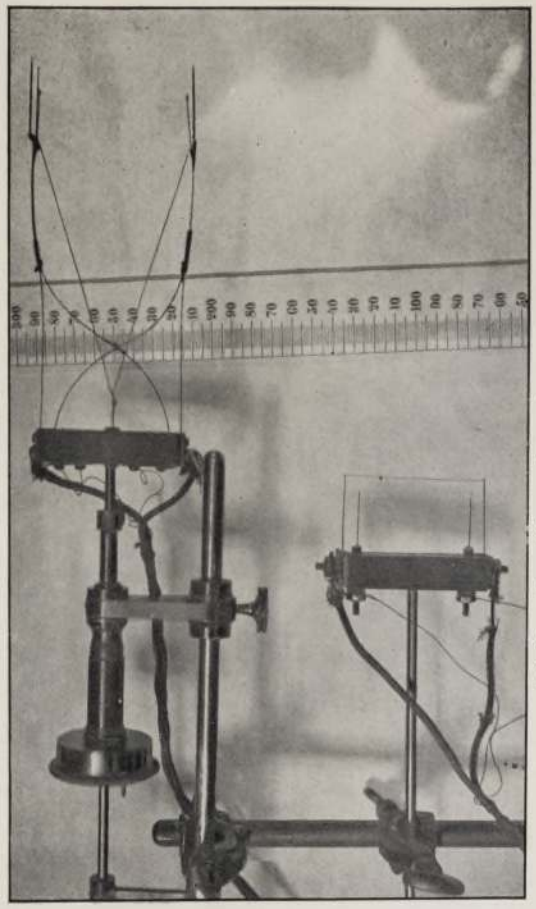
The two methods just described suffer the disadvantage that the contact resistances of the anemometer-wire are included in the resistance of the wire itself, and is likely to be uncertain if the wire is to be employed at high temperatures. The potential terminals and the Kelvin-bridge connections are recommended for use as the most satisfactory and should be employed if possible.



(a) General disposition of apparatus.



(b) Portable hot-wire anemometer. Details of mercury slip-rings.



(c) Details of linear hot-wire anemometers.