On the cyclicality of the interest rate in emerging economy models: solution methods matter[*]

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Abstract

We study the workhorse sovereign default model that has been used to explain the cyclical behavior of interest rates in emerging market economies. We show that the use of the discrete state space technique is likely to introduce spurious interest rate movements, and that these spurious movements can be eliminated using continuous methods. We find that the interest rate is less countercyclical when shocks to the growth rate of income predominate. This contrast with the results obtained by Aguiar and Gopinath (2006) using the discrete state space technique. For example, Aguiar and Gopinath (2006) report a correlation between the interest rate spread and income of 0.5 when shocks to the income level predominate. Using continuous methods, we find that this correlation is around -0.6. Moreover, the interest rate volatility is about one fourth of the one reported by Aguiar and Gopinath (2006). The discrete state space technique is unstable in settings of this sort because the model features a high sensitivity of the interest rate to the borrowing level.

JEL classification: F34, F41.

Keywords: Emerging economies, sovereign debt, default, discrete state space, interpolation.

1   Introduction

Business cycles in small emerging economies differ from those in developed economies. Emerging economies feature higher, more volatile and countercyclical interest rates, higher output volatility, higher volatility of consumption relative to income, and more countercyclical net exports (see, for example, Aguiar and Gopinath (2007), Neumeyer and Perri (2005), and Uribe and Yue (2006)). In order to account for these features, a state-dependent interest rate schedule is commonly used in emerging economy models. Some studies assume an exogenous interest rate schedule (see, for example, Aguiar and Gopinath (2007), Neumeyer and Perri (2005), Schmitt-Grohé and Uribe (2003), and Uribe and Yue (2006)). In contrast, recent quantitative models of sovereign default provide microfoundations for the interest rate schedule based on the risk of default.[1] Many articles in the growing literature on sovereign default rely on the discrete state space technique— constraining the economy to choose borrowing levels from a finite and invariant set—partly motivated by its simplicity and its widespread use in other areas in macroeconomics. We show that the use of the discrete state space technique (hereafter referred by DSS) is likely to introduce spurious interest rate movements that may distort the results, and that these spurious movements can be eliminated using continuous methods.

Figures 1 and 2 present an example that illustrates the origin of the distortions implied by DSS in models of sovereign default. The solid lines represent the actual equilibrium functions and the dots represent an approximate solution obtained using DSS. Figure 1 shows the optimal saving level as a function of income y. In the example, the economy borrows more in "good times", that is, when income is high.[2] Figure 2 shows the interest rate paid in equilibrium for different income levels when the borrowing level for each income is the optimal borrowing level

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[1] See, for example, Aguiar and Gopinath (2006), Arellano (2008), Arellano and Ramanarayanan (2006), Bai and Zhang (2006), Benjamin and Wright (2008), Cuadra and Sapriza (2006, 2008), D'Erasmo (2008), Eyigungor (2006), Hatchondo and Martinez (2008), Hatchondo et al. (2007, 2008), Lizarazo (2005, 2006), Mendoza and Yue (2007), and Yue (2005). The models in these studies extend the framework proposed by Eaton and Gersovitz (1981) and share blueprints with the models used in quantitative studies of household bankruptcy—see, for example, Athreya (2002), Athreya et al. (2007a,b), Chatterjee et al. (2007a), Chatterjee et al. (2007b), Li and Sarte (2006), Livshits et al. (2007), and Sánchez (2008). Tomz and Wright (2007) document 250 sovereign defaults by 106 countries between 1820 and 2004. Some of the latest episodes are Russia in 1998, Ecuador in 1999, and Argentina in 2001.

[2] This is in line with the findings in Neumeyer and Perri (2005) and Uribe and Yue (2006), who document that borrowing in emerging economies is procyclical.
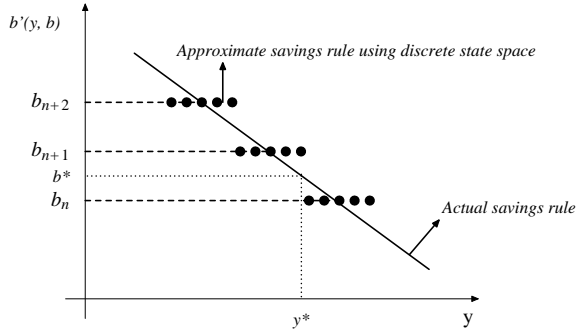
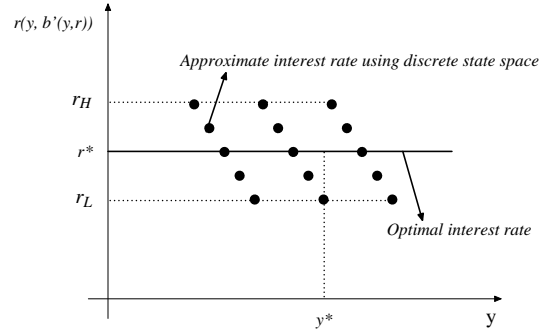Figure 1: Optimal savings and approximation using DSS.

Figure 2: Interest rate paid in equilibrium and approximation obtained using the DSS savings levels.

plotted in Figure 1. The example assumes that the interest rate schedule faced by the economy is increasing in the borrowing level and decreasing in income.[3] For expositional simplicity, our example also assumes that the extra borrowing that follows an increase in income is such that the interest rate paid in equilibrium does not depend on income (in Figure 2, it is always equal to r^*). Figure 2 illustrates how the DSS distortions to the optimal savings plotted in Figure 1 introduce distortions in the interest rate paid in equilibrium obtained using the DSS savings levels.

In the example, DSS leads to an overestimation of the interest rate volatility. While the true solution implies a constant interest rate, Figure 2 shows that the interest rate computed using the DSS borrowing levels displays volatility. With DSS, borrowing is not always allowed to adjust to changes in income. Recall that in our example (as in models of sovereign default), the increase in borrowing implied by an increase in income moderates the decrease in the interest rate implied by an increase in income. Consequently, when the borrowing level is not allowed to change, interest rates movements are exacerbated. In contrast, when one solves models of sovereign default using a continuous method, borrowing is allowed to adjust to income. We will show that this eliminates the spurious interest rate movements generated by DSS. Thus, the behavior of the interest rate computed using continuous methods is more accurate than the one computed using DSS.

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[3]In models of sovereign default, the interest rate is increasing with respect to the amount borrowed and decreasing with respect to income because the default probability is increasing with respect to the debt level and decreasing with respect to income.

Furthermore, the correlation between income and the interest rate paid in equilibrium may be overestimated or underestimated when it is computed using the interest rates implied by the DSS borrowing levels. First, consider income variations such that the corresponding optimal savings obtained using DSS lie on one step of the function plotted in Figure 1. Figure 2 shows that for these income levels, the interest rate computed using the DSS borrowing levels is negatively correlated with income—recall that the interest rate is increasing in the borrowing level and DSS does not allow borrowing to increase with income for these income levels. Second, consider income levels in a neighborhood of a value for which there is a change in the optimal saving level obtained with DSS—for instance, y^* in Figure 1. Figure 2 shows that for these income levels the interest rate implied by DSS borrowing levels is positively correlated with income. There is no reason to expect that these two forces should cancel each other out. The correlation between income and the interest rate computed using the DSS solution may be positive or negative depending on the probability distribution of income realizations.

The previous paragraphs explain the potential for DSS approximation errors to influence the results of the model simulations. While this is clearly a theoretical possibility, it remains to be established whether such inaccuracies are significant enough to misguide the conclusions of the research agenda. We demonstrate that this is the case by using Chebychev collocation and cubic spline interpolation to solve the baseline models presented by Aguiar and Gopinath (2006), an influential paper in the sovereign default literature.[4] When we repeat the exercises conducted by Aguiar and Gopinath (2006), we find that the behavior of the interest rate in our simulations stands in sharp contrast with the behavior they obtain using DSS.

We show that DSS, Chebychev collocation and spline interpolation produce similar approximations of optimal saving and default decisions. However, the high sensitivity of the equilibrium interest rate to the borrowing level in models of sovereign default implies that relatively small imprecisions in the approximation of optimal borrowing levels obtained using DSS introduce spurious movements in the interest rate that contaminate results and may direct us to misleading conclusions.

We find that in the model where the income process is such that shocks to the income level

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[4]See Judd (1998) for a description of the methods.

predominate ("Model I"), the standard deviation of the interest rate spread (margin of extra yield over the risk-free rate) is 0.01 (in percentage terms). In the model where the income process is such that shocks to the growth rate predominate ("Model II"), the spread volatility is 0.07. Aguiar and Gopinath (2006) report that the spread volatility is 0.04 in Model I and 0.32 in Model II. One of the main challenges of sovereign default models is to be able to replicate the high interest rate volatility observed in the data. Our results indicate that the discrepancy between the interest rate volatility generated by the models and the one observed in the data is larger than previously thought. Around 75% of the volatility reported by Aguiar and Gopinath (2006) is due to spurious movements in the spread implied by DSS.

In addition, we find that the correlation between the spread and income is around -0.60 in Model I and 0.08 in Model II. Aguiar and Gopinath (2006) report that this correlation is 0.51 in Model I and -0.03 in Model II. Thus, our results also cast doubt on the conclusion presented by Aguiar and Gopinath (2006) about income processes with shocks to the growth rate helping models of sovereign default generate a countercyclical interest rate and, therefore, helping these models generate the positive correlation between the interest rate and the current account observed in the data. Our findings imply that the ability of the model to fit the data does not necessarily improve when one assumes an income process with shocks to the growth rate instead of the standard process with shocks to the level. It should be mentioned that Aguiar and Gopinath (2006) also show that assuming shocks to the growth rate also allows the model to generate higher default rates and a more volatile interest rate spread. We find that this result is not sensitive to the numerical method. However, in a related article (see Hatchondo et al. (2007)) we show that these findings are not robust to the assumption that defaulting countries are exogenously excluded from capital markets: without that assumption, default rates are slightly higher and the interest rate spread is more volatile in the model with shocks to the income level. Overall, our findings cast doubt on the comparative advantages of using the model with shocks to the growth rate of income as a benchmark.

Potentially, the spurious spread movements introduced by DSS could be reduced using finer grids. We have increased the number of grid points using the same code used by Aguiar and Gopinath (2006), but the code hits memory restrictions in Matlab before being able to signifi-

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cantly reduce the spurious spread movements introduced by DSS. In addition, we show that the results generated by DSS are not robust to changes in the width and the density of the grids. In contrast, we find that results obtained using continuous methods appear to be robust. Our results with Chebychev collocation are similar to our results with spline interpolation. Moreover, our results with Chebychev collocation are robust to using more polynomials, and our results with spline interpolation are robust to using more grid points. This indicates that continuous methods may be a more reliable technique to study the behavior of the interest rate spread.

Our findings (that obtaining reliable results using DSS may be difficult when solving models of sovereign default) are also relevant for other versions of the baseline model of sovereign default used in recent quantitative studies. For example, in our experience with different extensions of the baseline model (see Hatchondo and Martinez (2008), and Hatchondo et al. (2007, 2008)) we found that it is difficult to eliminate the significant distortions that DSS introduces in the behavior of the interest rate spread.

The article is organized as follows. Section 2 presents the model. Section 3 shows the results. Section 4 concludes.

2   The Model

We solve the model presented by Aguiar and Gopinath (2006), which is based on the work of Eaton and Gersovitz (1981). This is a stylized version of the workhorse model of sovereign default that has been used in recent quantitative studies.

Consider a small open economy that receives a stochastic endowment stream of a single tradable good,

$$y_t = e^{z_t} \Gamma_t,$$

where the transitory component

$$z_t = (1 - \rho_z) \mu_z + \rho_z z_{t-1} + \varepsilon_t^z$$

follows an AR(1) process with long run mean μ_z, $|\rho_z| < 1$, and $\varepsilon_t^z \sim N\left(0, \sigma_z^2\right)$; and

$$\Gamma_t = g_t \Gamma_{t-1},$$

where

$$ln\left(g_t\right) = \left(1 - \rho_g\right)\left(ln\left(\mu_g\right) - m\right) + \rho_g ln\left(g_{t-1}\right) + \varepsilon_t,$$

$|\rho_g| < 1$, $\varepsilon_t \sim N\left(0, \sigma_g^2\right)$, and $m = \frac{1}{2}\frac{\sigma_g^2}{1-\rho_g^2}$.

The government's objective is to maximize the expected present discounted value of the future utility of the representative agent. The representative agent has CRRA preferences over consumption:

$$u\left(c\right) = \frac{c^{(1-\sigma)} - 1}{1 - \sigma},$$

where σ denotes the coefficient of relative risk aversion. Let β denote the discount factor. To ensure a well defined problem it is assumed that $E\left\{\lim_{t\to\infty}\beta^t\left(y_t\right)^{(1-\sigma)}\right\} = 0$.

The government makes two decisions. First, it decides whether to refuse to pay previously issued debt. Defaults imply a total repudiation of government debt (Yue (2005) studies partial defaults). Second, the government decides how much to borrow or save for the following period.

It is assumed that there are two costs of defaulting. First, the country is excluded from capital markets (Hatchondo et al. (2007) study the effects of eliminating the exclusion punishment from this framework). In each period after the default period, the country regains access to capital markets with probability $\phi \in [0, 1]$. Second, it is assumed that if a country has defaulted on its debt, it faces an "output loss" of λ percent in every period in which it is excluded from capital markets (Arellano (2008) allows λ to depend on the income level).

The government can choose to save or borrow using one-period bonds (Hatchondo and Martinez (2008) study long-duration bonds). These assets are priced in a competitive market. There is a large number of identical, infinitely lived foreign lenders. Each lender can borrow or lend at the risk-free rate r and can lend in a perfectly competitive market to the small open economy. Lenders are risk neutral (Lizarazo (2005) assumes risk-averse lenders). Creditors have perfect information regarding the economy's endowment.

Let b denote the current position in bonds. A negative value of b denotes that the country was an issuer of bonds in the previous period.

The bond price is determined as follows. First, the government announces how many bonds it wants to issue—each bond is a promise to deliver one unit of the good next period. Then,

lenders offer a price for these bonds. Finally, the government sells the bonds to the lenders who offered the highest price. Thus, in equilibrium lenders offer a price

$$q\left(b', z, \Gamma, g\right) = \frac{1}{1+r}\left[1 - \int \int d\left(b', z', \Gamma, g'\right) F_Z\left(dz' \mid z\right) F_G\left(dg' \mid g\right)\right] \qquad (1)$$

that satisfies their zero profit condition when the government issues b' bonds, and the optimal default rule is represented by the indicator function $d\left(b, z, \Gamma, g\right)$. The default rule takes a value of 1 it is optimal for the government to default, and takes a value of 0 otherwise.

Let F_Z and F_G denote the cumulative distribution functions for z and g. The value function for an economy that participates in financial markets is given by

$$V\left(b, z, \Gamma, g\right) = \max_{d \in \{0,1\}}\left\{\left(1 - d\right) V_0\left(b, z, \Gamma, g\right) + dV_1\left(z, \Gamma, g\right)\right\}, \qquad (2)$$

where

$$V_1\left(z, \Gamma, g\right) = u\left(y\left(1 - \lambda\right)\right) + \beta \int \int \left[\phi V\left(0, z', g'\Gamma, g'\right) + \left(1 - \phi\right) V_1(z', g'\Gamma, g')\right] F_Z\left(dz' \mid z\right) F_G\left(dg' \mid g\right) \qquad (3)$$

denotes the value function of an excluded economy, and

$$V_0\left(b, z, \Gamma, g\right) = \max_{b'}\left\{u\left(y + b - q\left(b', z, \Gamma, g\right) b'\right) + \beta \int \int V(b', z', g'\Gamma, g') F_Z\left(dz' \mid z\right) F_G\left(dg' \mid g\right)\right\} \qquad (4)$$

denotes the Bellman equation when the country has decided to pay back its debt.

Definition 1 *A recursive equilibrium consists of the following elements:*

1. *A set of value functions $V\left(b, z, \Gamma, g\right)$, $V_1\left(z, \Gamma, g\right)$, and $V_0\left(b, z, \Gamma, g\right)$.*

2. *A set of policies for asset holdings $b'\left(b, z, \Gamma, g\right)$ and default decisions $d\left(b, z, \Gamma, g\right)$.*

3. *A bond price function $q\left(b', z, \Gamma, g\right)$.*

   Such that:

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(a) $V(b, z, \Gamma, g)$, $V_1(z, \Gamma, g)$, and $V_0(b, z, \Gamma, g)$ satisfy the functional equations (2), (3), and (4), respectively;

(b) the default policy $d(b, z, \Gamma, g)$ solves problem (2), and the policy for asset holdings $b'(b, z, \Gamma, g)$ solves problem (4);

(c) the bond price function $q(b', z, \Gamma, g)$ is given by equation (1).

3 Results

We first solve the model using Chebychev collocation and spline interpolation and compare the implied behavior of the interest rate over the business cycles with the results obtained by Aguiar and Gopinath (2006) using DSS. Later, we discuss how the results with continuous methods compare to the results obtained using DSS and finer grids.

3.1 Parameterization

As in Aguiar and Gopinath (2006), this article solves two special cases of the model presented above. Model I corresponds to the case when only the transitory component of the endowment is stochastic (and g_t is constant). Model II corresponds to the case when only the growth rate g_t is stochastic (and z_t is constant). We use the parameter values assumed by Aguiar and Gopinath (2006). Each period refers to a quarter. Parameter values that are the same for Model I and Model II are presented in Table 1. Parameter values that are different for the two models are specified in Table 2. In order to find the solutions, the Bellman equations are recast in de-trended form, normalizing all variables by $\mu_g y_{t-1}$.

3.2 Computation

We solve the models numerically using value function iteration and continuous methods, and compare our results with the ones computed by Aguiar and Gopinath (2006) using DSS. The models are solved using Chebychev collocation and cubic spline interpolation. The algorithm finds the value functions V_0 and V_1. While the function V presents a kink because of the default

Risk aversion	σ	2
Interest rate	r	1%
Loss of output	λ	2%
Probability of redemption	ϕ	10%
Mean growth rate	μ_g	1.006
Mean (log) transitory productivity	μ_z	$(-1/2)\sigma_z^2$
Discount factor	β	0.8

Table 1: Parameter values common to both models.

	Model I	Model II
σ_z	3.4%	0
ρ_z	0.9	NA
σ_g	0	3%
ρ_g	NA	0.17

Table 2: Model specific parameter values.

choice, we find that V_0 and V_1 are well-behaved and smooth functions. Fifteen polynomials on the asset space and ten on the endowment shock are used when the model is solved using Chebychev collocation. Fifty grid points on the asset and endowment space are used when the model is solved using cubic spline interpolation. Our results are robust to using more Chebychev polynomials or more grid points when using spline interpolation. We find that the three methods (DSS, Chebychev collocation, and cubic spline interpolation) produce similar approximations of optimal saving and default decisions. However, the high sensitivity of the equilibrium interest rate to the borrowing level implies that relatively small imprecisions in the approximation of optimal borrowing levels obtained using DSS imply significant spurious movements in the equilibrium interest rate.

3.3   The imprecisions implied by DSS

Table 3 reports the business cycles statistics obtained when the model is solved using continuous methods and DSS. In order to facilitate comparisons, the statistics are computed as in Aguiar and Gopinath (2006). The model is simulated for 5,000,000 periods: 500 samples of 10,000 periods each. Statistics are computed using the last 500 periods of each sample. The logarithm of income and consumption are denoted by y and c respectively. The trade balance (output minus consumption, TB) is expressed as a fraction of income (Y), and the interest rate spread (margin of extra yield over U.S. Treasuries, R_s) is expressed in annual terms. All series are HP filtered with a smoothing parameter of 1600. Standard deviations are denoted by σ and are reported in percentage terms; correlations are denoted by ρ. The table also presents statistics computed by Aguiar and Gopinath (2006) using Argentine data for the 1983-2000 period.

Table 3 illustrates the quantitative importance of the numerical errors introduced by DSS. The table shows that when the equilibrium policy functions obtained using continuous methods are used in the simulations, the spread is countercyclical in the model with shocks to the income level and procyclical in the model with shocks to the growth rate. This contradicts the "conventional wisdom" in the literature.[5]

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[5]It is more appropriate to compare the data with the statistics computed using continuous methods than with the ones computed using DSS. Most likely, the movements of the spread in the data are not affected by a

| | Model I | | | Model II | | | Argentina |
	DSS	Cheb coll.	Spline	DSS	Cheb coll.	Spline	Data
$\sigma(y)$	4.32	4.34	4.35	4.45	4.43	4.44	4.08
$\sigma(c)$	4.37	4.47	4.48	4.71	4.67	4.69	4.85
$\sigma(TB/Y)$	0.17	0.49	0.49	0.95	0.92	0.95	1.36
$\sigma(R_s)$	0.04	0.01	0.01	0.32	0.07	0.07	3.17
$\rho(c,y)$	0.99	0.99	0.99	0.98	0.98	0.98	0.96
$\rho(TB/Y,y)$	-0.33	-0.30	-0.31	-0.19	-0.19	-0.18	-0.89
$\rho(R_s,y)$	0.51	-0.61	-0.59	-0.03	0.08	0.09	-0.59
$\rho(R_s,TB/Y)$	-0.21	0.69	0.70	0.11	0.53	0.52	0.68

Table 3: Simulation results with DSS and continuous methods.

The table also shows that DSS leads to an overestimation of the spread volatility. One of the main challenges of these models is to be able to replicate the spread volatility observed in the data. Table 3 indicates that the discrepancy between the spread volatility generated by the models and the spread volatility observed in the data is larger once the spurious volatility introduced by DSS is eliminated.

Figures 3-6 compare the optimal savings and the equilibrium bond prices obtained using DSS with the ones obtained using continuous methods—we present the equilibrium functions derived for Model II but the same rationale applies to Model I. The DSS figures where constructed with the code used by Aguiar and Gopinath (2006), which was made available by the authors. The figures show that for low enough growth rates, the country decides to default and is excluded from capital markets, i.e., it borrows zero. Following Aguiar and Gopinath (2006), Figures 5 and 6 impute the price of the risk free bond when the country defaults and is excluded from capital markets. Figures 3-6 show that the source of the imprecisions implied by DSS is the same that is illustrated in Figures 1 and 2 in the introduction: with DSS, borrowing is not always allowed to adjust to changes in income (see Figure 3) and this generates spurious movements in

restriction that limits governments to choose issuance volumes from a finite set of values (as in the DSS solution).
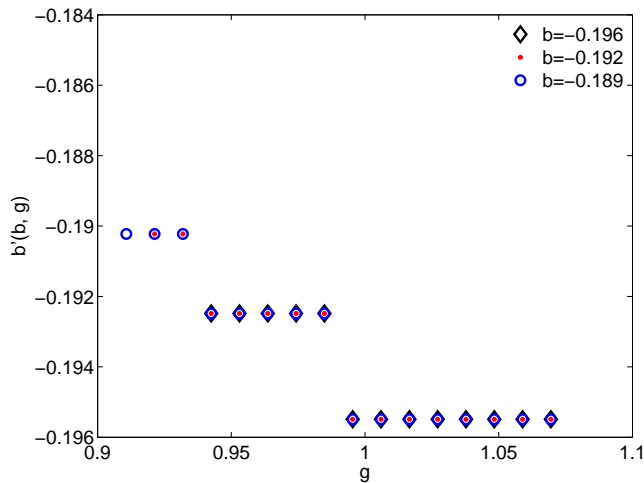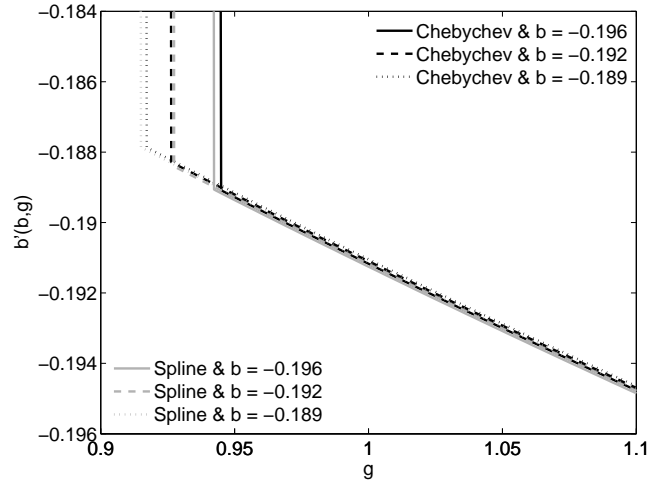
Figure 3: Optimal savings computed using DSS.

Figure 4: Optimal savings computed using continuous methods.

spread (see Figure 5). Recall that in the model, the increase in borrowing implied by an increase in income moderates the decrease in the interest rate implied by an increase in income. Thus, when borrowing is not allowed to change, interest rates movements are exacerbated. In contrast, when we solve the model using continuous methods, we always allow borrowing to adjust to income (see Figure 4).[6] This indicates to us that the behavior of the interest rate computed using continuous methods is more accurate than the one computed using DSS.

3.4   Results with different DSS grids

This section discusses how the solution obtained using DSS improves when we consider more dense and wider grids. The results presented below were obtained using the same code that was used by Aguiar and Gopinath (2006). We find that memory restrictions in Matlab limit the extent to which we can improve results using DSS.

The original specification in Aguiar and Gopinath (2006) uses 25 points in the grid for endowment shocks, 400 points in the grid for assets, a width of the grid for endowment shocks in

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[6]First, we find a candidate value for the optimal borrowing level using a global search procedure. Then, that candidate value is used as an initial guess in the optimization routine DUVMIF from the IMSL Fortran library. The routine uses a quasi-Newton method to find the maximum value of a function.
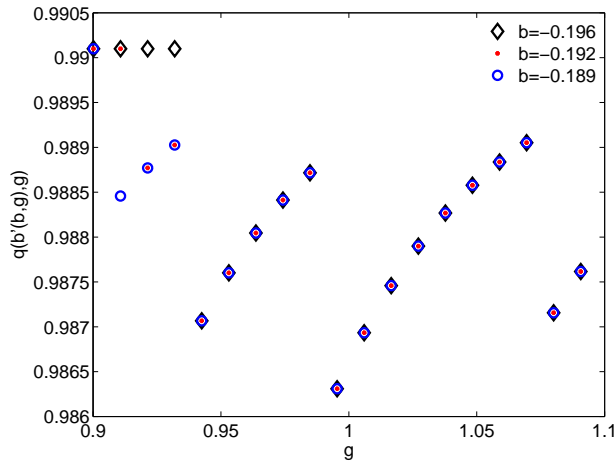
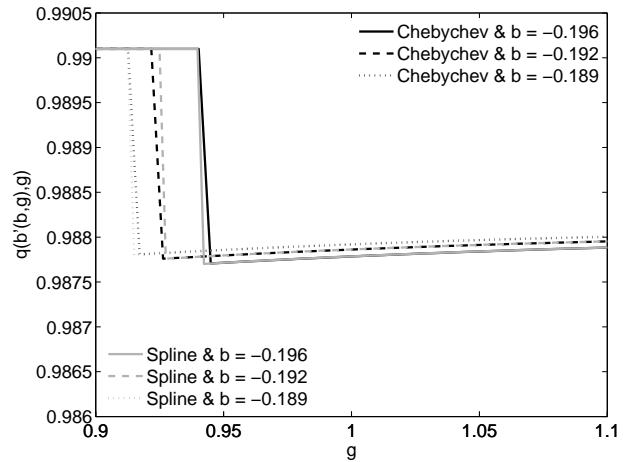Figure 5: Bond price paid in equilibrium computed using DSS.

Figure 6: Bond price paid in equilibrium computed using continuous methods.

Model I equal to $5\sigma_z$, and a width of the grid for endowment shocks in Model II equal to $8.3\sigma_g$ (the grids for endowment shocks are centered around the unconditional mean). We consider grids with more points and also wider grids for the endowment shocks. We increase the number of grid points as much as we can given the technology available to us. We use -0.35 and 0 as the minimum and the maximum values in the grid for assets.

Tables 4 and 5 report business cycle statistics for Model I and Model II computed using DSS and different grid specifications. The tables show that we are not able to reproduce the results obtained using continuous methods. Repeating the exercise presented in Figures 3-6 for the other grid specifications, one can see that the spurious volatility than contaminates the results reported by Aguiar and Gopinath (2006) cannot be easily eliminated when using DSS. In Model I, this is particularly true when a wider grid for the endowment shock is used (note that the volatility of trade balances obtained with continuous methods is only reproduced when a wider grid is used). Overall, DSS results do not appear to be robust to changes in the specification of the grid.

In contrast, we find that results obtained using continuous methods appear to be robust. As illustrated in Table 3, our Chebychev collocation results are similar to our spline interpolation

	(400,25,5)	(400,75,5)	(600,25,5)	(400,25,12)	(400,75,12)	(600,25,12)
$\sigma(y)$	4.32	4.27	4.30	4.54	4.33	4.53
$\sigma(c)$	4.37	4.31	4.34	4.67	4.45	4.66
$\sigma(TB/Y)$	0.17	0.17	0.16	0.52	0.44	0.52
$\sigma(R_s)$	0.04	0.07	0.06	0.26	0.11	0.26
$\rho(c,y)$	0.99	0.99	0.99	0.99	0.99	0.99
$\rho(TB/Y,y)$	-0.33	-0.35	-0.35	-0.29	-0.31	-0.29
$\rho(R_s,y)$	0.51	0.30	0.52	-0.24	0.014	-0.23
$\rho(R_s,TB/Y)$	-0.21	-0.22	-0.35	-0.03	-0.08	-0.03

Table 4: Simulation results for Model I computed using DSS with different grid specifications. Columns are parameterized by a vector. The first component indicates the number of grid points in the asset space, the second component indicates the number of grid points for the endowment shock and the third component indicates the distance between the maximum and the minimum values of the grid for endowment shocks expressed in terms of the standard deviation of the endowment shocks.

results. Furthermore, our Chebychev collocation results are robust to using more polynomials and our spline interpolation results are robust to using more grid points. This indicates to us that continuous methods may be more accurate for computing the behavior of the interest rate.

4   Conclusions

We show that approximation errors implied by DSS influence the results presented by Aguiar and Gopinath (2006), an influential paper in the sovereign default literature. When we solve their models using continuous methods, we find that 75% of the interest rate volatility obtained when solving the models using DSS results from approximation errors. This implies that the discrepancy between the interest volatility generated by the models and the one observed in the data is larger than previously thought. Furthermore, we show that the imprecisions implied by DSS are what lead to conclude that income processes with shocks to the growth rate help models of sovereign default generate a countercyclical interest rate and thus, it helps these models generate the positive correlation between the interest rate and the current account observed in the data.

In the growing literature on sovereign default, models are usually solved using DSS. Even

	(400,25,5)	(400,75,8.3)	(600,25,8.3)	(400,25,12)	(400,75,12)	(600,25,12)
$\sigma(y)$	4.45	4.36	4.39	4.42	4.38	4.42
$\sigma(c)$	4.71	4.61	4.64	4.67	4.62	4.67
$\sigma(TB/Y)$	0.95	0.8	0.94	0.91	0.89	0.91
$\sigma(R_s)$	0.32	0.18	0.32	0.38	0.27	0.38
$\rho(c,y)$	0.98	0.98	0.98	0.98	0.98	0.98
$\rho(TB/Y,y)$	-0.19	-0.21	-0.19	-0.21	-0.20	-0.20
$\rho(R_s,y)$	-0.03	-0.12	0.07	-0.13	-0.04	-0.13
$\rho(R_s,TB/Y)$	0.11	0.46	0.04	0.34	0.14	0.34
Mean debt output ratio (%)	19	19	19	19	19	19
Rate of default	23	21	23	21	21	21

Table 5: Simulation results for Model II computed using DSS with different grid specifications. Columns are parameterized by a vector. The first component indicates the number of grid points in the asset space, the second component indicates the number of grid points for the endowment shock and the third component indicates the distance between the maximum and the minimum values of the grid for endowment shocks expressed in terms of the standard deviation of the endowment shocks.

though we illustrate the message we want to convey solving the models presented by Aguiar and Gopinath (2006), our findings are also relevant for other models of sovereign default. For example, we find that it is difficult to eliminate the significant distortions that DSS introduces in the behavior of the interest rate spread for the models presented in Hatchondo and Martinez (2008) and Hatchondo et al. (2007, 2008).

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